Designation: D 2915 – 03

Standard Practice for Evaluating Allowable Properties for Grades of Structural Lumber[1]

This standard is issued under the fixed designation D 2915; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

INTRODUCTION

The mechanical properties of structural lumber depend upon natural growth characteristics and manufacturing practices. Several procedures can be used to sort lumber into property classes or stress grades, the most widely used being the visual methods outlined in Practice D 245. With each, a modulus of elasticity and a set of from one to five allowable stresses may be associated with each stress grade. The allowable stresses are extreme fiber stress in bending, tension parallel to the grain, compression parallel to the grain, shear, and compression perpendicular to the grain. This test method for evaluation of the properties of structural lumber defines an allowable property as the value of the property that would normally be published with the grade description.

This practice is useful in assessing the appropriateness of the assigned properties and for checking the effectiveness of grading procedures.

For situations where a manufactured product is sampled repeatedly or lot sizes are small, alternative test methods as described in Ref **(1)**[2] may be more applicable.

1. Scope

1.1 This practice covers sampling and analysis procedures for the investigation of specified populations of stress-graded structural lumber. Depending on the interest of the user, the population from which samples are taken may range from the lumber from a specific mill to all the lumber produced in a particular grade from a particular geographic area, during some specified interval of time. This practice generally assumes that the population is sufficiently large so that, for sampling purposes, it may be considered infinite. Where this assumption is inadequate, that is, the population is assumed finite, many of the provisions of this practice may be employed but the sampling and analysis procedure must be designed to reflect a finite population. The statistical techniques embodied in this practice provide procedures to summarize data so that logical judgments can be made. This practice does not specify the action to be taken after the results have been analyzed. The action to be taken depends on the particular requirements of the user of the product.

1.2 The values stated in inch-pound units are to be regarded as the standard.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 198 Test Methods of Static Tests of Timber in Structural Sizes[3]

D 245 Practice for Establishing Structural Grades and Related Allowable Properties for Visually Graded Lumber[3]

D 1990 Practice for Establishing Allowable Properties for Visually-Graded Dimension Lumber from In-Grade Tests of Full-Size Specimens[3]

E 105 Practice for Probability Sampling of Materials[4]

3. Statistical Methodology

3.1 Two general analysis procedures are described under this practice, parametric and nonparametric. The parametric approach assumes a known distribution of the underlying population, an assumption which, if incorrect, may lead to

[1] This practice is under the jurisdiction of ASTM Committee D07 on Wood and is the direct responsibility of Subcommittee D07.02 on Lumber and Engineered Wood Products.

Current edition approved April 10, 2003. Published June 2003. Originally approved in 1970 as D 2915 – 70 T. Last previous edition approved in 2002 as D 2915 – 02.

[2] The boldface numbers in parentheses refer to the list of references at the end of this practice.

[3] *Annual Book of ASTM Standards*, Vol 04.10.

[4] *Annual Book of ASTM Standards*, Vol 14.02.

inaccurate results. Therefore, if a parametric approach is used, appropriate statistical tests shall be employed to substantiate this choice along with measures of test adequacy **(2, 3, 4, 5, 6, 7).** Alternatively a nonparametric approach requires fewer assumptions, and is generally more conservative than a parametric procedure.

3.2 *Population*:

3.2.1 It is imperative that the population to be evaluated be clearly defined, as inferences made pertain only to that population. In order to define the population, it may be necessary to specify (*1*) grade name and description, (*2*) geographical area over which sampling will take place (nation, state, mill, etc.), (*3*) species or species group, (*4*) time span for sampling (a day's production, a month, a year, etc.), (*5*) lumber size, and (*6*) moisture content.

3.2.2 Where possible, the sampling program should consider the location and type of log source from which the pieces originated, including types of processing methods or marketing practices with respect to any influence they may have on the representative nature of the sample. Samples may be collected from stock at mills, centers of distribution, at points of end use or directly from current production at the grading chains of manufacturing facilities.

3.3 *Sampling Procedure*:

3.3.1 *Random Sampling*— The sampling unit is commonly the individual piece of lumber. When this is not the case, see 3.3.3. The sampling shall assure random selection of sampling units from the population described in 3.2 with all members of the population sharing equal probability of selection. The principles of Practice E 105 shall be maintained. When sampling current production, refer to Practice E 105 for a recommended sampling procedure (see Appendix X3 of this practice for an example of this procedure). If samples are selected from inventory, random number tables may be used to determine which pieces will be taken for the sample.

3.3.2 *Sampling with Unequal Probabilities*—Under some circumstances, it may be advisable to sample with unequal but known probabilities. Where this is done, the general principles of Practice E 105 shall be maintained, and the sampling method shall be completely reported.

3.3.3 *Sequential Sampling*—When trying to characterize how a certain population of lumber may perform in a structure, it may be deemed more appropriate to choose a sampling unit, such as a package, that is more representative of how the lumber will be selected for use. Such a composite sampling unit might consist of a sequential series of pieces chosen to permit estimation of the properties of the unit as well as the pieces. Where this is done, the principles in 3.3.1 and 3.3.2 apply to these composite sampling units and the sampling method shall be completely reported.

3.4 *Sample Size*:

3.4.1 Selection of a sample size depends upon the property or properties to be estimated, the actual variation in properties occurring in the population, and the precision with which the property is to be estimated. For the five allowable stresses and the modulus of elasticity various percentiles of the population may be estimated. For all properties, nonparametric or parametric techniques are applicable. Commonly the mean modulus of elasticity and the mean compression perpendicular to the grain stress for the grade are estimated. For the four other allowable stresses, a near-minimum property is generally the objective.

3.4.2 Determine sample size sufficient for estimating the mean by a two-stage method, with the use of the following equation. This equation assumes the data is normally distributed and the mean is to be estimated to within 5 % with specified confidence:

$$n = (ts/0.05\,\bar{X})^2 = \left(\frac{t}{0.05}CV\right)^2 \tag{1}$$

where:
n = sample size,
s = standard deviation of specimen values,
$\bar{X}$ = specimen mean value,
CV = coefficient of variation, $s/\bar{X}$,
0.05 = precision of estimate, and
t = value of the t statistic from Table 1.

Often the values of s, $\bar{X}$, and t or CV and t are not known before the testing program begins. However, s and $\bar{X}$, or CV, may be approximated by using the results of some other test program, or they may simply be guessed (see example, Note 1).

NOTE 1—An example of initial sample size calculation is:
Sampling a grade of lumber for modulus of elasticity (E). Assuming a 95 % confidence level, the t statistic can be approximated by 2.
s = 300 000 psi (2067 MPa)
$\bar{X}$ = assigned E of the grade = 1 800 000 psi (12 402 MPa)
CV = (300 000/1 800 000) = 0.167
t = 2
$n = \left(\frac{2}{0.05} \times 0.167\right)^2 = 44.622$ (45 pieces)
Calculate the sample mean and standard deviation and use them to estimate a new sample size from Eq 1, where the value of t is taken from Table 1. If the second sample size exceeds the first, the first sample was insufficient; obtain and test the additional specimens.

NOTE 2—More details of this two-stage method are given in Ref **(8).**

3.4.3 To determine sample size based on a tolerance limit (TL), the desired content (C) (Note 3) and associated confidence level must be selected. The choice of a specified content and confidence is dependent upon the end-use of the material, economic considerations, current design practices, code requirements, etc. For example, a content of 95 % and a confidence level of 75 % may be appropriate for a specific property of structural lumber. Different confidence levels may be suitable for different products or specific end uses. Appropriate content and confidence levels shall be selected before the sampling plan is designed.

NOTE 3—The content, *C*, is an estimate of the proportion of the population that lies above the tolerance limit. For example, a tolerance limit with a content of 95 % describes a level at which 95 % of the population lies above the tolerance limit. The confidence with which this inference is to be made is a separate statement.

3.4.3.1 To determine the sample size for near-minimum properties, the nonparametric tolerance limit concept of Ref **(8)**

TABLE 1 Values of the t Statistics Used in Calculating Confidence Intervals[A]

df $n - 1$	*CI* = 75 %	*CI* = 95 %	*CI* = 99 %
1	2.414	12.706	63.657
2	1.604	4.303	9.925
3	1.423	3.182	5.841
4	1.344	2.776	4.604
5	1.301	2.571	4.032
6	1.273	2.447	3.707
7	1.254	2.365	3.499
8	1.240	2.306	3.355
9	1.230	2.262	3.250
10	1.221	2.228	3.169
11	1.214	2.201	3.106
12	1.209	2.179	3.055
13	1.204	2.160	3.012
14	1.200	2.145	2.977
15	1.197	2.131	2.947
16	1.194	2.120	2.921
17	1.191	2.110	2.898
18	1.189	2.101	2.878
19	1.187	2.093	2.861
20	1.185	2.086	2.845
21	1.183	2.080	2.831
22	1.182	2.074	2.891
23	1.180	2.069	2.807
24	1.179	2.064	2.797
25	1.178	2.060	2.787
26	1.177	2.056	2.779
27	1.176	2.052	2.771
28	1.175	2.048	2.763
29	1.174	2.045	2.756
30	1.173	2.042	2.750
40	1.167	2.021	2.704
60	1.162	2.000	2.660
120	1.156	1.980	2.617
∞	1.150	1.960	2.576

[A] Adapted from Ref **(8)**. For calculating other confidence levels, see Ref **(8)**.

TABLE 2 Sample Size and Order Statistic for Estimating the 5 % Nonparametric Tolerance Limit, NTL[A]

75 % Confidence		95 % Confidence		99 % confidence	
Sample Size[B]	Order Statistic[C]	Sample Size	Order Statistic	Sample Size	Order Statistic
28	1	59	1	90	1
53	2	93	2	130	2
78	3	124	3	165	3
102	4	153	4	198	4
125	5	181	5	229	5
148	6	208	6	259	6
170	7	234	7	288	7
193	8	260	8	316	8
215	9	286	9	344	9
237	10	311	10	371	10
259	11	336	11	398	11
281	12	361	12	425	12
303	13	386	13	451	13
325	14	410	14	478	14
347	15	434	15	504	15
455	20	554	20	631	20
562	25	671	25	755	25
668	30	786	30	877	30
879	40	1013	40	1115	40
1089	50	1237	50	1349	50

[A] Adapted from Ref **(12)**. For other tolerance limits or confidence levels, see Ref **(12)** or **(8)**.
[B] Where the sample size falls between two order statistics (for example, 27 and 28 for the first order statistic at 75 confidence), the larger of the two is shown in the table, and the confidence is greater than the nominal value.
[C] The rank of the ordered observations, beginning with the smallest.

may be used (Table 2). This will provide the sample size suitable for several options in subsequent near-minimum analyses. Although the frequency with which the tolerance limit will fall above (or below) the population value, corresponding to the required content, is controlled by the confidence level selected, the larger the sample size the more likely the tolerance limit will be close to the population value. It is, therefore, desirable to select a sample size as large as possible commensurate with the cost of sampling and testing (see also 4.7).

3.4.3.2 If a parametric approach is used, then a tolerance limit with stated content and confidence can be obtained for any sample size; however, the limitation expressed in 3.4.3.1 applies. That is, although the frequency that the tolerance limit falls above (or below) the population value, corresponding to the required content is controlled, the probability that the tolerance limit will be close to the population value depends on the sample size. For example, if normality is assumed, the parametric tolerance limit (PTL) will be of the form PTL = $\bar{X} - Ks$, (see Ref **(8)**), and the standard error (SE) of this statistic may be approximated by the following equation:

$$SE = s\sqrt{\frac{1}{n} + \frac{K^2}{2(n-1)}} \quad (2)$$

where:
s = standard deviation of specimen values,
n = sample size, and
K = confidence level factor.

The sample size, *n*, may be chosen to make this quantity sufficiently small for the intended end use of the material (Note 4).

NOTE 4—An example of sample size calculation where the purpose is to estimate a near minimum property is shown in the following calculation:

Estimate the sample size, *n*, for a compression parallel strength test in which normality will be assumed. A CV of 22 % and a mean C_{11} of 4600 psi are assumed based on other tests. The target PTL of the lumber grade is 2700 psi. The PTL is to be estimated with a content of 95 % (5 % PTL) and a confidence of 75 %.

CV = 0.22
$\bar{X}$ = 4600 psi (31.7 MPa)
s = (0.22) (4600) = 1012 psi (7.0 MPa)
K = $(\bar{X} - \text{PTL})/s = 1.877$

From Table 3:
K = 1.869 for $n = 30$

Therefore $n \approx 30$ specimens.

$$SE = 1012\sqrt{\frac{1}{30} + \frac{1.877^2}{2(30-1)}} \quad (3)$$

$$= 310.5 \text{ psi (2.1 MPa)}$$

Consequently, although 30 specimens is sufficient to estimate the 5 % PTL with 75 % confidence, the standard error (approximately 12 % of the PTL) illustrates that, with this size sample, the PTL estimated by test may not be as close to the true population fifth percentile as desired. A larger *n* may be desirable.

TABLE 3 K Factors for One-Sided Tolerance Limits for Normal Distributions[A]

	75 % Confidence (γ = 0.25)				95 % Confidence (γ = 0.05)				99 % Confidence (γ = 0.01)			
1 − p	0.75	0.90	0.95	0.99	0.75	0.90	0.95	0.99	0.75	0.90	0.95	0.99
n												
3	1.464	2.501	3.152	4.397	3.805	6.156	7.657	10.555	8.726	13.997	17.374	23.900
4	1.255	2.134	2.681	3.726	2.617	4.162	5.145	7.044	4.714	7.381	9.085	12.389
5	1.151	1.962	2.464	3.422	2.149	3.407	4.203	5.742	3.453	5.362	6.580	8.941
6	1.087	1.859	2.336	3.244	1.895	3.007	3.708	5.063	2.847	4.412	5.407	7.336
7	1.043	1.790	2.251	3.127	1.732	2.756	3.400	4.643	2.490	3.860	4.729	6.413
8	1.010	1.740	2.189	3.042	1.617	2.582	3.188	4.355	2.253	3.498	4.286	5.813
9	0.984	1.702	2.142	2.978	1.532	2.454	3.032	4.144	2.083	3.241	3.973	5.390
10	0.964	1.671	2.104	2.927	1.465	2.355	2.912	3.982	1.954	3.048	3.739	5.075
11	0.946	1.646	2.074	2.886	1.411	2.276	2.816	3.853	1.852	2.898	3.557	4.830
12	0.932	1.625	2.048	2.852	1.366	2.210	2.737	3.748	1.770	2.777	3.411	4.634
13	0.919	1.607	2.026	2.823	1.328	2.156	2.671	3.660	1.702	2.677	3.290	4.473
14	0.908	1.591	2.008	2.797	1.296	2.109	2.615	3.585	1.644	2.593	3.189	4.338
15	0.899	1.577	1.991	2.776	1.267	2.069	2.566	3.521	1.595	2.522	3.103	4.223
16	0.890	1.565	1.977	2.756	1.242	2.033	2.524	3.465	1.552	2.460	3.028	4.124
17	0.883	1.555	1.964	2.739	1.220	2.002	2.487	3.415	1.514	2.405	2.963	4.037
18	0.876	1.545	1.952	2.724	1.200	1.974	2.453	3.371	1.480	2.357	2.906	3.961
19	0.869	1.536	1.942	2.710	1.182	1.949	2.424	3.331	1.450	2.314	2.854	3.893
20	0.864	1.528	1.932	2.697	1.166	1.926	2.396	3.296	1.423	2.276	2.808	3.832
21	0.858	1.521	1.924	2.686	1.151	1.906	2.372	3.263	1.398	2.241	2.767	3.777
22	0.854	1.514	1.916	2.675	1.138	1.887	2.349	3.234	1.376	2.209	2.729	3.727
23	0.849	1.508	1.908	2.666	1.125	1.869	2.329	3.207	1.355	2.180	2.695	3.682
24	0.845	1.502	1.901	2.657	1.113	1.853	2.310	3.182	1.336	2.154	2.663	3.640
25	0.841	1.497	1.895	2.648	1.103	1.838	2.292	3.159	1.319	2.129	2.634	3.602
30	0.825	1.475	1.869	2.614	1.058	1.778	2.220	3.064	1.247	2.030	2.516	3.447
35	0.812	1.458	1.849	2.588	1.025	1.732	2.167	2.995	1.194	1.958	2.430	3.335
40	0.802	1.445	1.834	2.568	0.999	1.697	2.126	2.941	1.154	1.902	2.365	3.249
45	0.794	1.434	1.822	2.552	0.978	1.669	2.093	2.898	1.121	1.857	2.312	3.181
50	0.788	1.426	1.811	2.539	0.960	1.646	2.065	2.863	1.094	1.821	2.269	3.125
60	0.777	1.412	1.795	2.518	0.932	1.609	2.023	2.808	1.051	1.764	2.203	3.039
70	0.769	1.401	1.783	2.502	0.911	1.581	1.990	2.766	1.019	1.722	2.153	2.974
80	0.762	1.393	1.773	2.489	0.894	1.560	1.965	2.733	0.994	1.689	2.114	2.924
90	0.757	1.386	1.765	2.479	0.881	1.542	1.944	2.707	0.974	1.662	2.083	2.884
100	0.753	1.380	1.758	2.470	0.869	1.527	1.927	2.684	0.957	1.639	2.057	2.850
120	0.745	1.371	1.747	2.456	0.851	1.503	1.900	2.650	0.930	1.604	2.016	2.797
140	0.740	1.364	1.739	2.446	0.837	1.485	1.879	2.623	0.909	1.577	1.985	2.758
160	0.736	1.358	1.733	2.438	0.826	1.471	1.862	2.602	0.893	1.556	1.960	2.726
180	0.732	1.353	1.727	2.431	0.817	1.460	1.849	2.585	0.879	1.539	1.940	2.700
200	0.729	1.350	1.723	2.425	0.809	1.450	1.838	2.570	0.868	1.524	1.923	2.679
250	0.723	1.342	1.714	2.414	0.794	1.431	1.816	2.542	0.846	1.496	1.891	2.638
300	0.719	1.337	1.708	2.406	0.783	1.417	1.800	2.522	0.830	1.476	1.868	2.609
350	0.715	1.332	1.703	2.400	0.775	1.407	1.788	2.507	0.818	1.461	1.850	2.586
400	0.712	1.329	1.699	2.395	0.768	1.398	1.778	2.495	0.809	1.449	1.836	2.568
450	0.710	1.326	1.696	2.391	0.763	1.391	1.770	2.484	0.801	1.438	1.824	2.553
500	0.708	1.324	1.693	2.387	0.758	1.385	1.763	2.476	0.794	1.430	1.815	2.541
600	0.705	1.320	1.689	2.382	0.750	1.376	1.753	2.462	0.783	1.416	1.799	2.521
700	0.703	1.317	1.686	2.378	0.745	1.369	1.744	2.452	0.775	1.406	1.787	2.506
800	0.701	1.315	1.683	2.374	0.740	1.363	1.738	2.443	0.768	1.398	1.777	2.493
900	0.699	1.313	1.681	2.371	0.736	1.358	1.732	2.436	0.762	1.391	1.769	2.483
1000	0.698	1.311	1.679	2.369	0.733	1.354	1.728	2.431	0.758	1.385	1.763	2.475
1500	0.694	1.306	1.672	2.361	0.722	1.340	1.712	2.411	0.742	1.365	1.741	2.447
2000	0.691	1.302	1.669	2.356[B]	0.715	1.332	1.703	2.400[B]	0.733	1.354	1.727	2.431[B]
2500	0.689	1.300[B]	1.666[B]	2.353[B]	0.711	1.326	1.697[B]	2.392[B]	0.727	1.346	1.719[B]	2.419[B]
3000	0.688	1.299[B]	1.664[B]	2.351[B]	0.708	1.323[B]	1.692[B]	2.386[B]	0.722	1.340[B]	1.712[B]	2.411[B]
inf	0.674	1.282	1.645	2.326	0.674	1.282	1.645	2.326	0.674	1.282	1.645	2.326

[A] Obtained from a noncentral *t* inverse approach; see Ref **(15)**.
[B] Computed using formula X5.2.

3.4.4 Often the objective of the evaluation program will be to estimate mean and near-minimum properties simultaneously. When this is the case, only one sample size need be used. It should be the greater of the two obtained in accordance with 3.4.2 and 3.4.3.

3.4.5 If a sampling unit other than an individual piece of lumber is to be used, as provided for in 3.3.3, then the required sample size must be determined by procedures that are statistically appropriate for the sampling method chosen. In the case of multisource data, as in the sampling of some or all mills in a defined region, special procedures may be required, for example, those based on the methodology introduced in Ref **(9)**. In all cases, the procedures shall be fully described.

4. Analysis and Presentation of Results

4.1 The results of the tests performed in accordance with Test Methods D 198 or other standard testing procedures shall be analyzed and presented as (*1*) a set of summarizing statistics, and (*2*) an appendix of unadjusted individual test specimen results. If parametric procedures are to be used, a description of the selection procedures and a tabulation of distribution parameters shall be provided. Any "best-fit" judgment (Note 5) between competing distributions shall be documented.

NOTE 5—A best-fit procedure should recognize the low power of some published procedures. To check the fit, the series of tests outlined in Ref **(10)** represents several alternatives. Also, tests based on the Anderson-Darling statistic **(2, 3, 4)** have been shown to be among the more powerful tests **(6, 7)**. It should be noted, however, that not all tests are valid for all distributions and that these procedures are effective for checking central tendency. For instance, revised standard tables of the Kolmogorov-Smirnov statistic are presently available only for the normal, logistic, and exponential distributions **(5)**.

4.2 Properties shall be adjusted to a single moisture content appropriate for the objective of the testing program. Although test results can be adjusted for moisture content, these adjustments decrease in accuracy with increasing change in moisture content. For this reason, it is suggested that the specimens be conditioned as closely as possible to the target moisture content prior to test, and that adjustments for more than five percentage points of moisture content are to be avoided. When adjustments are required, the procedures given in Practice D 1990 shall be used for dimensions, bending strength, modulus of elasticity, tensile strength, and compressive strength parallel to the grain. When adjusting shear strength and compressive strength perpendicular to the grain the procedures of Practice D 245 shall be used.

4.3 Modulus of elasticity values of primary concern are apparent values, E_{ai}, used in deflection equations that attribute all deflection to moment. These apparent moduli may be standardized for a specific span-depth ratio and load configuration. Standardization should reflect, as far as possible, conditions of anticipated end use.[5,6] When tests at standardized conditions of load and span are not possible, to adjust E_{ai} to standardized conditions, it is necessary to account for the effect of shear deflection on beam deflection. Factors to adjust E_{ai} for span-depth ratio and load configuration may be derived from Eq 4, (Ref **(11)**). To determine the apparent modulus of elasticity, E_{ai2}, based on any set of conditions of span-depth ratio and load configuration, when the modulus, E_{ai}, based on some other set of conditions is known, solve the equation:

$$E_{ai2} = \frac{1 + K_1\left(\frac{h_1}{L_1}\right)^2\left(\frac{E}{G}\right)}{1 + K_2\left(\frac{h_2}{L_2}\right)^2\left(\frac{E}{G}\right)} E_{ai} \quad (4)$$

where:
h = depth of the beam,
L = total beam span between supports,
E = shear free modulus of elasticity,
G = modulus of rigidity, and
K_i = values are given in Table 4.

The equations were derived using simple beam theory for a simply supported beam composed of isotropic, homogeneous material. Experimental evidence suggests that these equations produce reasonable results with solid wood when converting between load conditions at a fixed span-depth ratio. Care must be exercised when converting between different span-depth ratios to assure that the adjustments are appropriate for the end use.

4.3.1 Often, lumber is not homogeneous within a piece with respect to modulus of elasticity. The apparent modulus, therefore, may be affected by the location of growth characteristics, such as knots, with respect to loads and supports. It is further cautioned that conversions may be less appropriate when converting between edgewise and flatwise specimen orientations.

4.3.2 If modulus of elasticity results are not measured at standardized conditions, separate justification shall be provided for factors used to adjust test values to standardized conditions.

4.3.3 In calculations using Eq 4 and that involve mean trends of large populations, a single E/G ratio is usually assumed.[7] If this assumption is critical for the intended

[5] Spans, which customarily serve as a basis for design range, go from 17 to 21 times the depth of the specimen.

[6] A uniform load distribution is commonly encountered in use. This load configuration is difficult to apply in testing, but may be closely approximated by applying the load at the one-third points of the span, if the span-to-depth ratio is the same.

[7] When using these conversion equations with solid wood, historically it has been assumed that the modulus of rigidity (G) is one sixteenth the shear free modulus of elasticity (E). Limited data indicate the ratio of E/G for individual pieces of lumber can vary significantly from this value depending upon the number, size, and location of knots present, the slope of grain in the piece, and the spans over which deflections are measured **(13).**

TABLE 4 K Factors for Adjusting Apparent Modulus of Elasticity of Simply Supported Beams[A]

Loading	Deflection measured at	K_i
Concentrated at midspan	midspan	1.200
Concentrated at third points	midspan	0.939
Concentrated at third points	load points	1.080
Concentrated at outer quarter-points	midspan	0.873
Concentrated at outer quarter-points	load points	1.20
Uniformly distributed	midspan	0.960

[A] See Appendix X4 for an example of use of Table 4.

application, it is recommended that the moduli of elasticity and rigidity of the individual pieces be measured (see Methods D 198).

4.4 The adjustment factors used to reduce the test statistics to the level of allowable properties depend on the property and are shown in Table 5. They are taken from Practice D 245, which includes a safety factor and a 10-year cumulative duration of load effect (normal loading).

4.5 Statistics shall be shown with three significant digits. Adequate significant digits shall be maintained in all intermediate calculations to avoid rounding errors in the statistics.

4.5.1 The sample mean is calculated as follows:

$$\bar{X} = \sum_{i=1}^{n} x_i/n \tag{5}$$

where:

x_i = individual observations, and
n = sample size.

The sample mean is an unbiased estimator of the true population mean.

4.5.2 The sample standard deviation is calculated as follows:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i^{\,2} - [(\Sigma x_i)^2/n]}{n-1}} \text{ or} \tag{6}$$

$$= \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{X})^2}{n-1}}$$

4.5.3 The confidence interval (CI) for the mean is calculated as follows:

$$CI = \bar{X} \pm (ts/\sqrt{n}) \tag{7}$$

where t depends on the sample size and confidence level, and is given in Table 1. A CI of this type provides that, if the population is normally distributed, a given percent of all intervals found in this manner are expected to contain the true population mean.

4.5.4 The sample nonparametric percent point estimate (NPE) may be interpolated from the sample. To perform the interpolation, arrange the test values in ascending order. Symbolically, call them x_1, x_2, x_3, ... x_n. Beginning with the lowest value (that is, first order statistic, see Note 6), calculate $i/(n + 1)$, where i is the order of the value, for each successively higher value until $i/(n + 1) \geq k/100$, call it the j^{th} value, equals or exceeds the sample k percentile point estimate. Interpolate the nonparametric k percentage point estimate by:

$$NPE = \left[\frac{k}{100}(n + 1) - (j - 1)\right] [x_j - x_{(j-1)}] + x_{(j-1)} \tag{8}$$

where k is the desired percentile point estimate sought.

NOTE 6—Order statistics are ranked test values from the lowest to the highest. For example, the first order statistic is the lowest test value or the weakest piece in the sample, the second order statistic is the second weakest piece, etc.

4.5.5 The nonparametric lower tolerance limit (NTL) of a specified content is the m^{th} order statistic, where m depends upon the sample size and confidence level. Table 2 depicts the order statistic required to determine the lower-5 % NTL at a given sample size and three confidence levels. For example, if the sample size was 93 and the confidence level was chosen to be 95 %, m = 2. That is, the lower-5 % NTL with at least 95 % confidence would be the second order statistic. If other lower percentiles are estimated, the corresponding NTLs can be determined **(8, 12)**.

4.5.6 If parametric methods are used, the parametric point estimate (PPE) and lower parametric tolerance limit (PTL) shall be estimated by procedures documented as adequate for the method adopted **(1, 8, 12)**.

4.5.7 A histogram, or an empirical cumulative distribution function, or both, shall be presented. The class widths for a histogram depend on the property; maximum widths are given in Table 6. If parametric procedures are used for analysis, either a cumulative distribution function or a probability density function can be shown superimposed on the empirical cumulative distribution function or the histogram respectively.

NOTE 7—Two examples of typical test data and a summary of the results that meet the requirements of 4.1-4.5 are given in Appendix X1 and Appendix X2.

4.5.8 If a sampling unit other than an individual piece of lumber is used, then the calculation of sample means, standard deviations, confidence intervals, tolerance limits, and exclusion limits must be made in a manner statistically consistent with the sampling procedure chosen.

4.6 If the purpose of the testing program is to evaluate the accuracy of existing allowable properties for the population sampled, this is done using the results of 4.5.3, 4.5.5, 4.5.6, or 4.5.8. If an allowable mean property for a population falls within the confidence interval obtained in accordance with 4.5.3, the testing program bears out the value allowed for the population with the associated confidence. The accuracy of existing near-minimum properties may be assessed using the results of 4.5.4, or 4.5.5, 4.5.6, and 4.5.8, or combination thereof. If the existing property falls at or below the point estimate as calculated in 4.5.4, the testing program may bear out the existing values, but no confidence statement may be

TABLE 5 Reduction Factors to Relate Test Statistics to Allowable Properties

Property	Factor
Modulus of elasticity	1
Bending strength	1/2.1
Tensile strength	1/2.1
Compressive strength parallel to grain	1/1.9
Shear strength	1/2.1
Compressive strength perpendicular to grain	1/1.67

TABLE 6 Maximum Class Width to Be Used in Histogram Plots

Property	Class Width, psi (MPa)
Modulus of elasticity	100 000 (690)
Bending strength	500 (3.4)
Tensile strength	500 (3.4)
Compressive strength parallel to grain	500 (3.4)
Shear strength	50 (0.34)
Compressive strength perpendicular to grain	50 (0.34)

associated with this conclusion. In order to associate a confidence statement, the existing value must fall below the tolerance limit as calculated in 4.5.5, 4.5.6, or 4.5.8.

4.7 If the purpose of the testing program is to establish allowable properties for the population, this is done using the results of 4.5.1, 4.5.4, 4.5.5, 4.5.6, or 4.5.8. The allowable value of modulus of elasticity shall be the sample mean of 4.5.1, if the width of the confidence interval is a sufficiently small fraction of the mean (for example, if $ts/(\bar{X}\sqrt{n}) \leq \lambda$, where λ, predetermined by the user will normally be in the range from 0.01 to 0.10). If this condition is not satisfied, additional samples must be taken as described in 3.3 until the condition holds. Generally, the allowable value of any near-minimum allowable stress shall be the sample 5 % NPE of Section 4.5.4, if the relative difference between the NPE and the NTL is sufficiently small, (that is, if $(NPE - NTL)/NPE < \delta$, where δ will normally be in the range from 0.01 to 0.10). This condition is essentially that of having sufficiently narrow confidence interval for the NPE. If this condition is not satisfied, additional samples may be taken until the condition holds, or the NTL may be used for the allowable value. If the latter course is chosen, one should be cognizant of the imprecision in the NTL consequent on the sample size (see 3.4.3.1). Alternatively, the PPE and PTL of the parametric procedures provided for in 4.5.6 may be employed in a parallel manner.

5. Applications

5.1 The results, reduced to the level of allowable properties, may be used to evaluate the accuracy of existing allowable properties or to establish allowable properties.

5.2 Where properties have been previously assigned to a lumber population, one purpose of this practice is to provide a format for evaluation of this assignment through full-size lumber tests. Provisions are made for estimating both the mean and near-minimum property values.

5.3 Results obtained following the procedures and analyses of this practice may also be used to characterize the population sampled for establishing design values. The specific characterization with respect to the population, such as the mean or a near-minimum property, depends on the objective, the content, and confidence associated with the test sample. The representativeness and size of the sample influence how the characterization can be made. Contemporary practice is reflected in 4.7, however, other interpretations may be appropriate.

5.4 The end use of a specific product will dictate the specification requirement. Indeed this practice addresses itself to the procedures for sampling specified populations and procedures for interpreting the results. It cannot be implemented without the selection of values for the confidence levels and degree of precision needed at various stages of the procedures. These values should be given careful consideration so that they are compatible with the anticipated end use, the risks that surround imprecise estimates, or incorrect decisions, and the costs of sampling, testing, and analysis.

6. Keywords

6.1 lumber; structural lumber; wood

APPENDIXES

(Nonmandatory Information)

X1. TYPICAL EXAMPLE—COMMODITY LUMBER

X1.1 *Population Description*—Selected at random, from one mill, were 80 No. 2 grade Hem-Fir two-by-fours (current lumber agency grade rules). The 80 test specimens were equilibrated to an average of 15 % moisture content (see Note 4 and Practice D 245).

X1.2 The purpose of the test was to evaluate the bending modulus of elasticity, *E*, and tensile strength, F_t, of a one-mill sample relative to present design values. Consequently the fifth percentile estimate will be considered for strength and the mean value for *E* (see 4.7).

X1.3 The design value for the grade and species sampled is given in Table X1.1. A table of test statistics is given in Table X1.2.

X1.4 Histograms and fitted normal, lognormal, and Weibull distributions of edgewise bending *E* and tensile strength are shown in Fig. X1.1 and Fig. X1.2.

X1.5 Several of the individual test results are shown, only as an example of data that is typically recorded (Table X1.3). It may be desirable to tabulate additional information, such as specific gravity, knot location, etc., depending on purpose. Note that tensile strength data is ordered in ascending order.

X1.6 If appropriate best fit tests have been carried out and documented, only the best fit distribution need be illustrated; however, illustration of other options is instructive (see Table X1.4). Note that the nonparametric estimates in Table X1.4 for tensile strength can be estimated directly from Table X1.3, but the estimates for modulus of elasticity are based on data, most of which, is not shown in Table X1.3.

X1.7 Using 4.3, 4.5.1, 4.5.2, 4.5.3, and 4.6, the confidence interval for the mean *E* value (Table X1.2) did not contain the value as printed in Table X1.1. Consequently, it was decided this sample *E* did not verify the design *E*. Analysis of the tension strength values was conducted in accordance with 4.5.4 and 4.5.5. After adjusting the nonparametric lower -5 % tolerance limit to an allowable design value (that is, 1152/

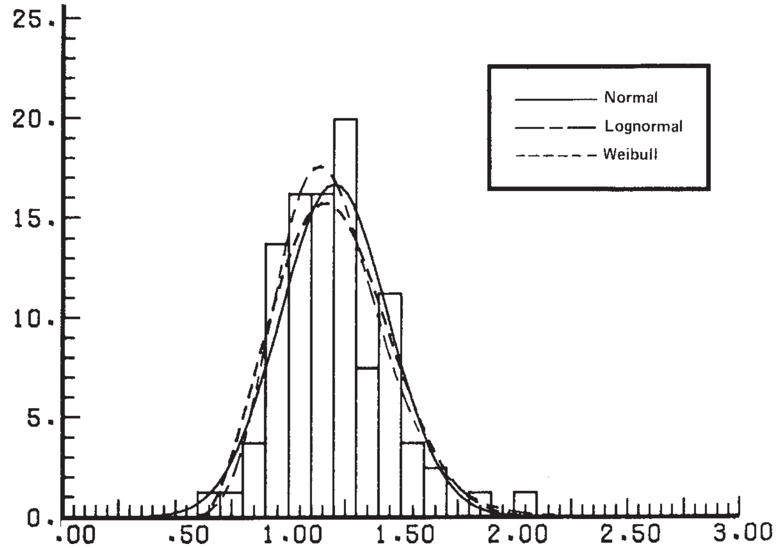

FIG. X1.1 Static Edgewise Modulus of Elasticity (10^6 psi)

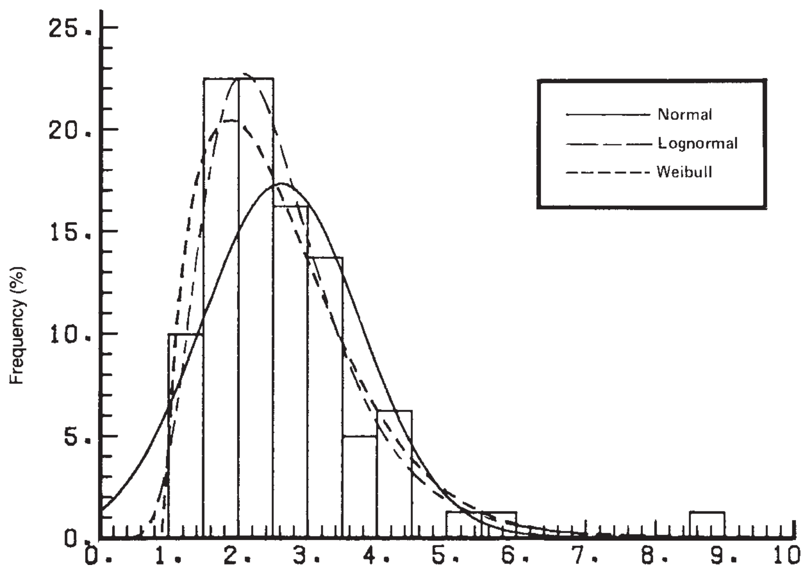

FIG. X1.2 Tensile Strength (1000 psi)

2.1 = 548.6 psi (3.8 MPa)), it can be seen that this value is below the value shown in Table X1.1 (F_t = 675 psi (4.6 MPa)); therefore, the sample tension values do not verify the design tension value.

X1.8 Similar analyses could be performed using parametric procedures and employing the values shown in Table X1.4.

TABLE X1.1 Design Values for No. 2 Grade Hem-Fir Two-by-Fours[A]

Species/Grade	Design Values	
	F_t, psi (MPa)	E, psi (MPa)
Hem-Fir No. 2	675 (4.6)	1 400 000 (9 646)

[A] National Design Specification for Wood Construction.

TABLE X1.2 Example Test Results for No. 2 Grade Hem-Fir Two-by Fours[A]

Property	Mean, psi (MPa)	Confidence Interval for Mean, psi (MPa)[B]	Standard Deviation, psi (MPa)	Sample Size
Static edgewise modulus of elasticity[C]	1 201 600(8 279)	1 148 500(7 113)–1 254 700(8 645)	238 500(1 643)	80
Tensile strength	1 250(8.6)	1 100(7.6)–1 350(9.3)	547(3.8)	80

[A] All statistics in psi; all adjusted to 15 % moisture content in accordance with 4.2; reductions in accordance with 4.4 (not rounded).
[B] 95 % confidence.
[C] Adjusted to ℓ/d of 21 and uniform load.

TABLE X1.3 Example of Test Results Ordered by Tensile Strength—Two-by-Four Sample

Specimen Number	Moisture Content at Test, %	Tensile Strength, psi (MPa)	Edgewise Modulus of Elasticity 10^2, psi (MPa)[A]	Width, in. (mm)[B]	Thickness, in. (mm)[B]	Bending Strength Ratio, %[C]
1 P 43	15.0	1004 (6.9)	994 (6849)	3.47 (88)	1.47 (37)	13
1 P 1	15.0	1092 (7.5)	959 (6607)	3.47 (88)	1.51 (38)	13
1 P 15	13.0	1152 (7.9)	1061 (7310)	3.42 (87)	1.50 (38)	52
1 P 28	15.0	1169 (8.0)	667 (4596)	3.45 (88)	1.46 (37)	47
1 P 22	16.0	1257 (8.7)	950 (6545)	3.46 (88)	1.49 (38)	52

[A] Test ℓ/d of 44, quarter-point load, corrected to ℓ/d of 21 and a uniform load.
[B] At test moisture content.
[C] Obtained by 5.3.4.1 of Practice D 245.

TABLE X1.4 Estimates of Population Parameters for Two-by-Four Sample

Parameter	Static Edgewise Modulus Elasticity 10^6, psi (MPa)	Tensile Strength[A], psi (MPa)
Weibull:		
5 % point estimate	0.8255 (5688)	1.230 (8.5)
Lognormal:		
5 % point estimate	0.8549 (5890)	1.270 (8.7)
5 % TL (75 %)	0.8340 (5746)	1.208 (8.3)
Normal:		
Mean	1.2016 (8279)	2.616 (18.0)
Standard deviation	0.2385 (1643)	1.149 (7.9)
5 % point estimate	0.8091 (5575)	0.726 (5.0)
5 % TL (75 %)	0.7790 (5367)	0.580 (4.0)
Nonparametric:		
5 % point estimate	0.8745 (6025)	1.169 (8.0)
5 % TL (75 %)	0.8490 (5850)	1.152 (7.9)

[A] Not reduced to allowable property.

X2. TYPICAL EXAMPLE—LADDER RAIL STOCK

X2.1 *Population Description*—(Species) ladder rail stock graded in accordance with the (Grading Rules) as "V.G. Ladder Rails." Two hundred pieces of 1⅜ by 2¾-in. by 8 ft were selected randomly from stock at a ladder manufacturer in (location). Specimens were equilibrated in a conditioning room. Actual average moisture content of specimens equaled 11.2 %. The standard deviation was 1.4 %. The purpose of the sampling, testing, and analysis was to obtain the bending modulus of rupture (MOR) and modulus of elasticity (E) of typical ladder rails for use in a research study on ladder rail properties. Only mean and lower tail properties estimated by nonparametric procedures were of interest. The 95 % confidence level was deemed appropriate for both E and MOR in this study.

X2.2 Data reduced to summary statistics are shown in Table X2.1. Examples of individual specimen data are shown in Table X2.2; Table X2.3 contains estimates of near-minimum values. Histograms of test results are shown in Fig. X2.1 and Fig. X2.2. Empirical cumulative distribution functions are

TABLE X2.1 Ladder Rail Test Statistics[A]

Property	Mean, psi (MPa)	Confidence Interval for Mean, psi (MPa)[B]	Standard Deviation, psi (MPa)	Sample Size
Static edgewise modulus of elasticity[C]	1 755 300 (12 094)	1 713 200 (11 804)–1 797 400 (12 384)	301 500 (2077)	200
Modulus of rupture[D]	9 758 (67)	9 520 (66)–10 014 (69)	1 836 (12.6)	200

[A] All statistics in psi.
[B] 95 % confidence.
[C] Adjusted to ℓ/d of 21, uniform load, and 12 % moisture content.
[D] Adjusted to 12 % moisture content.

TABLE X2.2 Sample Test Results—Ladder Rail

Specimen	Moisture Content at Test, %	Modulus of Rupture, psi (MPa)[A]	Edgewise Static E, 10^6psi (MPa) [A,B]	Width at Test, in. (mm)	Thickness at Test, in. (mm)
103	12.8	14 343 (99)	2.51 (17 294)	2.753 (70)	1.366 (35)
111	11.0	11 423 (79)	1.80 (12 402)	2.760 (70)	1.381 (35)
114	8.6	6 505 (45)	1.37 (9 439)	2.784 (71)	1.406 (36)
121	11.6	9 708 (69)	2.17 (14 951)	2.762 (70)	1.386 (35)

[A] Statistics adjusted to 12 % moisture content in accordance with 4.2; not adjusted to allowable properties.
[B] Adjusted to ℓ/d of 21 and uniform load; actual conditions were ℓ/d of 33 and center point load.

TABLE X2.3 Estimates of Near-Minimum Population Parameters of Ladder Rail

Property	5 % Point Estimates	5 % Tolerance Limits 75 % Confidence	5 % Tolerance Limits 95 % Confidence	5 % Tolerance Limits 99 % Confidence
Edgewise modulus of elasticity[A]	1.30 (8957)	1.29 (8888)	1.23 (8475)	1.16 (7992)
Modulus of rupture[B]	6518 (45)	6072 (42)	5364 (37)	5353 (37)

[A] 10^6 psi (MPa); adjusted to ℓ/d of 21 and uniform load in accordance with 4.5.1; adjusted to 12 % moisture content in accordance with 4.2; not reduced to allowable property.
[B] psi (MPa); adjusted to 12 % in accordance with 4.2; not reduced to allowable property.

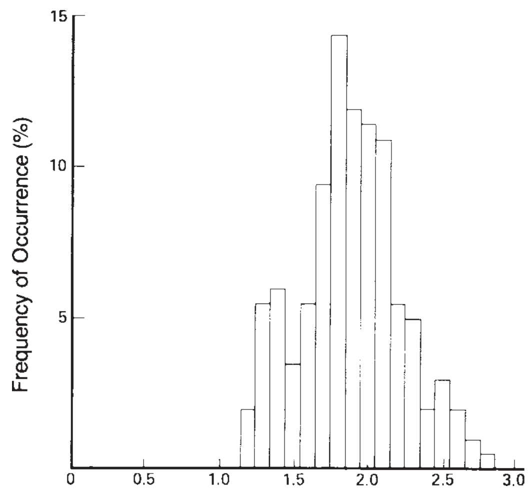

FIG. X2.1 Edgewise *E* (10^6 psi)

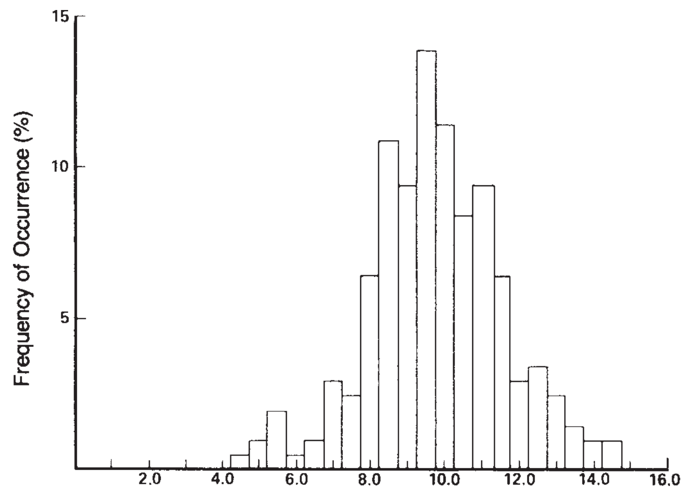

FIG. X2.2 Bending Strength MOR (1000 psi)

shown in Fig. X2.3 and Fig. X2.4.

X2.3 Following the procedures of 4.7 it was determined that the dispersion of E (static edgewise) measurements met the 5 % requirement (that is, $ts/\bar{X}\sqrt{n} = 0.024 \leq 0.05$) with 95 % confidence. Consequently, the research suggested an edgewise E of 1.7×10^6 psi could be used as a design value (Practice D 245 rounding rule would round the test value to 1.8 but this would be out of the confidence interval for the mean, thus 1.7 was chosen).

X2.4 Continuing the procedures of 4.7 for the MOR, comparisons between the NPE and several NTL's can be made (Table X2.3). Maintaining the 10 % relative difference criterion (NPE-NTL/NPE < 0.10) the relative difference for the NTL at a 95 % confidence level does not meet the criterion (6518-5364/6518 = 0.17 > 0.10). Therefore, the 95 % confidence level goal of X2.1 for MOR is not met. Either more sampling (see 4.7) is required or the NTL (5364 psi (37 MPa)) may be used as the best estimate of the population MOR.

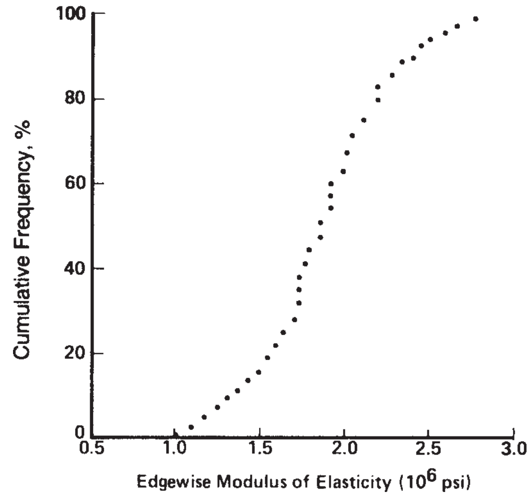

FIG. X2.3 Empirical Cumulative Distribution Function for *E*

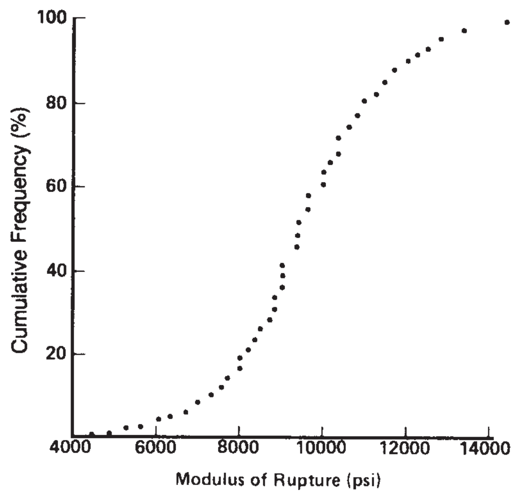

FIG. X2.4 Empirical Cumulative Distribution Function for *R*

X3. EXAMPLE—SAMPLING PROCEDURE

X3.1 When sampling from current production (that is, from the green chain) at a manufacturing facility, the following procedure allows the estimation of a standard error (SE) of the estimate as well as some information about the within-and-between sample variance.

X3.2 Following the procedure outlined in Practice E 105 (A1.6) k is generally chosen to be five or greater. Let $k = 5$, therefore, $10k = 50$. Select ten random numbers between 1 and 50. These are the ten random start points; 3, 9, 14, 29, 31, 36, 40, 42, 47, and 50 (Table X3.1). Systemically select test specimens using an interval length of $10k$ beginning at each of the random start points (that is, random start $x + 10k$).

TABLE X3.1 Test Specimens to Be Selected[A]

3	9	14	29	31	36	40	42	47	50
53	59	64	79	81	86	90	92	97	100
103	109	114	129	131	136	140	142	147	150
153	159	164	179	181	186	190	192	197	200
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...

[A] This process is continued until the desired sample size is obtained.

X4. EXAMPLE OF USE OF TABLE 5 TO ADJUST MODULUS OF ELASTICITY (MOE) TO STANDARD CONDITIONS

X4.1 An average apparent modulus of elasticity was obtained by testing simply supported beams loaded at the center and having a span-depth (L/h) ratio of 14:1. The MOE value obtained was 1.60 million psi. Assuming an E/G ratio of 16:1, what would be the apparent MOE for loads applied at the one-third points of the span with a span-depth ratio of 21:1? Deflections were measured at the center of the span.

From Table 4:

$$h_1/L_1 = 1{:}14 \quad (X4.1)$$

$$h_2/L_2 = 1{:}21$$

$$E_1 = 1.60 \text{ million psi}$$

$$E/G = 16$$

$$K_1 = 1.20$$

$$K_2 = 0.939$$

Therefore,

$$E_2 = (1.09796/1.034070)*1.60 \text{ million psi} \quad (X4.2)$$

$$E_2 = 1.70 \text{ million psi}$$

X5. ONE-SIDED TOLERANCE LIMITS FOR A NORMAL DISTRIBUTION

X5.1 A one-sided tolerance limit, *PTL*, is a value about which it may be said with confidence 1-γ, that at least a proportion, 1-p, of the population is greater than *PTL*. The formula is as follows:

$$PTL = \overline{X} - Ks \quad (X5.1)$$

where $\overline{X}$ and s are the mean and the standard deviation, respectively, calculated from the sample data. K depends upon sample size n, as well as percentile 100-p and confidence 1-γ. K values are given in Table 3 or they may be calculated from the following formula:

$$K = \frac{Z_p g + \sqrt{Z_p^{\,2} g^2 - [g^2 - Z_\gamma^{\,2}/(2(n-1))](Z_p^{\,2} - Z_\gamma^{\,2}/n)}}{g^2 - Z_\gamma^{\,2}/(2(n-1))} \quad (X5.2)$$

where:

$g = (4n - 5)/(4n - 4)$, and

Z_p and Z_γ are calculated with the following formula:

$$Z = T - (b_0 + b_1T + b_2T^2)/(1 + b_3T + b_4T^2 + b_5T^3) \quad (X5.3)$$

where:

$T = \sqrt{Ln(1/Q^2)}$ ($Q = p$ for Z_p and $Q = \gamma$ for Z_γ)

b_0 = 2.515517

b_1 = 0.802853

b_2 = 0.010328

b_3 = 1.432788

b_4 = 0.189269

b_5 = 0.001308

NOTE X5.1—K values computed using Eq X5.2 are approximations (see Ref **(15)**). For small values, the formula can seriously overestimate the K factors.

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