INTERNATIONAL

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Standard Practice for Calculation of Color Tolerances and Color Differences from Instrumentally Measured Color Coordinates[1]

This standard is issued under the fixed designation D 2244; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the Department of Defense.

ϵ^1 NOTE—Equation 38 was editorially corrected in July 2003.

INTRODUCTION

This practice originally resulted from the consolidation of a number of separately published methods for the instrumental evaluation of color differences. As revised in 1979, it included four color spaces in which color-scale values could be measured by instruments, many of which were obsolete, and the color differences calculated by ten equations for different color scales. The sections on apparatus, calibration standards and methods, and measurement procedures served little purpose in the light of modern color-measurement technology. The revision published in 1993 omitted these sections, and limited the color spaces and color-difference equations considered, to the three most widely used in the paint and related coatings industry. This revision adds two new color tolerance equations and puts two of the color difference equations from the 1993 version in an informative annex for historical purposes. The Hunter L_H, a_H, b_H and FMC-2 color difference equations are no longer recommended. This revision also changes the status of the standard from test method to practice.

1. Scope

1.1 This practice covers the calculation, from instrumentally measured color coordinates based on daylight illumination, of color tolerances and small color differences between opaque specimens such as painted panels, plastic plaques, or textile swatches. Where it is suspected that the specimens may be metameric, that is, possess different spectral curves though visually alike in color, Practice D 4086 should be used to verify instrumental results. The tolerances and differences determined by these procedures are

[1] This practice is under the jurisdiction of ASTM Committee E12 on Color and Appearance and is the direct responsibility of Subcommittee E12.04 on Color and Appearance Analysis.

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expressed in terms of approximately uniform visual color perception in CIE 1976 CIELAB opponent-color space **(1)**[2], CMC tolerance units **(2)**, CIE94 tolerance units **(3)**, the DIN99 color difference formula given in DIN 6176 **(4)**, or the new CIEDE2000 color difference units **(5)** . The color differences based on Hunter L_H, a_H, b_H opponent-color space **(6)**, or the Friele-MacAdam-Chickering (FMC-2) color space **(7)**, are no longer recommended for industrial practice.

1.2 For product specification, the purchaser and the seller shall agree upon the permissible color tolerance between test specimen and reference and the procedure for calculating the color tolerance. Each material and condition of use may require specific color tolerances because other appearance factors, (for example, specimen proximity, gloss, and texture), may affect the correlation between the magnitude of a measured color difference and its commercial acceptability.

1.3 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory requirements prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 1729 Practice for Visual Appraisal of Colors and Color Differences of Diffusely-Illuminated Opaque Materials[3]
D 4086 Practice for Visual Evaluation of Metamerism[3]
E 284 Terminology of Appearance[3]
E 308 Practice for Computing the Colors of Objects by Using the CIE System[3]
E 805 Practice for Identification of Instrumental Methods of Color or Color-Difference Measurement of Materials[3]
E 1164 Practice for Obtaining Spectrophotometric Data for Object-Color Evaluation[3]

2.2 *Other Standards:*

DIN 6176 Farbmetrische, Bestimmung von Farbabständen bei Körperfarben nach der DIN99-Formel[4]

3. Terminology

3.1 Terms and definitions in Terminology E 284 are applicable to this practice.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *colorimetric spectrometer*, *n*—spectrometer, one component of which is a dispersive element (such as a prism, grating or interference filter or wedge or tunable or discrete series of monochromatic sources), that is normally capable of producing as output colorimetric data (such as tristimulus values and derived color coordinates or indices of appearance attributes). Additionally, the colorimetric spectrometer may also be able to report the underlying spectral data from which the colorimetric data were derived.

3.2.1.1 *Discussion*—At one time, UV-VIS analytical spectrophotometers were used for colorimetric measurements. Today, while instruments intended for use in color measurements share many common components, UV-VIS analytical spectrophotometers are designed to optimize their use in chemometric quantitative analysis, which requires very precise spectral position and very narrow bandpass and moderate baseline stability. Colorimetric spectrometers are designed to optimize their use as digital simulations of the visual colorimeter or as the source of spectral and colorimetric information for computer-assisted color matching systems. Digital colorimetry allows more tolerance on the spectral scale and spectral bandwidth but demand much more stability in the radiometric scale.

3.2.2 *color tolerance equation*, *n*—a mathematical expression, derived from acceptability judgments, which distorts the metric of color space based on the coordinates in that color space, of a reference color, for the purpose of single number shade passing.

3.2.2.1 *Discussion*—The color tolerance equation computes a pass/fail value based on which of the pair of specimens is assigned the designation "standard." Thus, inter-changing the reference and test specimens will result in a change in the predicted level of acceptance between the specimens while the perceived difference is unchanged. A color difference equation quantifies distance in a color space using the metric of that space. Inter-changing the reference and test specimens does not change either the perceived or predicted color differences.

4. Summary of Practice

4.1 The differences in color between a reference and a test specimen are determined from measurements made by use of a spectral based or filter based colorimeter. Reflectance readings from spectral instruments are converted by computations to color-scale values according to Practice E 308, or these color-scale values may be read directly from instruments that automatically make the computations. Color-difference units are computed, from these color-scale values, and approximate the perceived color differences between the reference and the test specimen.

5. Significance and Use

5.1 The original CIE color scales based on tristimulus values X, Y, Z and chromaticity coordinates x, y are not uniform visually. Each subsequent color scale based on CIE values has had weighting factors applied to provide some degree of uniformity so that

[2] The boldface numbers in parentheses refer to the list of references at the end of this standard.
[3] *Annual Book of ASTM Standards*, Vol 06.01.
[4] Available from Beuth Verlag GmbH, 10772 Berlin, Germany.

color differences in various regions of color space will be more nearly comparable. On the other hand, color differences obtained for the same specimens evaluated in different color-scale systems are not likely to be identical. To avoid confusion, color differences among specimens or the associated tolerances should be compared only when they are obtained for the same color-scale system. There is no simple factor that can be used to convert accurately color differences or color tolerances in one system to difference or tolerance units in another system for all colors of specimens.

5.2 For uniformity of practice, the CIE recommended in 1976 the use of two color metrics. The CIELAB metric, with its associated color-difference equation, has found wide acceptance in the coatings, plastics, textiles and related industries. While, it has not completely displaced the use of the Hunter L_H, a_H, b_H and the FMC-2 scales, their performance versus experienced visual color judgements is so poor, compared to the newer optimized tolerance equations based on the CIELAB coordinates, that they can no longer be recommended. Therefore, the two older scales are included in an Annex in this practice only for historical purposes. It is anticipated that in a future revision of this practice, the Annex may be deleted as well. The CIELAB metric, by itself, is also not recommended in this practice for use in describing small and moderate color differences (differences with magnitude less than 5.0 ΔE^*_{ab} units). The four more recently defined equations, documented here, are highly recommended for use with color-differences in the range of 0.0 to 5.0 ΔE^*_{ab} units.

5.3 Users of color tolerance equations have found that, in each system, summation of three, vector color-difference components into a single scalar value is very useful for determining whether a specimen color is within a specified tolerance from a standard. However, for control of color in production, it may be necessary to know not only the magnitude of the departure from standard but also the direction of this departure. It is possible to include information on the direction of a small color difference by listing the three instrumentally determined components of the color difference.

5.4 Selection of color tolerances based on instrumental values should be carefully correlated with a visual appraisal of the acceptability of differences in hue, lightness, and saturation obtained by using Practice D 1729. The three tolerance equations given here have been tested extensively against such data for textiles and plastics and have been shown to agree with the visual evaluations to within the experimental uncertainty of the visual judgments. That implies that the equations themselves misclassify a color difference with a frequency no greater than that of the most experienced visual color matcher.

5.5 While color difference equations and color tolerance equations are routinely applied to a wide range of illuminants, they have been derived or optimized, or both, for use under daylight illumination. Good correlation with the visual judgments may not be obtained when the calculations are made with other illuminants. Use of a tolerance equation for other than daylight conditions will require visual confirmation of the level of metamerism as per Practice D 4086.

6. Description of Color-Difference and Color-Tolerance Equations

6.1 *CIE 1931 and 1964 Color Spaces*— The daylight colors of opaque specimens are represented by points in a space formed by three rectangular axes representing the lightness scale Y and chromaticity scales x and y, where:

$$x = \frac{X}{X + Y + Z} \tag{1}$$

$$y = \frac{Y}{X + Y + Z} \tag{2}$$

where X, Y, and Z are tristimulus values for either the 1931 CIE standard observer (2° observer) or the 1964 CIE standard observer (10° observer) and standard illuminant D_{65}, or other phase of daylight. These scales do not provide a perceptually uniform color space. Consequently, color differences are seldom if ever computed directly from differences in x, y, and Y.

6.2 *CIE 1976 L* a* b* Uniform Color Space and Color-Difference Equation* **(1, 8)—**This is an approximately uniform color space based on nonlinear expansion of the tristimulus values and taking differences to produce three opponent axes that approximate the percepts of lightness-darkness, redness-greenness and yellowness-blueness. It is produced by plotting in rectangular coordinates the quantities L^*, a^*, b^*, calculated as follows:

$$L^* = 116 \cdot \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - 16 \tag{3}$$

$$a^* = 500 \cdot \left(\frac{X}{X_n}\right)^{\frac{1}{3}} - \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} \tag{4}$$

$$b^* = 200 \cdot \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - \left(\frac{Z}{Z_n}\right)^{\frac{1}{3}} \tag{5}$$

$$\frac{X}{X_n}, \frac{Y}{Y_n}, \frac{Z}{Z_n} \geq 0.008856 \tag{6}$$

The tristimulus values X_n, Y_n, Z_n define the color of the nominally white object-color stimulus. Usually, the white object-color stimulus is given by the spectral radiant power of one of the CIE standard illuminants, for example, C, D_{65} or another phase of daylight, reflected into the observer's eye by the perfect reflecting diffuser. Under these conditions, X_n, Y_n, Z_n are the tristimulus values of the standard illuminant with Y_n equal to 100.

6.2.1 The total color-difference ΔE^*_{ab} between two colors each given in terms of L^*, a^*, b^* is calculated as follows:

$$\Delta E^*_{ab} = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \quad (7)$$

NOTE 1—The color space defined above is called the CIE 1976 L^* a^* b^* space and the color-difference equation the CIE 1976 L^* a^* b^* color-difference formula. The abbreviation CIELAB (with all letters capitalized) is recommended.

6.2.2 The CIE 1976 (L^* a^* b^*) metric fails to converge to zero appropriately when one or more of the ratios X/X_n, Y/Y_n, and Z/Z_n is less than 0.008856. In calculating L^*, values of Y/Y_n less than 0.008856 may still be utilized if the normal formula is used for values of Y/Y_n greater than 0.008856, and the following modified formula is used for values of Y/Y_n equal to or less than 0.008856.

$$L^* = 903.3\left(\frac{Y}{Y_n}\right), \quad \frac{Y}{Y_n} < 0.008856 \quad (8)$$

6.2.3 In calculating a^* and b^*, values of X/X_n, Y/Y_n, Z/Z_n less than 0.008856, may still be included if the normal equations are replaced by the following modified equations for calculations of a^* and b^*:

$$a^* = 500 \cdot f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \quad (9)$$

$$b^* = 200 \cdot f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \quad (10)$$

where:

$$f\left(\frac{X}{X_n}\right) = \left(\frac{X}{X_n}\right)^{\frac{1}{3}} \quad \text{if} \quad \frac{X}{X_n} \geq 0.008856 \quad (11)$$

$$f\left(\frac{X}{X_n}\right) = 7.787 \cdot \left(\frac{X}{X_n}\right) + \frac{16}{116} \quad \text{if} \quad \frac{X}{X_n} < 0.008856 \quad (12)$$

$$f\left(\frac{Y}{Y_n}\right) = \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} \quad \text{if} \quad \frac{Y}{Y_n} \geq 0.008856 \quad (13)$$

$$f\left(\frac{Y}{Y_n}\right) = 7.787 \cdot \left(\frac{Y}{Y_n}\right) + \frac{16}{116} \quad \text{if} \quad \frac{Y}{Y_n} < 0.008856 \quad (14)$$

$$f\left(\frac{Z}{Z_n}\right) = \left(\frac{Z}{Z_n}\right)^{\frac{1}{3}} \quad \text{if} \quad \frac{Z}{Z_n} \geq 0.008856 \quad (15)$$

$$f\left(\frac{Z}{Z_n}\right) = 7.787 \cdot \left(\frac{Z}{Z_n}\right) + \frac{16}{116} \quad \text{if} \quad \frac{Z}{Z_n} < 0.008856 \quad (16)$$

6.2.4 The magnitude, ΔE^*_{ab}, gives no indication of the character of the difference since it does not indicate the relative quantity and direction of hue, chroma, and lightness differences.

6.2.5 The direction of the color difference is described by the magnitude and algebraic signs of the components ΔL^*, Δa^*, and Δb^*:

$$\Delta L^* = L^*_B - L^*_S \quad (17)$$

$$\Delta a^* = a^*_B - a^*_S \quad (18)$$

$$\Delta b^* = b^*_B - b^*_S \quad (19)$$

where L^*_S, a^*_S, and b^*_S refer to the reference or standard, and L^*_B, a^*_B, and b^*_B refer to the test specimen or batch. The signs of the components ΔL^*, Δa^*, and Δb^* have the following approximate meanings **(9):**

$$+\Delta L^* = \text{lighter} \quad (20)$$

$$-\Delta L^* = \text{darker} \quad (21)$$

$$+\Delta a^* = \text{redder (less green)} \quad (22)$$

$$-\Delta a^* = \text{greener (less red)} \quad (23)$$

$$+\Delta b^* = \text{yellow (less blue)} \quad (24)$$

$$-\Delta b^* = \text{bluer (less yellow)} \quad (25)$$

6.2.6 For judging the direction of the color difference between two colors, it is useful to calculate their CIE 1976 metric hue angles h_{ab} and CIE 1976 metric chroma C^*_{ab} as follows:

$$h_{ab} = \tan^{-1}\left(\frac{b^*}{a^*}\right) \quad (26)$$

$$C^*_{ab} = \sqrt{(a^*)^2 + (b^*)^2} \quad (27)$$

Differences in hue angle $°h_{ab}$ between the test specimen and reference can be correlated with differences in their visually perceived hue, except for very dark colors **(10)**. Differences in chroma $\Delta°C^*_{a\ b}([C^*_{ab}]_{batch} - [C^*_{ab}]_{standard})$ can similarly be correlated with differences in visually perceived chroma.

6.2.7 For judging the relative contributions of differences in lightness, chroma, and hue to the total color difference between two colors, it is useful to calculate the CIE 1976 metric hue difference $°\Delta H^*_{ab}$ between them as follows:

$$\Delta H^*_{ab} = \sqrt{(\Delta E^*_{ab})^2 - (\Delta L^*)^2 - (\Delta C^*_{ab})^2} \tag{28}$$

where ΔE^*_{ab} is calculated as in 6.2.1 and $\Delta C^*_{a\ b}$ is calculated as in 6.2.6; then the equation

$$\Delta E^*_{ab} = \sqrt{(\Delta L^*)^2 + (\Delta C^*_{a\ b})^2 + (\Delta H^*_{ab})^2} \tag{29}$$

contains terms showing the relative contributions of lightness difference ΔL^*, chroma difference ΔC^*_{ab}, and hue difference $\Delta°H^*_{a\ b}$ to the total color difference ΔE^*_{ab}. This method of computing a metric hue difference loses the information about the sign of the hue difference (positive or negative) and can be unstable for pairs of colors near the neutral axis. An alternative method has been proposed **(18)** that corrects both problems:

$$\Delta H^*_{ab} = \frac{a^*_S \cdot b^*_B - a^*_B \cdot b^*_S}{\sqrt{0.5 \cdot (C^*_{ab,B} \cdot C^*_{ab,S} + a^*_B \cdot a^*_S + b^*_B \cdot b^*_S)}} \tag{30}$$

6.3 *CMC Color Tolerance Equation*—The Colour Measurement Committee of the Society of Dyers and Colourists undertook a task to improve upon the results of the JPC79 tolerance equation **(2)** developed at J & P Coates thread company in the United Kingdom. It was a combination of the CIELAB equation and local optimization based on the position of the standard used to derive the FMC-2 equation. It was based on the more intuitive perceptual variables of lightness, chroma and hue instead of the lightness, redness/greenness and yellowness/blueness of the older equation. It is intended to be used as a single-number shade-passing equation. There should not be a need to break the equation down into perceptual components—the CIELAB components of the model do that already. Fig. 1 **(16)** shows the CIELAB chromaticness plane (a^*,b^*) with a large number of CMC ellipsoids plotted on that plane. The figure clearly shows the change in area of the ellipses with increases in CIELAB metric chroma C^*_{ab} and with respect to changes in CIELAB metric hue angle h^*_{ab}. The CMC components and single number tolerances are computed as follows:

$$\Delta E_{CMC}(l:c) = cf \cdot \sqrt{\left(\frac{\Delta L^*}{l \cdot S_L}\right)^2 + \left(\frac{\Delta C^*}{c \cdot S_c}\right)^2 + \left(\frac{\Delta H^*}{S_H}\right)^2} \tag{31}$$

The parameters (l, c) are to compensate for systematic bias or parametric effects such as texture and sample separation. The most common values are (2:1) for textiles and plastics that are molded to simulate a woven material, implying that lightness differences carry half the importance of chroma and hue differences **(17)**. The values (1:1), often assumed to represent a just perceptible difference, should be applied to materials that require very critical tolerances or have glossy surfaces. For specimens that are matte, randomly rough, or mildly textured, values intermediate between (1:1) and (2:1) can be used, with the value (1.3:1) being reported most frequently. The parameter *cf* is a commercial factor **(19)**, used to adjust the total volume of the tolerance region so that accept/reject decisions can be made on the basis of a unit value of the tolerance. The color dependent functions are defined as:

$$S_L = \frac{0.040975 \cdot L^*}{(1 + 0.01765 \cdot L^*)'} \quad \text{for} \quad L^* \geq 16 \tag{32}$$

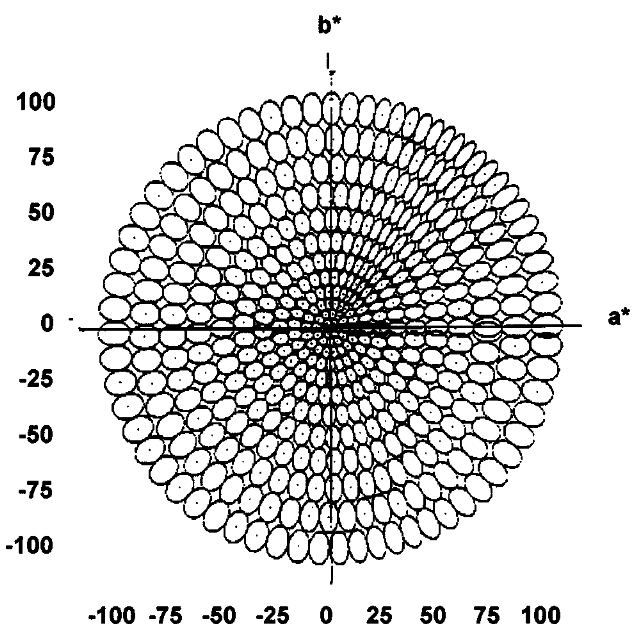

FIG. 1 CMC Ellipse Distribution in the CIELAB (*a, *b**) Plane**

$$S_L = 0.511, \qquad \text{for} \quad L^* < 16$$

$$S_C = \frac{0.0638 \cdot C^*}{(1 + 0.0131 \cdot C^*)} + 0.638$$

$$S_H = S_C (T \cdot f + 1 - f)$$

where,

$$f = \left\{ \frac{(C^*)^4}{(C^*)^4 + 1900} \right)^{\frac{1}{2}}$$

$$T = 0.56 + |0.2 \cos (h + 168°)|, \qquad \text{if } 164° < h < 345°$$

else,

$$T = 0.36 + |0.4 \cos (h + 35°)|$$

All angles are given in degrees but will generally need to be converted to radians for processing on a digital computer.

6.4 *CIE94 Color Tolerance Equation* **(3)**—The development of this color tolerance equation was prompted by the success of the CMC tolerance equation. It was derived primarily from visual observations of automotive paints on steel panels. Like, the CMC equation, it is based on the CIELAB color metric and uses the position of the standard in CIELAB color space to derive a set of analytical functions that modify the spacing of the CIELAB space in the region around the standard. Its weighting functions are much simpler than those of the CMC equation. CIE94 tolerances are computed as follows:

$$\Delta E^*_{94} = k_v \left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*}{k_H S_H}\right)^{2\frac{1}{2}} \tag{33}$$

Unlike many previous color difference equation, CIE94 comes with a well defined set of conditions under which the equation will provide optimum results and departures from this set of conditions will cause the agreement between the visually evaluated color-difference and the computed color-difference to be significantly poorer. Those conditions are given in Table 1. The

TABLE 1 Basis Conditions for CIE94 Tolerance Equation

Attribute	Requirement
Illumination	D65 source
Specimen Illuminance	1000 lx
Observer	Normal color vision
Background	Uniform neutral gray L^* = 50
Viewing Mode	Object
Sample Size	>4° subtended visual angle
Sample Separation	Minimum possible
Size of Color Differences	0 to 5 CIELAB units
Sample Structure	Visually homogenous

parameters k_L, k_C, k_H, are the parametric factors that can be used to compensate for texture and other specimen presentation effects while k_V is used to adjust the size of the tolerance volume for industrial bias. The parameters S_L, S_C, S_H are used to perform the local distortion of CIELAB color space, again based on the position of the standard specimen in that space. The are computed using the following equations:

$$S_L = 1 \tag{34}$$

$$S_C = 1 + 0.045 \cdot C^*$$

$$S_H = 1 + 0.015 \cdot C^*$$

6.5 *DIN99 Color Difference Equation*— The publication in 1996 of the paper by Rohner and Rich **(4)** prompted the German standards institute (DIN) to further develop and standardize a modified version as a new color difference formula that globally models color space using logarithms of the CIELAB coordinates rather than the linear and hyperbolic functions of CMC and CIE94. The equations derived and documented in standard DIN 6176 provides an axes rotation and the logarithmic expansion of the new axes to match that of the spacing of the CIE94 color tolerance formula without the need to make the specimen identified as standard the source of the distortion of distances in the CIELAB color space. Also, as neither the tristimulus values XYZ nor the CIELAB axes a^*, b^* are perceptual variables while the axes L^*, C^* and h^*_{ab} are correlates of the perceptions of lightness, chroma and hue, it seemed appropriate to scale the differences or distances in color space following the Weber-Fechner law of perception. This resulted in a formula which is easy to use and has equivalent performance to CMC or CIE94. It also eliminates the annoying reference-color based distortion of CIELAB. Thus computed color differences are based only on the Euclidean distance in the DIN99 space. The procedures for computing the DIN99 formula are:

Step 1

$$\text{Redness } e = \cos (16°)\, a^* + \sin (16°)\, b^* \tag{35}$$

$$\text{Yellowness } f = -0.7(-\sin(16°)\, a^* + \cos(16°)\, b^*)$$

$$\text{Chroma } G = (e^2 + f^2)^{0.5}$$

$$\text{Hue angle } h_{ef} = \arctan\left(\frac{f}{e}\right)$$

Step 2

$$\text{Chroma } C_{99} = \frac{(\log_e(1 + 0.045\, G))}{(0.045\, k_{CH} k_E)} \tag{36}$$

$$\text{Hue angle } h_{99} = h_{ef} \frac{180}{\pi}$$

$$\text{Redness } a_{99} = C_{99} \cos(h_{ef})$$

$$\text{Yellowness } b_{99} = C_{99} \sin(h_{ef})$$

$$\text{Lightness } L_{99} = 105.509\, [\log_e(1 + 0.0158\, L^*)]\, k_E$$

$$\text{Lightness } L_{99} = 105.509\, [\log_e(1 + 0.0158\, L^*)]\, k_E$$

Step 3

$$\Delta E_{99} = \sqrt{(\Delta L_{99})^2 + (\Delta a_{99})^2 + (\Delta b_{99})^2} \tag{37}$$

or

$$\Delta E_{99} = \sqrt{(\Delta L_{99})^2 + (\Delta C_{99})^2 + (\Delta H_{99})^2}$$

with,

$$\Delta C_{99} = C_{99,B} - C_{99,S}$$

$$\Delta H_{99} = \frac{(a_{99,S} \cdot b_{99,B} - a_{99,B} \cdot b_{99,S})}{\sqrt{0.5 \cdot (C_{99,B} \cdot C_{99,S} + a_{99,B} \cdot a_{99,S} + b_{99,B} \cdot b_{99,S})}}$$

Where subscripts S refers to the product standard and subscript B refers to the current product batch or test sample.

Default parameters are: $k_E = k_{CH} = 1$, k_E $(1 : k_{CH})$.

For textiles the following equivalence relations holds: To obtain an equivalent computed difference to a *CMC* ($l = 2$, $c = 1$) difference, use the parameters: 2 (1 : 0.5), which indicate that $k_E = 2$ and, $k_{CH} = 0.5$.

6.6 *CIEDE2000 Color Difference Equation* **(5)**—The development of this color difference equation grew out of the research being performed to try to determine which of the two color tolerances equations, CMC or CIE94, was the better formula. In the process, the researchers came to the conclusion that neither formula was truly optimum. Therefore the CIE set up a new technical committee, TC 1-47, Hue & Lightness Dependant Correction to Industrial Colour Difference Equations, to recommend a new equation that addresses the short-comings in both color tolerance equations. One of the major weaknesses of the color tolerance equations was using the position of the reference color in CIELAB color space for computing the local distortion of CIELAB color space. When the identifications of the two specimens are reversed (calling the original test specimen the reference and the original reference now the test specimen) the computation results in a different computed color difference. This is contrary to what is observed. Visually, there is no change in the magnitude of the difference between the specimens simply by switching roles. By using the position of the arithmetic average color between the two specimens to compute the local distortions to CIELAB color space, the roles of the two specimens may be switched without changing the magnitude of the computed color-difference, in full agreement with the visual assessments. The report from CIE TC 1-47 has shown that CIEDE2000 out-performs both CMC and CIE94 across a wide array of specimens. The CIEDE2000 color differences are computed from the following equations:

$$L' = L^* \qquad a' = (1 + G) \cdot a^* \qquad b' = b^* \tag{38}$$

$$C' = \sqrt{a'^2 - b'^2}$$

$$C' = \sqrt{a'^2 + b'^2}$$

$$h' = \arctan\left(\frac{b'}{a'}\right)$$

$$G = 0.5 \cdot \left(1 - \sqrt{\frac{\overline{C^*}^7}{\overline{C^*}^7 + 25^7}}\right)$$

where $\overline{C^*}$ is the arithmetic mean of the CIELAB C^* values for the pair of specimens (standard and batch).

$$\Delta L' = L'_B - L'_S$$

$$\Delta C' = C'_B - C'_S$$

$$\Delta H' = \frac{a'_S \cdot b'_B - a'_B \cdot b'_S}{\sqrt{0.5 \cdot (C'_S \cdot C'_B + a'_S \cdot a'_B + b'_S \cdot b'_B)}}$$

$$\Delta E_{00}{}^2 = \left(\frac{\Delta L'}{K_L \cdot S_L}\right)^2 + \left(\frac{\Delta C'}{K_C \cdot S_C}\right)^2 + \left(\frac{\Delta H'}{K_H \cdot S_H}\right)^2 + R_T \cdot \left(\frac{\Delta C' \cdot \Delta H'}{K_C \cdot S_C \cdot K_L \cdot S_H}\right)$$

$$\Delta E_{00} = \sqrt{\Delta E_{00}{}^2}$$

The specimen or industry dependent parameters are K_L, K_C, K_H and the color space dependent parameters are S_L, S_C, S_H and R_T. The three S terms operate on the, assumed orthogonal, CIELAB coordinates and the R_T term computes a rotation of the color difference volume in the blue and purple-blue regions of the CIELAB diagram. The four color space terms are computed as follows:

$$S_L = 1 + \frac{0.015 \cdot (\overline{L'} - 50)^2}{\sqrt{20 + (\overline{L'} - 50)^2}} \tag{39}$$

$$S_C = 1 + 0.045 \cdot \overline{C'}$$

$$S_H = 1 + 0.015 \cdot \overline{C'} \cdot T$$

$$R_T = -\sin(2 \cdot \Delta\theta) \cdot R_C$$

$$R_C = 2 \cdot \sqrt{\frac{\overline{C'}^7}{\overline{C'}^7 + 25^7}}$$

$$\Delta\theta = 30 \cdot \exp\left(-\frac{(\overline{h'} - 275°)_2}{25}\right)$$

$$\Delta\theta = 30 \cdot \exp\left(-\frac{(\overline{h'} - 275°)^2}{25}\right)$$

$$T = 1 - 0.17 \cdot \cos(\overline{h'} - 30°) + 0.24 \cdot \cos(2\overline{h'}) + 0.32 \cdot \cos(3\overline{h'} + 6°) - 0.20 \cdot \cos(4\overline{h'} - 63°)$$

While not obvious from this listing, all displayed angles are assumed to be given in degrees, including$\Delta\theta$ and thus must generally be converted into radians for trigonometric analysis on digital computers.

6.6.1 Using the arithmetic average of the CIELAB color coordinates of the reference and test specimens to compute the local distortion of CIELAB color space introduces a new problem. Current color tolerance difference equations which base the distortion of CIELAB space on the position of the standard allows a user to predefine the acceptance volume. This is convenient for certain textile sorting applications and for graphical quality control charting. Such a predetermination is not possible with CIEDE2000. Nor is it possible or reasonable to plot groups of colors in terms of the modified space coordinates, L^*,a', b^* since the meaning of a' is determined uniquely for each pair of colors. Thus the equation is highly optimized for pairwise comparison of a product standard to a production test specimen but not for statistical process control.

7. Test Specimens

7.1 This practice does not cover specimen preparation techniques. Unless otherwise specified or agreed, prepare specimens in accordance with appropriate test methods and practices.

8. Procedure

8.1 Select appropriate geometric conditions for color measurement in accordance with Practice E 805.

8.2 Operate the instrument in accordance with the manufacturer's instructions and the procedures given in Practice E 1164.

8.3 When a colorimetric spectrometer is used, obtain the reflectance values of the reference specimen and test specimens, in turn, at a sufficient number of wavelength intervals to permit accurate calculation of CIE tristimulus values. See Practice E 308.

8.4 Measure at least three portions of each specimen surface to obtain an indication of uniformity. Record the location where these measurements were made on the specimen.

9. Calculation

9.1 Calculate color-scale values L^*, a^*, b^*, and local tolerance weights (S_L, S_C, S_H) if not obtained automatically.

9.2 Calculate color differences ΔE^*_{ab},ΔE_{CMC} and their components, or ΔE_{94}, ΔE_{99}, or ΔE_{00}, if not obtained automatically, as described in 6.2-6.6, respectively.

10. Report

10.1 Report the following information:

10.1.1 Total color difference ΔE_{CMC}, or ΔE_{94}, ΔE_{99}, or ΔE_{00} of each test specimen from its reference.

10.1.2 For CIELAB color differences, L^*, a^*, b^* for the reference, ΔL^*, Δa^*, Δb^* and if desired Δh_{ab}, ΔC^*_{ab}, and ΔH^*_{ab} for each specimen.

10.1.3 For other color tolerance or color difference metrics, only the CIELAB coordinates should be reported as the local distortions do not necessarily provide continuous, visually correlated parameters.

10.1.4 For non-uniform specimens, range of color-difference magnitudes obtained for different areas of the specimens.

10.1.5 Description or identification of the method of preparing the specimens.

10.1.6 Identification of the instrument used, by the manufacturer's name and model number, and the color-scale system used.

11. Precision and Bias

11.1 Since the precision and bias of a test method cannot be separated from the effect of the specimens and materials and since this practice does not address the issues related to the preparation and presentation of specimens, no definitive statement about precision and bias can be made. The next section, uses data from a commercial collaborative testing program to illustrate precision for one material. Because of the many trigonometric functions and power functions involved in computing the color space parameters, all computations should be carried out in IEEE floating point format to greatest number of bits of precision available on the computational system, usually known as double precision.

11.2 *The Collaborative Testing Services Color and Color Difference Collaborative Reference Program* **(13)** has surveyed the precision of color and color-difference measurements by sending out pairs of painted chips exhibiting small color differences on a quarterly basis since 1971. In a typical recent survey (Report No. 111, February, 2000), 118 instruments were involved. Table 2 gives the mean color differences and their standard deviations for the groups of instruments considered separately in the intercomparison, together with the conditions of analysis and measurement.

11.2.1 *Reproducibility*—Based on the between-laboratory standard deviations, two color-difference results, obtained by operators in different laboratories measuring opaque, matte paint on sealed white paper stock should be considered suspect if they differ by more than the values shown in column R^* of Table 2.

11.3 *Precision*—Based on the within-laboratory standard deviations, the precision of color-difference measurements, summarized in Table 2, was equivalent to the precision of measured values of color as reported in the literature **(14,15)** and is thus likely to be representative of the precision obtainable for all production materials.

12. Keywords

12.1 color; color difference; color metrics; color spaces; color tolerances

ANNEX

(Mandatory Information)

A1. COLOR SPACES AND COLOR DIFFERENCE METRICS NO LONGER RECOMMENDED BUT STILL IN USE

A1.1 *Hunter L_H, a_H, b_H Color Space and Color-Difference Equation* **(6)**—This approximately uniform color space is produced by plotting in rectangular coordinates the quantities L_H, a_H, b_H calculated as follows:

$$L_H = 10(Y)^{\frac{1}{2}} \tag{A1.1}$$

$$a_H = 17.5(1.02X - Y)/(Y)^{\frac{1}{2}} \tag{A1.2}$$

$$b_H = 7.0(Y - 0.847Z)/(Y)^{\frac{1}{2}} \tag{A1.3}$$

where X, Y, and Z are tristimulus values for the CIE 1931 standard observer and standard illuminant C. The total difference ΔE_H between two colors each given in terms of L_H, a_H, b_H is calculated as follows:

$$\Delta E_H = [(\Delta L_H)^2 + (\Delta a_H)^2 + (\Delta b_H)^2]^{\frac{1}{2}} \tag{A1.4}$$

TABLE 2 Bias of Calculated Color Differences Determined for Various Conditions of Measurement and Analysis

Measurement Conditions			ΔE Equation	No. of Instruments	Mean ΔE	Standard Deviation	R^{*A}
Geometry	Illuminant	Observer					
45°/0°	D_{65}	1964	CIELAB	54	1.05	0.07	0.21
45°/0°	D_{65}	1964	CMC(2:1)	54	0.55	0.03	0.09
Sphere[B]	D_{65}	1964	CIELAB	282	1.00	0.06	0.18
Sphere[B]	D_{65}	1964	CMC(2:1)	282	0.53	0.03	0.09

[A] R^* is the approximate inter-laboratory precision = 3.0 × standard deviation.

[B] Specular component included for integrating-sphere measurements.

$$\Delta E_H = [(\Delta L_H)^2 + (\Delta a_H)^2 + (\Delta b_H)^2]^{\frac{1}{2}} \quad (A1.4)$$

The magnitude and direction of the color difference are described by considerations similar to those found in 6.2.4 and 6.2.5, respectively.

A1.2 *Friele-MacAdam-Chickering Color Space and Color-Difference Equation* **(7,11)—**This color space is more nearly perceptually uniform than the CIE 1931 space in terms of the MacAdam chromaticity-difference values **(12)**. It is produced by a linear transformation of CIE 1931 tristimulus values *X, Y, Z* into tristimulus values *P, Q, S* as follows:

$$P = 0.742X + 0.382Y - 0.098Z \quad (A1.5)$$

$$Q = -0.48X + 1.37Y + 0.1276Z \quad (A1.6)$$

$$S = 0.686Z \quad (A1.7)$$

Approximate lightness difference $\Delta L_{FMC\text{-}2}$ and chromatic differences ΔC ("yellow-blue") and C_3("red-green") are calculated as follows:

$$\Delta L_{FMC-2} = 0.279K_2(P\Delta P + Q\Delta Q)/aD \quad (A1.8)$$

$$\Delta C_1 = K_1 S(P\Delta P + Q\Delta Q)/bd^2 - K_1°S/b \quad (A1.9)$$

$$\Delta C_3 = K_1(Q\Delta P - P\Delta Q)/aD \quad (A1.10)$$

where:

a^2 = $17.3 \times 10^{-6}(P^2 + Q^2)/[1 + 2.73P^2Q^2/(P^4 + Q^4)]$,
b^2 = $3.098 \times 10^{-4}(S^2 + 0.2015Y^2)$
D = $(P^2 + Q^2)^{1/2}$,
K_1 = $0.55669 + 0.049434Y - 0.82575 \cdot 10^{-3} Y^2 + 0.79172 \times 10^{-5} Y^3 - 0.30087 \cdot 10^{-7} Y^4$, and
K_2 = $0.17548 + 0.027556Y - 0.57262 \cdot 10^{-3} Y^2 + 0.63893 \times 10^{-5} Y^3 - 0.26731 \cdot 10^{-7} Y^4$.

The correlation between $\Delta L_{FMC\text{-}2}$ and perceived lightness, between ΔC_1 and perceived yellowness-blueness, and between ΔC_3 and perceived redness-greenness, are not well established and should not be used unless confirmed by visual observations.

The total difference $\Delta E_{FMC\text{-}2}$ between two colors is calculated as follows:

~~$$\Delta E_{FMC-2} = [(\Delta L_{FMC-2})^2 + (\Delta C_1)^2 + (\Delta C_3)^2]^{\frac{1}{2}} \quad (A1.11)$$~~

$$\Delta E_{FMC-2} = [(\Delta L_{FMC-2})^2 + (\Delta C_1)^2 + (\Delta C_3)^2]^{\frac{1}{2}} \quad (A1.11)$$

APPENDIX

(Nonmandatory Information)

X1. EXAMPLE CALCULATIONS FOR COLOR TOLERANCES EQUATION

Color Coordinate	STD-1	BAT-1	STD-2	BAT-2	STD-3	BAT-3	STD-4	BAT-4	STD-5	BAT-5
X	19.4100	19.5525	22.4800	22.5833	28.9950	28.7704	4.1400	4.4129	4.9600	4.6651
Y	28.4100	28.6400	31.6000	31.3700	29.5800	29.7400	8.5400	8.5100	3.7200	3.8100
Z	11.5766	10.5791	38.4800	36.7901	35.7500	35.6045	8.0300	8.6453	19.5900	17.7848
*L**	60.2574	60.4626	63.0109	62.8187	61.2901	61.4292	35.0831	35.0232	22.7233	23.0331
*a**	-34.0099	-34.1751	-31.0961	-29.7946	3.7196	2.2480	-44.1164	-40.0716	20.0904	14.9730
*b**	36.2677	39.4387	-5.8663	-4.0864	-5.3901	-4.9620	3.7933	1.5901	-46.6940	-42.5619
*C**	49.7194	52.1857	31.6447	30.0735	6.5490	5.4474	44.2792	40.1031	50.8326	45.1188
~~h_{ab}~~	~~133.160~~	~~130.910~~	~~190.683~~	~~187.810~~	~~304.609~~	~~294.373~~	~~175.086~~	~~177.728~~	~~293.280~~	~~289.382~~
h_{ab}*	133.160	130.910	190.683	187.810	304.609	294.373	175.086	177.728	293.280	289.382
SL_{CMC}	1.1965		1.2224		1.2064		0.8878		0.6646	
SC_{CMC}	2.5589		2.0653		1.0228		2.4259		2.5848	
f	0.9998		0.9991		0.7014		0.9998		0.9999	
T	0.7515		0.7599		0.6369		0.7513		0.5991	
SH_{CMC}	1.9231		1.5700		0.7623		1.8229		1.5487	
~~Δ*L*~~		~~0.2052~~		~~-0.1922~~		~~0.1391~~		~~-0.0599~~		~~0.3098~~
Δ*L**		0.2052		-0.1922		0.1391		-0.0599		0.3098
~~Δ*C*~~		~~2.4663~~		~~-1.5712~~		~~-1.1016~~		~~-4.1761~~		~~-5.7138~~
Δ*C**		2.4663		-1.5712		-1.1016		-4.1761		-5.7138
~~Δ*H*~~		~~-1.9999~~		~~-1.5472~~		~~-1.0657~~		~~1.9430~~		~~-3.2580~~
Δ*H**		-1.9999		-1.5472		-1.0657		1.9430		-3.2580

Color Coordinate	STD-1	BAT-1	STD-2	BAT-2	STD-3	BAT-3	STD-4	BAT-4	STD-5	BAT-5
ΔE^*_{ab}		3.1819		2.2134		1.5390		4.6063		6.5847
ΔE_{CMC}(1:1)		1.4282		1.2549		1.7684		2.0258		3.0870
ΔE_{CMC}(2:1)		1.4205		1.2474		1.7656		2.0250		3.0604
SL_{94}	1.0000		1.00000		1.00000		1.00000		1.00000	
SC_{94}	3.23737		2.42401		1.29470		2.99256		3.28747	
SH_{94}	1.74579		1.47467		1.09824		1.66419		1.76249	
ΔE_{94}		1.3910		1.2481		1.2980		1.8204		2.5561
e	-22.6983	-21.9832	-31.5095	-29.7677	2.09015	0.79346	-41.3637	-38.0826	6.44406	2.66348
f	30.9657	33.1313	2.05161	2.99825	-4.34462	-3.77258	11.0634	8.80059	-35.2962	-31.5285
G	38.3939	39.7611	31.5762	29.9184	4.82125	3.85513	42.8177	39.0863	35.8796	31.6408
h_{ef}	126.242	123.565	176.275	174.249	295.692	281.878	165.026	166.988	280.347	274.829
k_E	1	1	1	1	1	1	1	1	1	1
k_{CH}	1	1	1	1	1	1	1	1	1	1
C_{99}	22.2993	22.7950	19.6478	18.9522	4.36339	3.55497	23.8646	22.5517	21.3579	19.6745
h_{99}	2.20334	2.15662	3.07657	3.04121	5.16080	4.91969	2.88024	2.91449	4.89297	4.79667
a_{99}	-13.1833	-12.6029	-19.6063	-18.8568	1.89166	0.73168	-23.0542	-21.9726	3.83592	1.65617
b_{99}	17.9850	18.9941	1.27658	1.89928	-3.93203	-3.47886	6.16626	5.07769	-21.0106	-19.6046
L_{99}	70.5738	70.7489	72.8994	72.7388	71.4521	71.5698	46.5330	46.4688	32.3670	32.7463
ΔE_{99}(Lab)		1.1772		0.98756		1.25091		1.53592		2.62143
ΔC_{99}		0.49568		-0.69558		-0.80842		-1.31296		-1.68341
ΔH_{99}		-1.05329		-0.68237		-0.94729		0.79439		-1.97335
ΔL_{99}		0.17512		-0.16065		0.11774		-0.06425		0.37933
ΔE_{99}(LCH)		1.1772		0.98756		1.25091		1.53592		2.62143
L^*ave	60.3600		62.9148		61.3597		35.0532		22.8782	
C^*ave	50.9525		30.8591		5.9982		42.1911		47.9757	
G	0.0017		0.0490		0.4966		0.0063		0.0026	
a'	-34.0678	-34.2333	-32.6195	-31.2542	5.5669	3.3643	-44.3939	-40.3237	20.1424	15.0118
C'	49.7590	52.2238	33.1428	31.5202	7.7488	5.9950	44.5557	40.3550	50.8532	45.1317
h'	133.21	130.96	190.20	187.45	315.92	304.14	175.12	177.74	293.33	289.43
$C'ave$	50.9914		32.3315		6.8719		42.4554		47.9924	
$h'ave$	132.084		188.822		310.031		176.429		291.381	
ΔL^*	0.2052		-0.1922		0.1391		-0.0599		0.3098	
$\Delta C'$	2.4648		-1.6226		-1.7538		-4.2007		-5.7215	
$\Delta H'$	-2.0018		-1.5490		-1.3995		1.9430		-3.2653	
SL	1.1427		1.1831		1.1586		1.2148		1.4014	
SC	3.2946		2.4549		1.3092		2.9105		3.1597	
SH	1.9951		1.4560		1.0717		1.6476		1.2617	
RC	1.9932		1.8527		0.0218		1.9759		1.9897	
$\Delta\theta$	0.0000		0.0002		4.2110		0.0000		19.5282	
RT	0.0000		0.0000		-0.0032		0.0000		-1.2537	
T	1.3010		0.9402		0.6952		1.0168		0.3636	
ΔE_{00}	1.2644		1.2630		1.8731		1.8645		2.0373	

Color Coordinate	STD-6	BAT-6	STD-7	BAT-7	STD-8	BAT-8	STD-9	BAT-9	STD-10	BAT-10	
X	15.6000	15.9148	73.0000	73.9351	73.9950	69.1762	0.7040	0.6139	0.2200	0.0933	
Y	9.2500	9.1500	78.0500	78.8200	78.3200	73.4000	0.7500	0.6500	0.2300	0.1000	
Z	5.0200	4.3872	81.8000	84.5156	85.3060	79.7130	0.9720	0.8510	0.3250	0.1453	
L^*	36.4612	36.2715	90.8027	91.1528	90.9257	88.6381	6.7747	5.8714	2.0776	0.9033	
a^*	47.8580	50.5065	-2.0831	-1.6435	-0.5406	-0.8985	-0.2908	-0.0985	0.0795	-0.0636	
b^*	18.3852	21.2231	1.4410	0.0447	-0.9208	-0.7239	-2.4247	-2.2286	-1.1350	-0.5514	
C^*	51.2680	54.7844	2.5329	1.6441	1.0677	1.1538	2.4421	2.2308	1.1378	0.5550	
h_{ab}^*	21.0148	22.7924	145.326	178.441	239.583	218.857	263.160	267.469	274.004	263.419	
SL	1.4295		1.4303		0.5110		0.5110		0.9090		
SC	0.7944		0.7052		0.7890		0.7095		2.5947		
f	0.1456		0.0261		0.1356		0.0297		0.9999		
T	0.7600		0.6949		0.6246		0.5878		0.5836		
SH	0.7666		0.6996		0.7488		0.7008		1.5144		
~~ΔL~~		~~-0.1897~~		~~0.3501~~		~~-2.2876~~		~~-0.9033~~		~~-1.1743~~	~~-0.1897~~
ΔL^*		-0.1897		0.3501		-2.2876		-0.9033		~~-1.1743~~	~~-0.1897~~ -1.1743
~~ΔC~~		~~-0.8888~~		~~-0.8888~~		~~0.0861~~		~~-0.2117~~		~~-0.5828~~	3.5164
ΔC^*		3.5164		-0.8888		0.0861		-0.2117		~~-0.5828~~	~~3.5164~~ -0.5828
~~ΔH~~		~~1.6441~~		~~1.1631~~		~~-0.3993~~		~~0.1766~~		~~-0.1444 1.6441~~	
ΔH^*		1.6441		1.1631		-0.3993		0.1766		-0.1444	
~~ΔE^*_{ab}~~		~~3.8864~~		~~1.5051~~		~~2.3238~~		~~0.9444~~		~~1.3189~~	
ΔE^*_{ab}		3.8864		1.5051		2.3238		0.9441		1.3191	
ΔE_{CMC}(1:1)		1.9009		1.7026		1.8034		2.4491		1.7490	
ΔE_{CMC}(2:1)		1.8890		0.9901		0.9533		1.4274		1.7396	
SL_{94}	1.00000		1.00000		1.00000		1.00000		1.00000		
SC_{94}	3.30706		1.11398		1.04805		1.10989		1.05120		
SH_{94}	1.76902		1.03799		1.01602		1.03663		1.01707		
ΔE_{94}		1.4249		1.4194		2.3226		0.9388		1.3063	
e	51.0729	54.4010	-1.60532	-1.56759	-0.77341	-1.06323	-0.94785	-0.70772	-0.23643	-0.21163	
f	3.13861	4.53729	1.37149	0.34716	-0.51529	-0.31376	-1.57548	-1.48059	-0.77907	-0.35911	
G	51.1692	54.5899	2.11140	1.60557	0.92934	1.10855	1.83863	1.64104	0.81415	0.41684	
h_{ef}	3.51660	4.76770	139.491	167.513	213.674	196.441	238.968	244.452	253.118	239.488	
k_E	1	1	1	1	1	1	1	1	1	1	

Color Coordinate	STD-6	BAT-6	STD-7	BAT-7	STD-8	BAT-8	STD-9	BAT-9	STD-10	BAT-10
k_{CH}	1	1	1	1	1	1	1	1	1	1
C_{99}	26.5492	27.5616	2.01703	1.55022	0.91044	1.08179	1.76652	1.58328	0.79959	0.41297
h_{99}	0.06138	0.08321	2.43458	2.92365	3.72931	3.42855	4.17078	4.26650	4.41774	4.17986
a_{99}	26.4992	27.4662	-1.53356	-1.51355	-0.75767	-1.03756	-0.91067	-0.68281	-0.2322	-0.20967
b_{99}	1.62847	2.29081	1.31019	0.33520	-0.5048	-0.30618	-1.51369	-1.42847	-0.76514	-0.35579
L_{99}	48.0009	47. 8000	93.8837	94.1231	93.9679	92.3911	10.7292	9.36013	3.40777	1.49518
ΔE_{99}(Lab)		1.18914		1.00416		1.61372		1.39052		1.95603
ΔC_{99}		1.01234		-0.46681		0.17135		-0.18325		-0.38662
ΔH_{99}		0.59066		0.85621		-0.29736		0.16002		-0.13638
ΔL_{99}		-0.20088		0.23942		-1.5768		-1.36907		-1.91259
ΔE_{99}		1.18914		1.00416		1.61372		1.39052		1.95603
L^*ave	36.3664		90.9778		89.7819		6.3230		1.4904	
C^*ave	53.0262		2.0885		1.1108		2.3362		0.8464	
G	0.0013		0.4999		0.5000		0.4999		0.5000	
a'	47.9197	50.5717	-3.1244	-2.4651	-0.8108	-1.3477	-0.4362	-0.1461	0.1192	-0.0931
C'	51.3256	54.8444	3.4407	2.4655	1.2269	1.5298	2.4637	2.2330	1.1412	0.5593
h'	20.99	22.77	155.24	178.96	228.63	208.24	259.80	266.25	275.99	260.42
$C'ave$	53.0850		2.9531		1.3784		2.3483		0.8503	
$h'ave$	21.8781		167.101		218.436		263.025		268.207	
ΔL^*	-0.1897		0.3501		-2.2876		-0.9033		-1.1743	
$\Delta C'$	3.5189		-0.9751		0.3029		-0.2306		-0.5820	
$\Delta H'$	1.6444		1.1972		-0.4850		0.2638		-0.2165	
SL	1.1943		1.6110		1.5930		1.6517		1.7246	
SC	3.3888		1.1329		1.0620		1.1057		1.0383	
SH	1.7357		1.0511		1.0288		1.0336		1.0099	
RC	1.9949		0.0011		0.0001		0.0005		0.0000	
$\Delta\theta$	0.0000		0.0000		0.1794		23.8495		27.8649	
RT	0.0000		0.0000		0.0000		-0.0004		0.0000	
T	0.9239		1.1546		1.3916		0.9549		0.7787	
ΔE_{00}	1.4146		1.4440		1.5381		0.6386		0.9076	

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