Designation: E 2029 – 99

Standard Test Method for Volumetric and Mass Flow Rate Measurement in a Duct Using Tracer Gas Dilution[1]

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1. Scope

1.1 This test method describes the measurement of the volumetric and mass flow rate of a gas stream within a duct, stack, pipe, mine tunnel, or flue using a tracer gas dilution technique. For editorial convenience all references in the text will be to a duct, but it should be understood that this could refer equally well to a stack, pipe, mine tunnel, or flue. This test method is limited to those applications where the gas stream and the tracer gas can be treated as ideal gases at the conditions of the measurement. In this test method, the gas stream will be referred as air, though it could be any another gas that exhibits ideal gas law behavior.

1.2 This test method is not restricted to any particular tracer gas although experimental experience has shown that certain gases are used more readily than others as suitable tracer gases. It is preferable that the tracer gas not be a natural component of the gas stream.

1.3 Use of this test method requires a knowledge of the principles of gas analysis and instrumentation. Correct use of the formulas presented here requires consistent use of units.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and to determine the applicability of regulatory limitations prior to use.* For specific precautionary statements, see Section 7.

2. Referenced Documents

2.1 *ASTM Standards:*

D 3154 Test Method for Average Velocity in a Duct (Pitot Tube Method)[2]

D 3464 Test Method for Average Velocity in a Duct Using a Thermal Anemometer[2]

2.2 *ANSI/ASME Standard:*

ANSI/ASME TC 19.1–1985 (1994) Measurement Uncertainty: Instrument Apparatus[3]

3. Terminology

3.1 *Definitions of Terms Specific to This Standard:*

3.1.1 *ideal gas*, *n*—a gas or gas mixture for which the ratio of the pressure divided by product of the density and temperature is a constant.

3.1.2 *mass flow*, *n*—the total mass of air passing the sampling point per unit time (kg/s, lb/min).

3.1.3 *tracer gas*, *n*—a gas that can be mixed with air and measured in very low concentrations.

3.1.4 *tracer gas analyzer*, *n*—a device that measures the concentration of tracer gas in an air sample.

3.1.5 *tracer gas mass concentration*, *n*—the ratio of the mass of tracer gas in air to the total mass of the air-tracer mixture. For an ideal gas, the mass concentration is independent of temperature and pressure.

3.1.6 *tracer gas molar concentration*, *n*—the ratio of the number of moles of tracer gas in air to the total number of moles of the air-tracer mixture.

3.1.7 *tracer gas volume concentration*, *n*—the ratio of the volume of tracer gas in air to the total volume of the air-tracer mixture. For an ideal gas, the volume concentration is independent of temperature and pressure and is equal to the molar concentration.

3.1.8 *volumetric flow*, *n*—the total volume of air passing the sampling point per unit time (m^3/s, ft^3/min).

3.2 *Symbols:*

C = mass concentration[4] of tracer gas (ppb-mass, ppm-mass, ppt-mass)
C_U = upstream mass concentration[4] of tracer gas (ppb-mass, ppm-mass, ppt-mass)
C_D = downstream mass concentration[4] of tracer gas (ppb-mass, ppm-mass, ppt-mass)
C_I = injection stream mass concentration[4] of tracer gas (ppb-mass, ppm-mass, ppt-mass)
c = volume concentration[4] of tracer gas (ppb, ppm, ppt)

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[1] This test method is under the jurisdiction of ASTM Committee E06 on Performance of Buildings and is the direct responsibility of Subcommittee E06.41 on Air Leakage and Ventilation.

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[2] *Annual Book of ASTM Standards*, Vol 11.03.

[3] Available from American National Standards Institute, 11 W. 42nd Street, 13th floor, New York, NY 10036.

c_U = upstream volume concentration[4] of tracer gas (ppb, ppm, ppt)
c_D = downstream volume concentration[4] of tracer gas (ppb, ppm, ppt)
c_I = injection volume concentration[4] of tracer gas (ppb, ppm, ppt)
F = mass flow rate[5](kg/s, g/min, lb/min)
F_I = injection mass flow rate[5](kg/s, g/min, lb/min)
F_U = upstream mass flow rate[5](kg/s, g/min, lb/min)
F_D = downstream mass flow rate[5](kg/s, g/min, lb/min)
f = volumetric flow rate[5](m^3/s, L/min, cfm)
f^{std} = volumetric flow rate at standard conditions[5](m^3/s, L/min, cfm)
f_I = injection volumetric flow rate[5](m^3/s, L/min, cfm)
f_U = upstream volumetric flow rate[5](m^3/s, L/min, cfm)
f_D = downstream volumetric flow rate[5](m^3/s, L/min, cfm)
f_I^{std} = injection volumetric flow rate[5] at standard conditions (m^3/s, L/min, cfm)
f_U^{std} = upstream volumetric flow rate[5] at standard conditions (m^3/s, L/min, cfm)
f_D^{std} = downstream volumetric flow rate[5] at standard conditions (m^3/s, L/min, cfm)
ρ = density[6](kg/m^3, g/L, lb/ft^3)
ρ_a = density[6] of gas stream without any tracer (kg/m^3, g/L, lb/ft^3)
ρ_t = density[6] of the tracer gas (kg/m^3, g/L, lb/ft^3)
ρ_I = density[6] of the injection gas mixture (kg/m^3, g/L, lb/ft^3)
ρ_U = density[6] of the upstream gas mixture (kg/m^3, g/L, lb/ft^3)
ρ_D = density[6] of the downstream gas mixture (kg/m^3, g/L, lb/ft^3)
$\rho^t{}_U$ = density[6] of the tracer gas at upstream conditions (kg/m^3, g/L, lb/ft^3)
$\rho^t{}_D$ = density[6] of the tracer gas at downstream conditions (kg/m^3, g/L, lb/ft^3)

4. Summary of Test Method

4.1 This test method describes the use of a tracer gas dilution technique to infer the volumetric flow rate through a duct. In practice, tracer gas is injected into a duct at a known mass or volumetric flow rate. Downstream of the injection point gas samples are taken and are analyzed for the resulting tracer concentration. The ratio of the injection flow rate and the downstream concentration represents the dilution volume per unit time or volumetric flow rate in the duct.

5. Significance and Use

5.1 The method presented here is a field method that may be used to determine mass and volume flow rates in ducts where flow conditions may be irregular and nonuniform. The gas flowing in the duct is considered to be an ideal gas. The method may be especially useful in those locations where conventional pitot tube or thermal anemometer velocity measurements are difficult or inappropriate due either to very low average flow velocity or the lack of a suitable run of duct upstream and downstream of the measurement location.

5.2 This test method can produce the volumetric flow rate at standard conditions without the need to determine gas stream composition, temperature, and water vapor content.

5.3 This test method is useful for determining mass or volumetric flow rates in HVAC ducts, fume hoods, vent stacks, and mine tunnels, as well as in performing model studies of pollution control devices.

5.4 This test method is based on first principles (conservation of mass) and does not require engineering assumptions.

5.5 This test method does not require the measurement of the area of the duct or stack.

5.6 The test method does not require flow straightening.

5.7 The test method is independent of flow conditions, such as angle, swirl, turbulence, reversals, and hence, does not require flow straightening.

5.8 The dry volumetric airflow can be determined by drying the air samples without measuring the water vapor concentration.

6. Apparatus

6.1 The apparatus includes a source of tracer gas, means for distributing the tracer gas in the duct, means for obtaining air samples from the duct, and a gas analyzer to measure tracer gas concentrations in the air samples.

6.2 *Tracer Gas*—See Appendix X1 for information on tracer gases and equipment used to measure their concentrations. Appendix X1 also contains tracer gas target concentrations and safety information.

6.3 *Tracer Gas Injection Source*—This normally is a cylinder of compressed tracer gas either pure or diluted in a carrier such as air or nitrogen. Tracer release from the cylinder is controlled by a critical orifice or nozzle, a metering valve, an electronic mass flow meter or mass flow controller, or other gas flow rate measurement and control device. A rotameter is not recommended for this measurement unless of special design, calibration, and a corresponding decrease in measurement accuracy is acceptable.

6.4 *Tracer Gas Distribution*—A single tube or a tubing network is inserted into the duct to dispense tracer gas. The tube or tubes may have either a single or multiple release points for tracer gas. For large cross-section ducts a network that distributes tracer gas over a wide area will facilitate measurement.

6.5 *Tracer Sampling*—This is performed using tubing inserted into the duct downstream of the injection point. A single tube is inserted into the duct. Air samples are removed from the duct by means of a sampling pump to distribute tracer laden air to the analyzer either directly or by means of syringe samples.

6.6 *Gas Analyzer*—This device must be suited for the tracer gas used and the concentrations expected in the duct being measured. It should be calibrated properly and exhibit a accuracy of better than ± 3 % at concentrations employed in the measurement.

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[4] Equations in this test method assume that all mass or volume concentrations are in the same units.

[5] Equations in this test method assume that all mass or volume flow rates are in the same units.

[6] Equations in this test method assume that all densities are in the same units.

7. Hazards

7.1 Safety is the responsibility of the user of this test method. Tracer gases have safe maximum concentration limits due to health and, in some cases, explosive potential. Table 1 presents, as a guide, the maximum allowable concentration in air for some tracer gasses that can be used for airflow measurements. The tracer gas supplier must provide a Material Safety Data Sheet (MSDS) that will provide information about health, fire, and explosion hazards.

7.2 *Health Limitations*—Use current OSHA information on the permissible exposure limit (PEL), or the ACGIH threshold limit value (TLV) if the particular tracer is not listed with a PEL, to determine the safe concentration for the gas chosen for the test. Never exceed the maximum safe concentration. It is good practice to use a concentration that is at most one tenth of the maximum safe concentration. Avoid using tracer gases for which no PEL or TLV exists.

7.3 *Compressed Gas Equipment*—Observe the supplier's safety information and CGA information on the transportation, use, and storage of compressed gas cylinders, regulators, and related equipment.

8. Procedure for Measuring Mass and Volumetric Flowrate

8.1 Inject tracer of known concentration, $C_I(c_I)$, and at a known rate, $F_I(f_I)$, into a flowing duct using procedures provided in Section 9.

TABLE 2 Minimum Number of Down Stream Sample Locations

Duct Cross Sectional Area $m^2(ft^2)$	Number of Areas	Number of Samples
Less then 0.2 (2)	4	5
0.2 to 2.3 (2 to 25)	12	13
Greater than 2.3 (25)	20	21

8.1.1 If the tracer gas analyzer is field calibrated using a single point method, the injection rate, or injection concentration, or a combination thereof, should be adjusted to produce a concentration at the sample location that is the same as the calibration concentration to within ± 20 %.

8.1.2 If the tracer gas analyzer is field calibrated using two calibration points, the injection rate, or injection concentration, or a combination thereof, should be adjusted to produce a concentration at the sample location that lies between the two calibration points.

8.1.3 If the tracer gas analyzer is field calibrated using more than two calibration points, the injection rate, or injection concentration, or a combination thereof, should be adjusted to produce a concentration at the sample location that lies at the approximate midpoint of the calibration range.

8.2 Obtain at least N measurements of the resulting concentrations, C^i_D, at least ten diameters, or equivalent hydraulic diameters for nonround cross section ducts, downstream of the injection at the center of N-1 equal areas of the duct cross section and one at the center of the duct. The number N is determined by Table 2 depending on the duct size.

TABLE 1 Tracer Gases and Safety Issues

Tracer Gas	TLV[A]	Toxicity	Chemical Reactivity	Comments
Hydrogen	Asphyxiant	Nontoxic	Highly reactive in presence of heat, flame, of O_2	Fire and explosion hazard when exposer to heat, flame, or O_2
Helium	Asphyxiant	Nontoxic	Inert	
Carbon Monoxide	25 ppm	Combines with hemoglobin to cause anoxia	Highly reactive with O_2	Fire and explosion hazard when exposed to heat or flame
Carbon Dioxide	5000 ppm	Can be an eye irritant	Reacts vigorously with some metals; soluble in water	
Sulfur Hexafluoride	1000 ppm	Nontoxic	Inert	Thermal decomposition yields highly toxic compounds
Nitrous Oxide	25 ppm	Moderately toxic	Violent reaction with aluminum; water soluble	Can form explosive mixture with air; ignites at high temperature
Ethane	Asphyxiant	Nontoxic	Flammable	Incompatible with chlorine and oxidizing materials
Methane	Asphyxiant	Nontoxic	Flammable	Incompatible with chlorine and oxidizing materials
Octofluorocyclobutane (Halocarbon C-318	1000 ppm	Low toxicity	Nonflammable	Thermal decomposition yields highly toxic compounds
Bromotrifluoromethane (Halocarbon 13B1)	500 ppm	Moderately toxid by inhalation	Incompatible with aluminum	Dangerous in a fire
Dichlorodifluoromethane (Halocarbon 12)	1000 ppm	Central nervous system and eye irritant; can be narcotic at high levels	Nonflammable; can react violently with aluminum	Thermal decomposition yields highly toxic compounds
Dichlorotetrafluoromethane (Halocarbon 116)	1000 ppm	Can be asphyxiant, mildly irritating, narcotic at high levels	Can react violently with aluminum	Thermal decomposition yields highly toxic compounds

[A] *Threshold Limit Values for Chemical Substances in the Work Environment*, American Conference of Governmental Industrial Hygienists (ACGIH), 1997.

8.3 If recirculation is possible or likely, N samples $C^i{}_U$ in the center of duct upstream of the injection point should be taken within 10 s of the time a downstream sample is taken. If recirculation does not exist, take at least one upstream sample before and after taking the downstream samples.

8.4 At each time a downstream sample is taken, the injection flow rate $F^i{}_I$ shall be recorded.

8.5 Calculate the following quantities in either mass, volume, or dry concentration depending on results desired:

8.5.1 The average downstream concentration C_D:

$$C_D = \frac{1}{N_D} \sum_{i=1}^{N_D} C_D^i \tag{1}$$

where N_D is the number of downstream sample locations.

8.5.2 The average upstream concentration C_U:

$$C_U = \frac{1}{N_U} \sum_{i=1}^{N_U} C_U^i \tag{2}$$

where N_U is the number of upstream sample locations.

8.5.3 The average injection flow F_I (or the corresponding volumetric flow rate):

$$F_I = \frac{1}{N_I} \sum_{i=1}^{N_I} F_I^i \tag{3}$$

where N_I is the number of flow rate measurements.

8.5.3.1 The mass flow rate in the duct is given by:

$$F_U = \frac{(C_I - C_D)}{(C_D - C_U)} F_I \text{ (mass concentrations)} \tag{4}$$

8.5.3.2 The volumetric flow rate in the duct is given by:

$$f_U = \frac{(C_I - C_D)}{(C_D - C_U)} \frac{\rho_I}{\rho_U} f_I \text{ (mass concentrations)} \tag{5}$$

$$f_U = \frac{c_I - r_{na}c_D - (1 - r_{na})c_I c_D}{c_D - c_U)} \cdot \frac{\rho_t^I}{\rho_t^U} \cdot f_I \text{ (volume concentrations)} \tag{6}$$

where $r_{na} \equiv \frac{\rho_n}{\rho_a}$ is the ratio of the density of the main constituent of the injection gas mixture to the density of the gas stream without any tracer. If $c_I = 1$, than $r_{na} = 1$.

8.6 The volumetric flow rate in the duct at standard conditions is given by:

$$f_U^{std} = \frac{c_I - r_{na}c_D - (1 - r_{na})c_I c_D}{c_D - c_U)} \cdot f_I^{std} \text{ (volume concentrations)} \tag{7}$$

8.7 The dry gas flow rate in the duct at standard conditions is given by:

$$f_U^{(d)std} = \frac{c_I}{(c_D^{(d)} - c_U^d)} \cdot f_I^{std} \text{ if } \frac{c_I}{c_D} < 0.001 \text{ (dry volume concentrations)} \tag{8}$$

where the superscript (d) refers to quantities at dry conditions.

8.7.1 Dry volume concentrations are obtained by drying the gas sample before analysis. It is important that the drying technique used should not remove any of the tracer gas. This can be checked by drying a sample of the calibration standard using the drying techniques and comparing the measured dry concentrations with the calibration standard. The two concentrations should be the same within the precision of the analyzer as determined in Appendix X2.

9. Procedures for Injecting Tracer Gas

9.1 Inject tracer gas at a known, constant rate using metered injection. To accomplish this a critical orifice, critical orifice metering valve, an electronic mass flow meter or an electronic mass flow controller may be used in conjunction with a source of pure or diluted tracer gas. The flow measuring device shall be calibrated and its accuracy certified by a method that is traceable to NIST. The calibration shall be performed with an injection gas of the same approximate concentration as will be used in an actual measurement.

9.1.1 Reliance on scale factors to convert a flow meter calibration using one gas to predict the calibration on a second gas is not acceptable unless the accuracy of the conversion factor has been demonstrated experimentally for the type of meter used.

9.1.2 The total uncertainty (uncertainty in flowrate and uncertainty in injection concentration) in the tracer gas injection rate shall be less than 3 %. The bias of the assumed injection rate shall be no more than 3 % of the true rate.

9.2 Injection of tracer gas may be via a single tube or via a manifold consisting of several tubes connected to the flow injection metering device. In the case of a manifold, flow through each branch of the manifold should be approximately equal. The tube or tubes may have either a single or multiple release points for tracer gas.

10. Procedures for Sampling Tracer Gas

10.1 Sampling is performed using a single tube connected to a pump that draws air samples to the exterior of the duct for analysis. Use a separate sampling tube for the downstream and for the upstream air sample.

10.2 Samples of air may be routed directly to the analyzer or grab samples using syringes, sample bags, or other appropriate containers may be taken for subsequent analysis.

10.3 If good mixing[7] is not obtained, enhance the mixing by one of the following procedures:

10.3.1 Move[8] the sample point further downstream from the tracer injection point. When flowing air encounters a rapid change in direction, mixing within the duct is assisted; hence, moving the sample point to a location past one or more bends in the flow will enhance mixing.

10.3.2 Move[8] the tracer injection point further upstream from the sample point. When flowing air encounters a rapid change in direction, mixing within the duct is assisted; hence, moving the injection point to a location past one or more bends in the flow will materially enhance mixing. Often moving the injection point upstream of an air handling fan can enhance mixing. Note that centrifugal fans are not as efficient at mixing as are vaneaxial fans.

10.3.3 Enhance the uniformity of the tracer injection by increasing the number of injection tubes or the number of injection holes in each tube.

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[7] The lack of good mixing results in an increase in the uncertainty in the determination of the duct airflow rate (see Eq 11). Good practice usually results in a variation in tracer concentration across the duct of less than 10 %.

[8] In moving the sample or injection locations, care should be taken so that there is no leakage into or out of the duct between the point of injection and the downstream sample locations.

11. Tracer Gas Analysis Requirements

11.1 The tracer gas analyzer shall be calibrated using calibration standards that bracket the anticipated measured concentration using the procedures given in Appendix X2.

11.2 Determine the accuracy of the gas analyzer in order to estimate the accuracy of a given concentration reading. Calibrate the analyzer with several tracer gas concentration standards to estimate bias. Biases of samples exhibiting low tracer gas concentrations (relative to the ultimate sensitivity of the analyzer) and deviations in the detector response from the empirical relationship between concentration and analyzer output all affect the accuracy of the gas analyzer. A gas analyzer whose bias is less than 1 % for all concentrations encountered in the measurement may be assumed to be unbiased for the purpose of this test method.

11.3 The precision of the gas analyzer is a measure of reproducibility. Make an estimate of the precision of the gas analyzer by using replicate measurements. See Appendix X2 for details on assessing analyzer precision.

11.4 The zero concentration response of the gas analyzer shall be demonstrated by injecting a zero tracer concentration air mixture into the analyzer and noting the analyzer response, if any.

11.5 It must be demonstrated using the procedure of Appendix X3 that the gas analyzer response is not affected by potential interference due to other gases that may be present in the duct. To demonstrate this one can sample the duct in the absence of tracer and note the analyzer response, if any.

12. Calculation of Test Errors

12.1 The uncertainty of the test results depends on the instrumentation used and on the mixing obtained in the test. The bias, ΔF, of the results is given by the following equation:

$$\frac{\Delta F}{F} = \sqrt{\left(\frac{\Delta C_I}{C_I}\right)^2 + \left(\frac{\Delta F_I}{F_I}\right)^2 + \frac{(\Delta C_D)^2 + (\Delta C_U)^2}{(C_D - C_U)^2}} \tag{9}$$

where:

ΔC_I = the uncertainty in the injection gas concentration,
ΔF_I = the uncertainty in the injection flow rate,
$\Delta\ C_D$ = the calibration uncertainty in the downstream concentration (Appendix X2), and
ΔC_U = the calibration uncertainty in the upstream concentration (Appendix X2).

12.2 The precision ∂F in the flow is given by the following equation:

$$\frac{\partial F}{F} = t(N - 1{,}0.95)\sqrt{\frac{\partial F_I^{\,2}}{(F_I)^2} + \frac{\partial(C_D - C_U)^2}{(C_D - C_U)^2}} \tag{10}$$

where:

$\partial F_I^2 = \frac{1}{N-1}\sum_{i=1}^{N}(F_I^i - F_I)^2$ is the square of the standard deviation of the injection flow rate, and

$\partial(C_D - C_U)_I^2 = \frac{1}{N-1}\sum_{i=1}^{N}(C_D^i - C_U^i - C_D + C_U)^2$ is the square of the standard deviation of the difference in the downstream and upstream tracer concentration and t (N-1,0.95) is the two-sided confidence limits of the Student Distribution for N-1 and probability 0.95.

The total uncertainty, $\frac{\partial_T F}{F}$, is the square root of the sum of the squares of the bias and the precision.

$$\frac{\partial_T F}{F} = \sqrt{\left(\frac{\Delta F}{F}\right)^2 + \left(\frac{\partial F}{F}\right)^2} \tag{11}$$

12.3 Note that Eq 11 is equivalent to the measurement uncertainty derived in ANSI/PTC 19.1.

13. Precision and Bias

13.1 The precision and error of the test results depend on the precision and error of the instrument and tracer gases used. The precision and bias in this test method can be assessed using the error analysis of Section 12 if the precision and error of each instrument and tracer gas is known.

14. Keywords

14.1 ducts; flow rate; mass flow; stacks; volumetric flow; tracer gas

APPENDIXES

(Nonmandatory Information)

X1. Tracer Gas Analysis Methods and Specifications

X1.1 Tracer gas analysis methods and specifications are listed in Table X1.1.

TABLE X1.1 Tracer Gas Analysis Methods and Specifications

Tracer Gas	Ambient Levels	Measurement Techniques	Detection Levels
Hydrogen	0.5 ppm	Katharometer	200 ppm
Helium	5.2 ppm	Katharometer	300 ppm
Carbon Monoxide	0.1 - 1ppm	Infrared absorption	5 ppm
Carbon Dioxide	320 ppm	Infrared absorption	1 ppm
Sulfur Hexafluoride	1 ppt	Electron capture detector	2 ppt
Nitrous Oxide	0.3 ppm	Infrared absorption	1 ppm
Ethane	1.5 ppb	Flame ionization detector	5 ppm
Methane	1.5 ppb	Flame ionization detector	5 ppm
Octofluorocyclobutane (Halocarbon C-318)	[A]	Electron capture detector	5 ppb
Bromotrifluoromethane (Halocarbon 13B1)	[A,B]	Electron capture detector	0.1 ppb
Dichlorodifluoromethane (Halocarbon 12)	[A]	Electron capture detector Flame ionization detector	0.6 ppm
Dichlorotetrafluoromethane (Halocarbon 116)	[A]	Electron capture detector Flame ionization detector	0.3 ppm

[A] Below detection limit.

[B] Locally variable.

X2. PROCEDURE FOR CALIBRATION OF THE GAS ANALYZER

X2.1 Detailed Calibration of the Gas Analyzer

X2.1.1 At an interval specified by the gas analyzer manufacturer, which is not more than one year, a complete calibration of the gas analyzer should be performed. The specific calibration procedure for a gas analyzer should be provided by the instrument manufacturer. After the calibration, the gas analyzer should be adjusted according to the instructions of the manufacture using certified gas calibration standards. After this adjustment, the following procedure should be performed. Using at least five certified calibration mixtures with concentrations approximately equally space over the intended range of use of the analyzer, perform the following:

X2.1.1.1 With each calibration standard $C^s{}_i$ obtain at least three readings $C^r{}_{i,j}$.

X2.1.1.2 Calculate the average $\bar{C}^r{}_i$ and the standard deviation $\partial C^r{}_i$ using the following equation:

$$\bar{C}_i^r = \frac{1}{m}\sum_{j=1}^{m} C_{i,j}^r \tag{X2.1}$$

$$(\partial C_i^r)^2 = \frac{1}{m-1}\sum_{j=1}^{m} (C_{i,j}^r - \bar{C}_i^r)^2 \tag{X2.2}$$

where m is the number of readings. The precision of the analyzer at this calibration point shall be considered as

$$prec^i = t(m-1{,}0.95)\sqrt{(\partial C_i^r)^2} \tag{X2.3}$$

where t (N,p) is the two-sided confidence limits of the Student Distribution.

TABLE X2.1 Two-Sided Confidence Limits T (p, N) for a Student Distribution

p	0.8	0.9	0.95	0.99	0.995	0.999
N						
1	3.078	6.3138	12.706	63.657	127.32	636.619
2	1.886	2.9200	4.3027	9.9248	14.089	31.598
3	1.638	2.3534	3.1825	5.8409	7.4533	12.924
4	1.533	2.1318	2.7764	4.6041	5.5976	8.610
5	1.476	2.0150	2.5706	4.0321	4.7733	6.869
6	1.440	2.0150	2.5706	4.0321	4.7733	6.869
7	1.415	1.8946	2.3646	3.4995	4.0293	5.408
8	1.397	1.8595	2.3060	3.2498	3.6897	4.781
9	1.383	1.8331	2.2622	3.2498	3.6897	4.781
10	1.372	1.8125	2.2281	3.1693	3.5814	4.587

X2.1.1.3 Calculate the uncertainty in the reading of the calibration standard using the following equation:

$$\Delta C_i^s = \sqrt{(\Delta C_i^s)^2 + (C_i^s - \bar{C}_i^r)^2} + t(m-1{,}0.95)\sqrt{(\partial C_i^r)^2} \tag{X2.4}$$

where t $(m-1{,}095)$ is the student t-function with a 0.95 confidence level; and, $\Delta C_i{}^s$ is the certified uncertainty in the calibration standard.

X2.1.1.4 Calculate the uncertainty $\frac{\Delta C}{C}$ in the calibration over the range of use of the instrument using the following equation:

$$\frac{\Delta C}{C} = avg\left(\frac{\Delta C_i^s}{C_i^s}\right) + t(n-1,0.95)std\left(\frac{\Delta C_i^s}{C_i^s}\right) \tag{X2.5}$$

where n is the number of calibration points and

$$avg\left(\frac{\Delta\ C_i^s}{C_i^s}\right) = \frac{1}{n}\sum_{i=1}^{n}\frac{\Delta C_i^s}{C_i^s} \tag{X2.6}$$

is the average of the calibration errors and

$$std\left(\frac{\Delta C_i^s}{C_i^s}\right) = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{\Delta\ C_i^s}{C_i^s} - avg\left(\frac{\Delta C_i^s}{C_i^s}\right)\right)^2} \tag{X2.7}$$

is the standard deviation of the calibration errors.

X2.2 Field Calibration of the Gas Analyzer

X2.2.1 *Single Point Field Calibration*—For use at a single flow rate, the gas analyzer should be field calibrated before it is used to make flow measurements having the resulting downstream concentration within 20 % of calibration standard. The field calibration standard should be within the range of standards used for the detailed calibration of X2.1. The single point field calibration should be done by taking at least three readings of a certified calibration gas and adjusting the analyzer to read the average of the reading. The standard deviation of the readings should be less then the laboratory determined precision at this calibration point.

X2.2.2 *Two Point Field Calibration*—For use over an extended rate range, the gas analyzer should be field calibrated before it is used to make flow measurements using two calibration standards. The two field concentrations should be within the range of the detailed calibration of X2.1. The resulting downstream concentrations should be greater than 0.75 times the concentration of the low calibration standard and less than 1.25 times the concentration of the high calibration standard. The two point field calibration should be done by taking at least three readings of each certified calibration gas standard and adjusting the analyzer to read the average of the each set of readings. The standard deviation of the each set of readings at the two calibration points should be less then the laboratory determined precision at each calibration point.

X3. Derivation of Tracer Dilution Equations for Duct Flow Measurements

X3.1 Determining Flow Rate

X3.1.1 The equations for determining the flow rate in a duct by the tracer dilution method can be derived by considering the following experiment in which one injects a tracer gas into a duct in order to determine the total flow of gas in the duct. In this experiment one will inject the tracer gas mixture of mass concentration C_I with a mass flow rate of F_I.

X3.1.2 The bases of the tracer dilution equations for a duct are the two integral conservation of mass equations:

Conservation of mass for air-tracer mixture:

$$\frac{d}{dt}\int_V \rho dv - \int_{\partial V} \rho \vec{v} \cdot d\,\vec{a} = F_I \tag{X3.1}$$

Conservation of mass for the tracer:

$$\frac{d}{dt}\int_V \rho C dv - \int_{\partial V} \rho C \vec{v} \cdot d\,\vec{a} = C_I F_I \tag{X3.2}$$

where:

ρ = the density of air-tracer mixture,
v = the air velocity,
C = the mass concentration of the tracer,
F_I = the mass injection rate of the tracer mixture into the duct,
C_I = the mass concentration of the injected tracer mixture,
V = a volume of duct enclosing the point of injection, and
∂V = the enclosing surface of V.

NOTE X3.1—In normal tracer experiments, the amount of tracer injected into the duct is so small that the term F_I on the right-hand side of (X3.1) is negligible, in comparison to the terms in surface integral on the left hand side; however, no additional complication occurs by including the term. Also, in principle the density ρ depends on the tracer concentration at a point; however, this dependence usually is very small. Again no additional complication is caused *by* including this dependence at this stage.

X3.1.3 If one selects a volume, V, which consists of two end cross sections and the side surfaces of the duct:

$$\partial V = S \cup A_D \cup A_U, \tag{X3.3}$$

then, under the assumption that no mass enters or leaves the duct through the side walls:

$$\vec{n} \cdot \vec{n} = 0 \text{ on } S\,, \tag{X3.4}$$

the surface integrals in Eq X3.1 and X3.2 reduce to:

$$\int_{\partial V} \rho \vec{v} \cdot d\,\vec{a} = \int_{A_D} \rho \vec{v} \cdot d\,\vec{a} - \int_{A_U} \rho \vec{v} \cdot d\,\vec{a} \tag{X3.5}$$

$$\int_{\partial V} \rho C \vec{v} \cdot d\,\vec{a} = \int_{A_D} \rho C \vec{v} \cdot d\,\vec{a} - \int_{A_U} \rho C \vec{v} \cdot d\,\vec{a} \tag{X3.6}$$

X3.1.4 Using the mean value theorem, the two integrals on the right hand side of (Eq X3.6) can be written as:

$$\int_{A_D} \rho C \vec{v} \cdot d\,\vec{a} = C_D \int_{A_D} \rho \vec{v} \cdot d\,\vec{a} \tag{X3.7}$$

$$\int_{A_U} \rho C \vec{v} \cdot d\,\vec{a} = C_U \int_{A_U} \rho \vec{v} \cdot d\,\vec{a} \tag{X3.8}$$

where:

C_D = the tracer concentration at some point on downstream area, A_D, and
C_U = the tracer concentration at some point on the upstream area, A_U.

X3.1.5 If one defines the quantities:

$$F_D = \int_{A_D} \rho \vec{v} \cdot d\,\vec{a} \tag{X3.9}$$

$$F_U = \int_{A_U} \rho \vec{v} \cdot d\,\vec{a} \qquad \text{(X3.10)}$$

where:

F_D = the downstream total mass flow rate through the duct, and

F_U = the upstream total mass flow rate through the duct and also introduces the quantity m_v, the total mass in the volume V is:

$$m_V = \int_V \rho dv \qquad \text{(X3.11)}$$

and uses the mean value theorem to express:

$$\int_V \rho C dv = C_V \int_V \rho dv = C_V m_V \qquad \text{(X3.12)}$$

where C_v is the tracer concentration at some point is the volume, V, Eq X3.1 and X3.2 can be written as follows:

$$\frac{d}{dt} m_V + F_D - F_U = F_I \qquad \text{(X3.13)}$$

and

$$m_V \frac{d}{dt} C_V + C_V \frac{d}{dt} m_V + C_D F_D - C_U F_U = C_I F_I \qquad \text{(X3.14)}$$

X3.1.6 Eliminating the downstream total mass flow from Eq X3. one obtains:

$$m_V \frac{d}{dt} C_V + (C_V - C_D) \frac{d}{dt} m_V + (C_D - C_U) F_U = (C_I - C_D) F_I \qquad \text{(X3.15)}$$

X3.1.7 The normal application of the tracer dilution technique is in the case of incompressible or steady total flow, or both, for which:

$$\frac{d}{dt} m_V = 0 \qquad \text{(X3.16)}$$

in which case, (Eq X3.15) reduces to:

$$m_V \frac{d}{dt} C_V + (C_D - C_U) F_U = (C_I - C_D) F_I \qquad \text{(X3.17)}$$

X3.1.8 The tracer gas is injected at a constant flow rate until the tracer concentration does not change with time:

$$\frac{d}{dt} C_V = 0 \qquad \text{(X3.18)}$$

in which case, (Eq X3.17) can be solved for F_U as follows:

$$F_U = \frac{(C_I - C_D)}{(C_D - C_U)} F_I \qquad \text{(X3.19)}$$

NOTE X3.2—In normal tracer dilution tests:

$$C_D << C_I \qquad \text{(X3.20)}$$

the C_D in the denominator of (Eq X3.19) can be neglected.

X3.2 Bounds on the Flow

X3.2.1 One can use (Eq X3.19) to develop bounds on the estimate of the flow F_U

$$\frac{(C_I^{min} - C_D^{max})}{(C_D^{max} - C_U^{min})} F_I^{min} < F_U < \frac{(C_I^{max} - C_D^{min})}{(C_D^{min} - C_U^{max})} F_I^{max} \qquad \text{(X3.21)}$$

X3.3 Volumetric Flow and Standard Conditions

X3.3.1 The mass flow rate can be expressed in terms of the volumetric flow rate at either test or standard conditions using the relations:

$$F_U = \rho_U f_U = \rho_U^{std} f_U^{std} \qquad \text{(X3.22)}$$

$$F_I = \rho_I f_I = \rho_I^{std} f_I^{std} \qquad \text{(X3.23)}$$

where:

f_U = the volumetric upstream flow rate at the upstream conditions,

f_I = the volumetric injection flow rate at the injection conditions,

f_U^{std} = the volumetric upstream flow rate at the standard conditions,

f_I^{std} = the volumetric injection flow rate at the standard conditions,

ρ_U = the upstream mass density at the upstream conditions,

ρ_I = the injection gas mass density at the injection conditions,

ρ_U^{std} = the upstream mass density at the standard conditions, and

ρ_I^{std} = the injection gas mass density at the standard conditions.

X3.3.2 Using these relations the volumetric flow can be obtained from:

$$f_U = \frac{(C_I - C_D)\ \rho_I}{(C_D - C_U)\rho_U} f_I \qquad \text{(X3.24)}$$

$$f_U^{std} = \frac{(C_I - C_D)\ \rho_I^{std}}{(C_D - C_U)\rho_U^{std}} f_I^{std} \qquad \text{(X3.25)}$$

X3.4 Concentration Units

X3.4.1 The above equations were expressed in mass concentrations. There are many other concentration units: volume concentrations, molar concentrations (or fraction), mass per unit volume or volume per unit mass. The following are the relationships between these concentrations:

$$C = \frac{\rho_t c}{\rho_a (1 - c) + \rho_t c} \qquad \text{(X3.26)}$$

where:

c = the volume concentration,

C = the mass concentration,

ρ_t = the density of the tracer, and

ρ_a = the density of the other component (air):

$$c = \frac{x v_t}{(1 - x) v_a + x v_t} \qquad \text{(X3.27)}$$

where:

x = the molar concentration of the tracer,

v_t = the molar volume of the tracer gas, and

v_a = the molar volume of the other component of the mixture (usually air).

According to Avogrado's principles, a mole of a gas at the same temperature and pressure has the same volume (1g/mole occupies 22.41 L at standard conditions of pressure and temperature). This is strictly true for ideal gases and true to better than 1 % for most gases; thus, for an ideal gas or two gases with the same molar volume:

$$c = x \text{ (ideal gas)} \qquad \text{(X3.28)}$$

Also, for two ideal gases, the ratio of the density of the gases is independent of pressure and temperature and equal to the ratio of the molecular weights of the gases; therefore, for ideal gases, the mass concentration, *C*, and the volume concentration, *c*, are independent of temperature and pressure and the volume concentration *c* is equal to the molar concentration *x*.

X3.5 Equations in Volume Concentrations

X3.5.1 If the injection gas mixture is a mixture of a gas *n* and the tracer *t* and the gas in the duct is a mixture of the gas *a* and the tracer *t*, then introducing the density ratios:

$$r_{ta} \equiv \frac{\rho_t}{\rho_a} \quad \text{(X3.29)}$$

$$r_{tn} \equiv \frac{\rho_t}{\rho_n} \quad \text{(X3.30)}$$

(Eq X3.24) can be written as follows:

$$F_U = \frac{c_I - r_{nt}^{I} r_{ta}^{D} + (r_{nt}^{I} \cdot r_{ta}^{D} - 1)c_I C_D}{r_{at}^{U} r_{ta}^{D} c_D - c_U + (1 - r_{at}^{U} \cdot r_{ta}^{D})c_U c_D} \cdot \frac{\rho_t^I}{\rho_t^U} \cdot f_I \quad \text{(X3.31)}$$

where the superscripts *I*, *D*, and *U* mean the properties at the injection conditions *I*, the downstream conditions *D* and the upstream conditions *U*. For mixtures of ideal gases, the ratio of the densities of the gases is independent of pressure and temperature:

$$r_{ta}^{D} = r_{ta}^{U} \equiv r_{ta} \quad \text{(X3.32)}$$

and (Eq X3.31) reduces to:

$$f_U = \frac{c_I - r_{na}c_D - (1 - r_{na})c_I c_D}{c_D - c_U} \cdot \frac{\rho_t^I}{\rho_t^U} \cdot f_I \text{ (ideal gases)} \quad \text{(X3.33)}$$

or at standard conditions, (Eq X3.33) reduces to:

$$f_U^{std} = \frac{c_I - r_{na}c_D - (1 - r_{na})c_I c_D}{c_D - c_U} \cdot f_I^{std} \text{ (ideal gases)} \quad \text{(X3.34)}$$

NOTE X3.3—Most flow meters assume that the gases measured are ideal gases and correct for non-standard conditions using ideal gas laws.

X3.5.2 In most tracer gas measurements, the dilution concentrations are several order of magnitudes less than the injection concentration:

$$c_U < c_D << c_I < 1 \quad \text{(X3.35)}$$

In this case, (Eq X3.34) becomes:

$$f_U^{std} = \frac{c_I}{c_D - c_U} \cdot f_I^{std} \text{ (ideal gas and (X3.))} \quad \text{(X3.36)}$$

X3.5.3 The error in using (Eq X3.36) instead of (Eq X3.34) is of the order c_D/ c_I, which in terms of flows, is of the oreder of f_I/ f_U.

X3.6 Dry Gas Flow and Concentrations

X3.6.1 In many applications, one is either interested in the dry gas volumetric flow (EPA methods), or one must (due to condensation problems) dry gas before measuring the tracer gas concentrations. If one considers a mixture of dry air, tracer and water vapor then:

$$c = \frac{v_t}{v_a + v_w + v_t} \quad \text{(X3.37)}$$

and:

$$c^{(d)} = \frac{v_t}{v_a + v_t} \quad \text{(X3.38)}$$

from which:

$$c = c^{(d)}(1 - c_w) \quad \text{(X3.39)}$$

where c_w is the water vapor volume concentration (or fraction):

$$c_w = \frac{v_w}{v_a + v_w + v_t} \quad \text{(X3.40)}$$

X3.6.2 If the upstream and downstream water vapor concentrations are same, then (Eq X3.34) becomes in terms of the dry tracer concentrations:

$$f_U^{std} = \frac{c_I}{(c_D^{(d)} - c_U^{(d)})(1 - c_w)} \cdot f_I^{std} \text{ (ideal gas and (Eq X3.35)} \quad \text{(X3.41)}$$

The dry volumetric air flow is defined as:

$$f_U^{(d)std} \equiv (1 - c_w) f_U^{std} \text{ dry volumetric air flow at standard condtions} \quad \text{(X3.42)}$$

and therefore:

$$f_U^{(d)std} = \frac{c_I}{(c_D^{(d)} - c_U^{d})} \cdot f_I^{std} \text{ (ideal gas and (Eq X3.35)} \quad \text{(X3.43)}$$

X3.6.3 The significance of (Eq X3.43) is that the dry volumetric air flow can be determined by drying the air samples and measuring the resulting dry tracer concentrations without measuring the water vapor concentration.

X4. ACCURACY OF COMPONENTS OF THE MEASUREMENT SYSTEMS

X4.1 Accuracy of Tracer Injection Concentration c_I

X4.1.1 Gas mixtures can be obtained in various grades of accuracy and certification. One only should use analyzed gas mixtures for injection which have a certificate of accuracy, which is traceable to NIST. The level of accuracy that can be achieved with current technology is given in Table X4.1.

TABLE X4.1 Accuracy of Tracer Injection Concentration, c_I

Gas Concentration	Accuracy
Pure	
Commercial pure	0.2 % wt
Instrument grade	0.003 % wt
>2 %	0.02 % absolute
2 % > 1ppm	1 % of composition
1ppm > 100 ppt	2 % of composition[A]

[A] It is possible now to achieve 1 % accuracy in the higher ppb range with special instruments, but this is not typical of what is currently being supplied.

X4.2 Accuracy of Injection Flow f_I

X4.2.1 The injection flow meters used in this test method should have a current calibration certificate traceable to NIST at an interval recommended by the manufacturer but no more than one year. Typical accuracies, which currently can be achieved, are indicated in Table X4.2.

TABLE X4.2 Accuracy of Injection Flow, f_I

Meter Type	NIST Traceable Calibration	Typical Manufacturer
Sonic nozzles	0.25 % of reading	1 % of reading
Electronic mass flow meters Full-scale from 0.5 cc/min to 100 4 min	1 % of reading from 20 % to 100 % full scale	2 % full scale

X4.2.2 Injection meters must be calibrated with the mixture of gas injected if c_I is greater than 0.1 %. The injection flow meters should be calibrated at temperatures and pressures near those used in the test.

X4.3 Accuracy of Tracer Calibration Gases

X4.3.1 Calibration gas mixtures can be obtained in various grades of accuracy and should be certified. One only should use analyzed gas mixtures for calibration, which have a certificate of accuracy, which is traceable to NIST. The level of accuracy, which can be achieved with current technology, is given in Table X4.3. Noncertified dilution mixtures should not be used for calibration.

TABLE X4.3 Accuracy of Tracer Calibration Gases

Tracer Gas Composition	Accuracy
> 1 ppm	1 % of composition
1 ppm > 100 ppt	2 % of composition[A]

[A] It is possible now to achieve 1 % accuracy down to the ppb range with special instruments and significantly increased expense, but this is not typical of what is currently being supplied.

X5. PROCEDURE FOR ASSESSING THE POTENTIAL FOR INTERFERENCE FROM CONSTITUENTS IN THE GAS STREAM TESTED

X5.1 Procedure:

X5.1.1 The potential for interference in the response of the analyzer to other components of the gas stream should be determined by the following procedure. After the analyzer has been field calibrated according to Appendix X2, three readings of a certified zero gas should be made. Then, three readings of the gas stream should be made without any tracer gas. The difference in the average of the gas stream reading and the average of the zero gas reading should be less than 0.01 times the high calibration reading obtained in Appendix X2.