Designation: F 81 – 00

DIN 50435

Standard Test Method for Measuring Radial Resistivity Variation on Silicon Wafers[1]

This standard is issued under the fixed designation F 81; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method[2] provides procedures for the determination of relative radial variation of resistivity of semiconductor wafers cut from silicon single crystals grown either by the Czochralski or floating-zone technique.

1.2 This test method provides procedures for using Test Method F 84 for the four-point probe measurement of radial resistivity variation.

1.3 This test method yields a measure of the variation in resistivity between the center and selected outer regions of the specimen. The amount of information obtained regarding the magnitude and form of the variation in the intervening regions when using the four-point probe array depends on the sampling plan chosen (see 7.2). The interpretation of the variations measured as radial variations may be in error if azimuthal variations on the wafer or axial variations along the crystal length are not negligible.

1.4 This test method can be applied to single crystals of silicon in circular wafer form, the thickness of which is less than one-half of the average probe spacing, and the diameter of which is at least 15 mm (0.6 in.). Measurements can be made on any specimen for which reliable resistivity measurements can be obtained. The resistivity measurement procedure of Test Method F 84 has been tested on specimens having resistivities between 0.0008 and 2000 Ω·cm for *p*-type silicon and between 0.0008 and 6000 Ω·cm for *n*-type silicon. Geometrical correction factors required for these measurements are included for the case of standard wafer diameters, and are available in tabulated form for other cases.[3]

NOTE 1—In the case of wafers whose thickness is greater than the average spacing of the measurement probes, no geometrical correction factor is available except for measurement at the center of the wafer face.

1.5 Several sampling plans are given which specify sets of measurement sites on the wafers being tested. The sampling plans allow differing levels of detail of resistivity variation to be obtained. One of these sampling plans shall be selected and agreed upon by the parties to the measurement. The basic resistivity measurements of Test Method F 84 are then applied at each site specified in the chosen sampling plan.

1.6 Results are expressed as relative changes in resistivity between the several measurement sites. To obtain absolute values of resistivity it is necessary to measure and correct for specimen temperature (see 11.1.4).

1.7 The values stated in SI units are to be regarded as the standard. The values given in parentheses are for information only.

1.8 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

F 84 Test Methods for Measuring Resistivity of Silicon Wafers with an In-Line Four-Point Probe[4]

2.2 *SEMI Standard:*

Specifications M 1, S for Polished Monocrystalline Silicon Wafers[5]

3. Summary of Test Method

3.1 Resistivity measurements are made at specified sites along one or two diameters of a semiconductor specimen in accordance with a sampling plan selected from the four given. Choice among the sampling plans is made on the basis of the extent of information required regarding possible resistivity variations. The measured resistivity values are corrected for specimen geometry and, if desired, for temperature, and suitable differences are taken to obtain the resistivity variation.

4. Significance and Use

4.1 The radial resistivity variation of bulk semiconductor material is an important materials acceptance requirement for

[1] This test method is under the jurisdiction of ASTM Committee F1 on Electronics, and is the direct responsibility of Subcommittee F01.06 on Silicon Materials and Process Control.

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[2] DIN 50435 is an equivalent method. It is the responsibility of DIN Committee NMP 221, with which Committee F-1 maintains close liaison. DIN 50435, Determination of the Radial Resistivity Variation of Silicon or Germanium Slices by Means of a Four-Point DC-Probe, is available from Beuth Verlag GmbH, Burggrafenstrasse 4-10, D-1000 Berlin 30,

[3] Swartzendruber, L. J., "Correction Factor Tables for Four-Point Probe Resistivity Measurements on Thin Circular Semiconductor Samples," *Technical Note 199*, NBTNA, National Bureau of Standards, April 15, 1964. Available as AD 683 408 from National Technical Information Service, Springfield, Va. 22161.

[4] *Annual Book of ASTM Standards*, Vol 10.05.

[5] Available from Semiconductor Equipment and Materials International, 805 East Middlefield Road, Mountain View, CA 94043.

semiconductor device fabrication and is also used for quality control purposes.

4.2 The four-point probe method provides a test that requires little specimen preparation and that is nondestructive in that the wafer is left intact. The method can be applied to wafers using the resistivity-measuring apparatus and procedures of Test Method F 84 if provisions are made for making measurements at several sites on the wafer (see 6.1). Appropriate correction factors must be applied to compensate for effects of the wafer geometry (see 11.1).

4.3 Radial resistivity variations are a function of the crystal growth process and dopant, both in characteristic shape and magnitude. Because no single sampling plan is adequate to characterize the resistivity variations of all crystal types or for all applications, four sampling plans are included in this test method.

5. Interferences

5.1 *Current Level*— The current levels as a function of resistivity recommended in Test Method F 84 have been found satisfactory for the specified probe spacing and specimen size range. However, should smaller than recommended probe spacing be used, or very long lifetime specimens be measured, the suitability of the recommended currents should be established by doubling and halving the recommended current and checking for a resulting doubling and halving of measured specimen voltage. It is then recommended that a current near the middle of the range giving a constant measure of resistivity be used.

5.2 *Longitudinal Resistivity Variations*—The local fluctuations in dopant density which cause resistivity variations on a cross section of a semiconductor crystal also cause axial resistivity variations down the length of the crystal. The four-point probe method measures averaged local resistivity values, and these averaged values are influenced by resistivity changes through the thickness of the specimens. The extent of this influence depends on probe spacing. Radial variation measurements on the front and back sides of a wafer may differ because of longitudinal variations.

5.3 *Accuracy of Probe Placement*—The position of the probe head may have a noticeable effect on the measured voltage-to-current ratio because of the proximity of the probe tips to a wafer boundary. Geometrical correction factors used to calculate the local resistivity from the measured voltage and current values are calculated for a particular position of the probe head with respect to the wafer center and wafer boundaries. Appendix X1 gives magnitudes of the error in the geometrical correction factor and in resulting local resistivity values if the position of a probe with a 1.59-mm probe tip spacing shifted the maximum allowed value, 0.15 mm (0.006 in.), toward the edge of the wafer. These errors decrease with decreasing probe spacing for all wafer sizes and measurement sites.

5.4 *Wafer-Geometry Related Errors*:

5.4.1 The geometrical correction factors used to calculate the local resistivity from the measured voltage and current ratios depend on the assumptions of full circular wafer geometry and of nonconducting wafer back side and edges. As a result, some error is introduced if measurements are made in proximity to an orientation flat on a wafer, or if the wafer surfaces are conducting.

5.4.2 Additional errors in the correction factor are encountered if the true wafer diameter is not used in calculating the correction factor. Use of the nominal diameter for all wafers of standard dimensions with diametral tolerances allowed by SEMI Specifications M 1 introduces negligible error if measurements are made no closer to the edge of the wafer than 6 mm. Appendix X2 gives magnitudes of the error in the geometrical correction factor and in the resulting local resistivity values which result when the nominal wafer diameter is used in the calculation for specimen which have the smallest diameter allowed by SEMI Specifications M 1.

5.4.3 The wafer thickness enters directly into the calculation of resistivity from the measured voltage-to-current ratio. Appendix X2 gives magnitudes of the error in the local resistivity values when the nominal wafer thickness is used in the calculation for wafers with the smallest center-point thickness allowed by SEMI Specification M 1 and a local thickness that deviates from the nominal value by (*1*) the maximum allowed by SEMI Specifications M 1 or (*2*) the 13 µm (0.0005 in.) allowed by Test Method F 84. If more accurate determinations of local resistivity are required, (*1*) the thickness at each measurement site should be determined and used in calculating the resistivity at that site, (*2*) wafers with smaller thickness variation should be employed, or (*3*) thicker wafers should be employed.

5.5 *Polished Surfaces*—Measurements on a polished rather than a lapped wafer surface as required in this method will in general give satisfactory measurement results.[6] However, the possibility of measurement errors due to surface conduction or to low surface recombination velocity requires the use of lapped wafer surfaces for referee measurements.

5.6 Temperature fluctuations of specimen temperature during the measurement time will affect the measurement. This can be corrected if the specimen temperature is known (see 11.1.4 and Note 4).

6. Apparatus

6.1 Apparatus as specified in Test Method F 84 is required for four-point probe measurement, except that the specimen support shall include an *x-y* stage with micrometer adjustment capable of positioning the probe head at specified points on the specimen with an accuracy of ±0.15 mm; it shall also include provision for rotating the specimen through 360° with a rotational accuracy of ±5°.

7. Sampling

7.1 The sampling plan for selection of wafers from a lot shall be agreed upon by the parties to the measurement.

7.2 This test method provides four sampling plans (see Fig. 1) for the selection of the sites where measurements are to be

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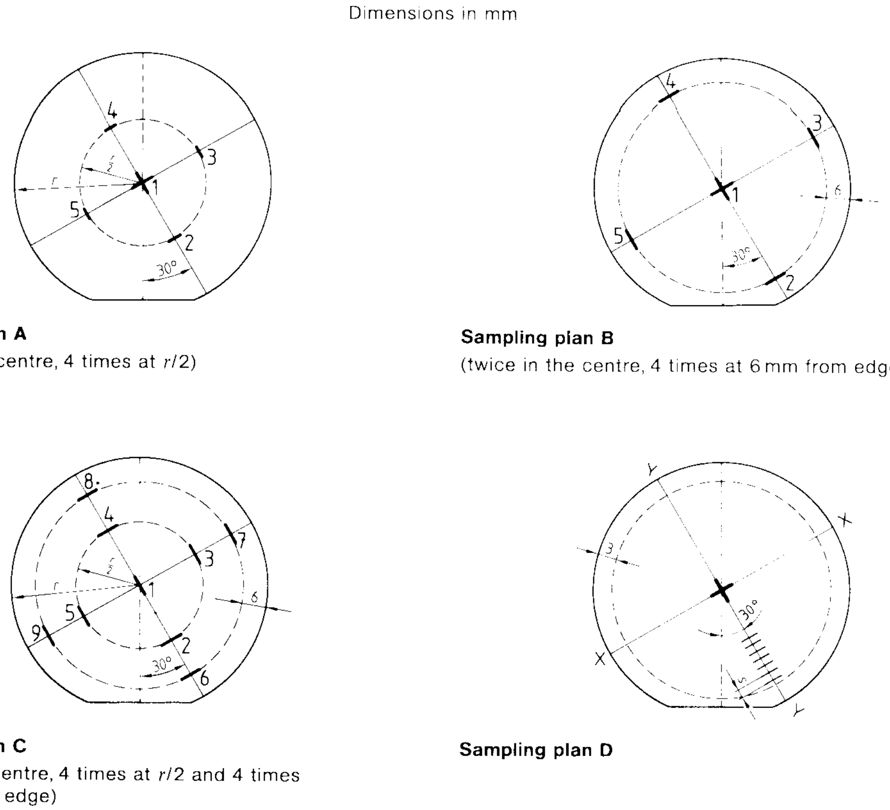
[6] Ehrstein, J. R., Brewer, F. H., Ricks, D. R., and Bullis, W. M.,“ Effects of Current, Probe, Force and Wafer Surface Condition on Measurement of Resistivity of Bulk Silicon Wafers by the Four-Probe Method,” Appendix E, “Methods of Measurement for Semiconductor Materials, Process Control, and Devices,” *Technical Note 773*, NBTNA, National Bureau of Standards, June 1973, pp. 43–49. Available as COM 73-50534 from National Technical Information Service, Springfield, Va. 22161.



NOTE—The flat shown at the bottom of each figure represents the primary flat or fiducial (see 8.1). Each of the small line segments is perpendicular to a slice diameter and shows the location and orientation of the four-point probe array for resistivity measurement at the site identified by the adjacent number.

FIG. 1 Sampling Plans for Four-Point Probe Measurement of Radial Resistivity Variation

made on a specimen and from which radial resistivity variations can be determined. A sampling plan shall be chosen from those given on the basis of device application, growth process, and dopant, and of the consequent level of resistivity information desired.

7.2.1 *Sampling Plan A, Small-Area Cross Pattern*—Six measurements are made: two at the center of the wafer and four at half radius (*R/2*) points.

7.2.2 *Sampling Plan B, Large-Area Cross Pattern*—Six measurements are made: two at the center of the wafer and four 6.0 mm (0.24 in.) from the wafer edge.

7.2.3 *Sampling Plan C, Small-Area and Large-Area Cross Patterns*—Nine measurements are made: one at the center of the wafer, four at half–radius (R/2) and four at 6.0 mm (0.24 in.) from the wafer edge.

7.2.4 *Sampling Plan D, Single-Diameter, High-Resolution Pattern*—Measurements are made at the center of the wafer and at as many additional sites as possible along a diameter at intervals of 2 mm between the center and each edge with the exclusion of the outer 3 mm of the sample at each end of the diameter.

NOTE 2—Because of the extent of the area over which the four-point probe array samples resistivity, little additional information is gained by using an interval smaller than 2 mm.

8. Test Specimen

8.1 Prepare the surface to be measured in accordance with the Preparation of Test Specimen Section of Test Method F 84. If the wafer does not have orientation flats as specified in SEMI Specifications M 1, place a reference mark on the periphery of the back surface. Use this mark in place of the principal orientation flat for purposes of wafer alignment during measurement. If a referee measurement is being made and if the wafer has only a single orientation flat, place a reference mark on the edge of the back side at the midpoint of the orientation flat.

8.2 Measure and note the diameter of the specimen along any three diameters separated by approximately 45° which do not intersect a wafer orientation flat. If each of these diameter values is within the range specified in SEMI Specifications M 1, record as the diameter the nominal standard value. If not, record as the diameter the average of the three measured values.

8.3 Using the thickness gage specified in the Apparatus Section of Test Method F 84, measure and record the specimen thickness at the nine sites of Sampling Plan C (Fig. 1C). Accept a specimen for measurement if the total thickness variation is less than 13 µm (0.0005 in.) (see 5.4.3).

9. Suitability of Test Equipment

9.1 Determine the suitability of the four-point probe and electronics for use in measuring resistivity in accordance with the Suitability of Test Equipment Section of Test Method F 84.

10. Procedure

10.1 Align the specimen so that the first measurement diameter is located 30° counterclockwise from the diameter perpendicular to the major orientation flat or from the diameter through the reference mark (see 8.1 and Fig. 1). For referee measurements, record the orientation of the measurement sites with respect to reference mark or flats (see 12.4).

10.2 Choose one of the sampling plans (see 7.2 and Fig. 1).

10.3 Measure and record the temperature of the specimens if absolute values of resistivity are desired.

10.4 For each site indicated for measurement on the chosen plan:

10.4.1 Place the four-point probe array on the surface of the specimen so that (*1*) the imaginary line joining the probe points is perpendicular to the specimen radius that passes through the site, and (*2*) the midpoint of the line is within ± 0.15 mm (±0.006 in.) of the site.

10.4.2 Measure the forward and reverse resistance once in accordance with the Procedure Section of Test Method F 84.

10.4.3 If the wafer has a nonstandard diameter, determine and record Δ , the distance from the center of the specimen to the midpoint of the probe pins.

11. Calculations

11.1 For each measurement site:

11.1.1 Calculate and note the mean value of resistance in accordance with the Calculation Section of Test Method F 84.

11.1.2 If the wafer has a standard diameter (see 8.2), determine the value of the correction factor, F_2, from Table 1.

11.1.3 If the diameter of the wafer is nonstandard:

11.1.3.1 Calculate Δ/r, the ratio of the distance between the measurement site and the wafer center (see 10.4.3) to one half of the average diameter of the wafer (see 8.2).

11.1.3.2 Determine the correction factor, F_2, from Table 1 of *NBS Technical Note 199*.[3]

NOTE 3—The procedure of 11.1.3 must also be used if the probe-tip spacing is not 1.59 mm (0.0625 in.).

11.1.4 If absolute values of resistivity are desired, calculate and record the resistivity at the temperature of the measurement in accordance with the Calculations Section of Test Method F 84.

NOTE 4—Temperature correction can generally be ignored if only a measure of change of resistivity with position is desired. Fluctuations of specimen temperature of no greater than 2°C during the course of measurement will cause an error in calculated resistivity variation that does not exceed 2 %.

11.2 If Sampling Plan A, B or C is chosen, calculate both the average percent variation of radial resistivity and the maximum percent variation of radial resistivity as follows:

$$\text{average percent variation} = 100\,|(\rho_a - \rho_c)/\rho_c| \quad (1)$$

where:

ρ_c = average of the two resistivity values at the center of the wafer, Ω·cm, and

ρ_a = average of the two resistivity values measured at a common distance from wafer center, such as for Plan A, at half radius, for Plan B, at 6 mm (0.24 in.) from the edge, and for Plan C, using separate applications of Eq 1 to measurements at half-radius and to measurements at 6 mm from the edge.

$$\text{maximum percent variation} = 100\,|(\rho_e - \rho_c)/\rho_c| \quad (2)$$

where:

ρ_c is the same for Eq 1, and

ρ_e = the single off-center measurement value that is the most different from the value at the center; for Plan C, it is chosen from among all eight off-center measurements without regard to location.

NOTE 5—Note that both Eq 1 and Eq 2 calculate the absolute value of the variation.

11.3 If Sampling Plan D was chosen, calculate the maximum percent variation of resistivity as follows:

$$\text{maximum percent variation} = [(\rho_M - \rho_m)/\rho_m] \times 100 \quad (3)$$

where:

ρ_M = maximum resistivity value measured,Ω· cm, and

ρ_m = minimum resistivity value measured, Ω·cm.

NOTE 6—It should be noted that Sampling Plan D includes measurement sites in the outer 6.0-mm (0.24-in.) annulus of the wafer; hence errors related to probe head position and wafer geometry may be appreciable (see Appendix X2).

12. Report

12.1 Report the following information:

12.1.1 Specimen identification,

12.1.2 Operator,

12.1.3 Date,

12.1.4 Sampling plan used,

12.1.5 Magnitude of measuring current, mA,

12.1.6 Probe-tip spacing, mm, and

12.1.7 Wafer diameter, mm.

NOTE 7—The diameter of standard 2 and 3-in. diameter wafers may be reported in inches.

12.2 If Sampling Plan A or B was chosen, report both the average percent-variation of radial resistivity and the maximum percent-variation of radial resistivity, (see 11.2).

12.3 If Sampling Plan C was chosen, report the average percent-variation of radial resistivity both for measurements at half-radius and for measurements 6 mm from the edge. Report also the maximum percent-variation of radial resistivity for the entire eight off-center measurements (see 11.2).

12.4 If Sampling Plan D was chosen report both a plot of all measurement values as a function of position along the diameter and the calculated maximum percent variation (see 11.3) together with the maximum and minimum values.

NOTE 8—The report may also include either of the following summaries of results as agreed to by the parties to the measurement: (*1*) calculated resistivity, Ω·cm, at each measurement site, or (*2*) calculated resistivity, Ω·cm, at the center of the wafer together with the maximum and minimum values of resistivity, Ω·cm and their location. In these cases the temperature for which the resistivity was determined should be given; measurement sites may be identified by the numbers shown in Fig. 1.

12.5 For referee purposes, the report shall identify by sketch of the measured surface, the diameter or diameters measured with respect to the reference fiducials (see 8.1).

13. Precision and Bias

13.1 The precision of the radial variation measurement is directly dependent on the precision of the individual resistivity measurements, and in this regard it is approximately inversely proportional to the size of the radial variation measured. If only the precision of individual measurements is considered a source of error, and probe position and wafer diameter are correct, the precision of the radial variations, as calculated in Eq 1 or Eq 3, can be computed (see Appendix X2). A summary of these results is given in Table X2.1. Values of expected precision of well-controlled resistivity measurements can be found in the Precision Section of Test Method F 84.

13.2 Errors in calculated resistivity values may result from errors in the correction factor, F_2, resulting from probe position error or diameter tolerance errors on standard wafers or from errors in wafer thickness. Values of errors due to these causes are given in Appendix X2 for the case of measurement with a probe-tip spacing of 1.59 mm (0.0625 in.).

TABLE 1 Correction Factor F_2 for Circular Slices with Standard Diameters and Probe-Tip Spacing of 1.59 mm (0.0625 in.)

NOTE 1—Values below line in each column correspond to measurement sites 6 mm or nearer to the wafer edge.

Measurement Site	Nominal Slice Diameter						
	2 in.	3 in.	100 mm	125 mm	150 mm	200 mm	300 mm
				Sampling Plans A, B, and C			
Center	4.494	4.515	4.522	4.526	4.528	4.530	4.531
Half-Radius	4.465	4.502	4.515	4.521	4.525	4.528	4.530
6 mm from Edge	4.328	4.345	4.352	4.357	4.360	4.363	4.367
Distance between Measurement Site and Wafer Center, mm					Sampling Plan D		
0	4.494	4.515	4.522	4.526	4.528	4.530	4.531
2	4.494	4.515	4.522	4.526	4.528	4.530	4.531
4	4.492	4.515	4.522	4.526	4.528	4.530	4.531
6	4.490	4.514	4.522	4.526	4.528	4.530	4.531
8	4.486	4.514	4.522	4.526	4.528	4.530	4.531
10	4.479	4.513	4.522	4.526	4.528	4.530	4.531
12	4.470	4.511	4.521	4.526	4.528	4.530	4.531
14	4.455	4.510	4.521	4.525	4.528	4.530	4.531
16	4.430	4.507	4.520	4.525	4.528	4.530	4.531
18	4.385	4.504	4.519	4.525	4.527	4.530	4.531
20	4.290	4.500	4.518	4.524	4.527	4.530	4.531
22	4.040	4.494	4.517	4.524	4.527	4.530	4.531
24	3.167	4.486	4.516	4.524	4.527	4.530	4.531
26		4.474	4.514	4.523	4.527	4.530	4.531
28		4.454	4.511	4.522	4.526	4.529	4.531
30		4.419	4.508	4.522	4.526	4.529	4.531
32		4.350	4.504	4.521	4.526	4.529	4.531
34		4.181	4.498	4.520	4.525	4.529	4.531
36		3.633	4.490	4.518	4.525	4.529	4.531
38			4.478	4.516	4.524	4.529	4.531
40			4.458	4.514	4.524	4.529	4.531
42			4.423	4.511	4.523	4.529	4.531
44			4.352	4.508	4.522	4.529	4.531
46			4.177	4.502	4.521	4.528	4.531
48			3.594	4.495	4.520	4.528	4.531
50				4.484	4.518	4.528	4.531
52				4.467	4.516	4.528	4.531
54				4.437	4.513	4.527	4.531
56				4.380	4.510	4.527	4.531
58				4.244	4.505	4.527	4.531
60				3.827	4.499	4.526	4.531
62					4.489	4.526	4.531
64					4.474	4.525	4.531
66					4.449	4.525	4.531
68					4.401	4.524	4.531
70					4.295	4.523	4.531
72					3.990	4.522	4.531
74					2.888	4.520	4.530
76						4.519	4.530
78						4.516	4.530
80						4.513	4.530
82						4.509	4.530
84						4.504	4.530
86						4.496	4.530
88						4.485	4.530
90						4.466	4.530
92						4.432	4.530

TABLE 1 *Continued*

Measurement Site	Nominal Slice Diameter						
	2 in.	3 in.	100 mm	125 mm	150 mm	200 mm	300 mm
94						4.363	4.529
96						4.191	4.529
98						3.614	4.529
100							4.529
102							4.529
104							4.528
106							4.528
108							4.528
110							4.527
112							4.527
114							4.526
116							4.526
118							4.525
120							4.524
122							4.523
124							4.521
126							4.520
128							4.518
130							4.515
132							4.511
134							4.506
136							4.498
138							4.487
140							4.468
142							4.435
144							4.367
146							4.196
148							3.620

14. Keywords

14.1 four-point probe; resistivity; resistivity variation; semiconductor; silicon; uniformity

APPENDIXES

(Nonmandatory Information)

X1. TABLES OF MEASUREMENT ERRORS RESULTING FROM PROBE POSITION ERROR AND FROM WAFER-GEOMETRY RELATED ERRORS

X1.1 Table X1.1 gives examples of worst-case errors in calculated resistivity which arise from probe location and diameter tolerance errors. Table X1.2 gives examples of worst-case errors in calculated resistivity if the local thickness deviates from the nominal value.

TABLE X1.1 Worst-Case Errors in Calculated Resistivity Resulting from Probe Location and Diameter Tolerance Errors

Nominal Wafer Diameter	Nominal Probe Location	Sampling Plan	E_1 [A]	E_2 [B]	E_3 [C]
2 in.	wafer center	A, B, C, D	0.0 %	0.0 %	0.0 %
2 in.	*R*/2	A, C	0.0 %	0.1 %	0.1 %
2 in.	6 mm from edge	B, C	0.2 %	0.3 %	0.5 %
2 in.	20 mm from center	D	0.3 %	0.4 %	0.7 %
2 in.	22 mm from center	D	0.9 %	1.1 %	2.0 %
2 in.	24 mm from center	D	3.8 %	4.9 %	8.8 %
3 in.	wafer center	A, B, C, D	0.0 %	0.0 %	0.0 %
3 in.	*R*/2	A, C	0.0 %	0.0 %	0.0 %
3 in.	6 mm from edge	B, C	0.2 %	0.4 %	0.7 %
3 in.	32 mm from center	D	0.2 %	0.4 %	0.7 %
3 in.	34 mm from center	D	0.5 %	1.2 %	1.8 %
3 in.	36 mm from center	D	2.2 %	4.9 %	7.5 %
100 mm	wafer center	A, B, C, D	0.0 %	0.0 %	0.0 %
100 mm	*R*/2	A, C	0.0 %	0.0 %	0.0 %
100 mm	6 mm from edge	B, C, D	0.2 %	0.3 %	0.5 %
100 mm	46 mm from center	D	0.6 %	1.0 %	1.6 %
100 mm	48 mm from center	D	2.5 %	4.4 %	6.9 %
125 mm	wafer center	A, B, C, D	0.0 %	0.0 %	0.0 %
125 mm	*R*/2	A, C	0.0 %	0.0 %	0.0 %
125 mm	56 mm from edge	D	0.1 %	0.2 %	0.4 %
125 mm	6 mm from center	B, C	0.2 %	0.3 %	0.5 %
125 mm	58 mm from center	D	0.3 %	0.7 %	1.1 %
125 mm	60 mm from center	D	1.5 %	2.9 %	4.4 %
150 mm	wafer center	A, B, C, D	0.0 %	0.0 %	0.0 %
150 mm	*R*/2	A, C	0.0 %	0.0 %	0.0 %
150 mm	6 mm from edge	B, C	0.2 %	0.2 %	0.4 %
150 mm	70 mm from center	D	0.3 %	0.3 %	0.6 %
150 mm	72 mm from center	D	1.3 %	0.6 %	1.9 %
150 mm	74 mm from center	D	4.4 %	2.2 %	6.6 %
200 mm	wafer center	A, B, C, D	0.0 %	0.0 %	0.0 %
200 mm	*R*/2	A, C	0.0 %	0.0 %	0.0 %
200 mm	6 mm from center	B, C, D	0.2 %	0.1 %	0.2 %
200 mm	96 mm from center	D	0.9 %	0.5 %	0.9 %
200 mm	98 min from center	D	3.7 %	1.9 %	3.7 %
300 mm	wafer center	A, B, C, D	0.0 %	0.0 %	0.0 %
300 mm	*R*/2	A, C	0.0 %	0.0 %	0.0 %
300 mm	6 mm from edge	B, C, D	0.2 %	0.2 %	0.4 %
300 mm	146 mm from center	D	0.9 %	0.9 %	1.8 %
300 mm	148 mm from center	D	2.1 %	0.0 %	2.1 %

[A] E_1 = magnitude of error in local resistivity calculated using the correction factors in Table 1 if the probe is displaced 0.15 mm (0.006 in.) toward the edge of the wafer.
[B] E_2 = magnitude of error in local resistivity calculated using the correction factors in Table 1 if the wafer has the smallest diameter allowed by SEMI Specifications M 1.
[C] E_3 = magnitude of error in local resisitivity calculated using the correction factors in Table 1 if the probe is displaced 0.15 mm (0.006 in.) toward the edge of the wafer and if the wafer has the smallest diameter allowed by SEMI Specifications M 1.

TABLE X1.2 Error in Calculated Resistivity if Local Thickness Deviates from Nominal Value for Wafers with Minimum Thickness Allowed by SEMI Specification M

Nominal Wafer Diameter	Minimum		Test Method F 84		SEMI M1	
	Thickness	Diameter	Δt	Error,%	Δt	Error, %
2 in.	254 µm	1.985 in.	13µ m	5.1	0.0005 in.	5.0
3 in.	356 µm	2.975 in.	13 µm	3.7	0.0010 in.	7.1
100 mm	505 µm	99.5 mm	13 µm	2.2	10 µm	1.9
125 mm	605µ m	124.5 mm	13 µm	2.2	10 µm	1.7
150 mm	655 µm	149.8 mm	13 µm	2.0	10 µm	1.5
200 mm	705 µm	199.8 mm	13µ m	1.8	10 µm	1.1
300 mm	750 µm	299.75 mm	13 µm	1.7	25 µm	3.3

X2. CALCULATION OF VARIANCE OF RADIAL RESISTIVITY VARIATION BASED ON THE VARIANCE OF INDIVIDUAL RESISTIVITY MEASUREMENTS

X2.1 This calculation is used to estimate the expected precision in radial resistivity variation measurements calculated in 11.2 or 11.3 resulting from the variability experienced in the resistivity measurements at individual sites. Results of this calculation for typical measurement situations are illustrated in Table X2.1.

X2.1.1 Errors in individual resistivity determinations due to errors in probe placement, wafer diameter, and wafer thickness are not accounted for here. Such errors may result in noticeably different estimates of radial variation, y, between different laboratories or in repeated measurements within a single laboratory. If such errors are present it is not meaningful to pool results using Eq X5.

X2.1.2 The effect of errors in probe placement, wafer

TABLE X2.1 Calculated Precision of Radial Resistivity Variation, σ (*y*), as a Function of Relative Radial Variation, *y*; Relative Standard Deviation, %, of Individual Resistivity Measurements, Σ(ρ); and Number of Measurements Taken, *J* and *K*

NOTE 1—The values *J* = 4 and *K* = 2 correspond to one set of data from either of the cross-pattern sampling plans, A or B. The values *J* = 8 and *K* = 4 are applicable either to a single laboratory's measurements using two sets of data from cross-pattern Sampling Plan A or B, or to a two-laboratory test, using cross-pattern plan A or B with one set of data from each laboratory. For other sampling plans, or if radial variation is calculated from maximum and minimum resistivity values, the values for *J* and *K* should be determined according to the definitions of *J* and *K* following Eq X2.2. If repeated sets of data are taken or multilaboratory results are used, use for *y* the grand mean value of the several measured relative radial variations, y_i, and inflate *J* and *K* according to the number of sets of measurements used.

Part A—Precision Expressed as Two Standard Deviations [2 σ (y)]

Σ(ρ)	*J* = 4, *K* = 2					*J* = 8, *K* = 4				
	y = 0.01	0.05	0.10	0.25	0.50	*y* = 0.01	0.05	0.10	0.25	0.50
0.5 %	0.0087	0.0090	0.0092	0.0102	0.0177	0.0062	0.0063	0.0065	0.0072	0.0083
1.0 %	0.0174	0.0179	0.0185	0.0203	0.0235	0.0123	0.0127	0.0131	0.0144	0.0166
1.5 %	0.0262	0.0269	0.0277	0.0305	0.0352	0.0185	0.0190	0.0196	0.0215	0.0249
2.0 %	0.0349	0.0358	0.0370	0.0406	0.0469	0.0247	0.0253	0.0262	0.0287	0.0332
2.5 %	0.0436	0.0448	0.0462	0.0508	0.0586	0.0308	0.0316	0.0327	0.0359	0.0415

Part B—Precision Expressed as Two Relative Standard Deviations {[2 σ (y) / y] × 100 %}

Σ(ρ)	*J* = 4, *K* = 2					*J* = 8, *K* = 4				
	y = 0.01	0.05	0.10	0.25	0.50	*y* = 0.01	0.05	0.10	0.25	0.50
0.5 %	87 %	18 %	9.3 %	4.1 %	2.3 %	62 %	13 %	6.5 %	2.9 %	1.7 %
1.0 %	174 %	36 %	19 %	8 %	5 %	124 %	26 %	13 %	6 %	3.4 %
1.5 %	261 %	54 %	28 %	12 %	7 %	186 %	39 %	20 %	9 %	5.1 %
2.0 %	348 %	72 %	37 %	16 %	9 %	248 %	52 %	26 %	12 %	6.8 %
2.5 %	435 %	90 %	46 %	20 %	12 %	310 %	65 %	33 %	15 %	8.5 %

diameter, and wafer thickness on individual resistivity determinations is tabulated for extreme cases in Appendix X2.

X2.2 *Derivation of Variance Relationship*—Divide average percent variation (Eq 1), or the maximum percent variation (Eq 2 and Eq 3), by 100, to obtain y, the relative radial variation of resistivity as a fraction. This may be expressed as follows:

$$y = \frac{\rho_2 - \rho_1}{\rho_1} = \left(\frac{\rho_2}{\rho_1}\right) - 1 \tag{X2.1}$$

where:

y = relative radial variation of resistivity,
ρ_2 = ρ_a in Eq 1, ρ_e in Eq 2, or ρ_M in Eq 3, and
ρ_1 = ρ_c in Eq 1 or Eq 2, or ρ_m in Eq 3.

X2.2.1 This expression may be written in a different form as follows:

$$y = \left(K \sum_{i=1}^{J} \rho_i / J \sum_{i=J+1}^{J+K} \rho_i\right) - 1 \tag{X2.2}$$

where:

J = number of measurements taken in positions of the type ρ_2,
K = number of measurements taken in positions of the type ρ_1, and
ρ_1 = resistivity measured at position *i*, Ω·cm.

X2.2.2 Then: [7]

$$\sigma^2(y) = \sum_{i=1}^{J+K} \left(\frac{\delta y}{\delta \rho_i}\right)^2 \cdot \sigma^2(\rho_i) \tag{X2.3}$$

where:

$\sigma^2(y)$ = variance of a radial variation measurement for any of the calculation forms used, Eq 1, Eq 2, or Eq 3, and
$\sigma^2(\rho_i)$ = variance of a measurement of ρ_i.

X2.2.3 Designate the ratio ρ_2/ρ_1 as *r*. Obtain the derivatives from Eq X2.2 and perform the summation in Eq X2.3 to find:

$$\sigma^2(y) = \left[\frac{\sigma^2(\rho)}{\rho_i^2}\right] \cdot \left(\frac{1}{J} + \frac{r^2}{K}\right) \tag{X2.4}$$

where it has been assumed that all the $\sigma^2(\rho_i)$ are equal and are given by $\sigma^2(\rho)$.

X2.2.4 Write σ(ρ), the absolute standard deviation of individual resistivity measurements, in terms of Σ(ρ), the relative standard deviation (percent) of the individual resistivity measurements:

$$\sigma(\rho) = \frac{\Sigma(\rho)\cdot\rho}{100} \approx \frac{\Sigma(\rho)\cdot\rho_1}{100} \tag{X2.5}$$

Eq X2.4 can be rewritten to remove dependence on the resistivity level of the specimen being considered as follows:

$$\sigma^2(y) = \left[\left(\frac{\Sigma(\rho)}{100}\right)^2\right]\left(\frac{1}{J} + \frac{r^2}{K}\right) \tag{X2.6}$$

Expected values of Σ(ρ) of well-controlled resistivity measurements are given in the Precision Section of Test Method F 84.

X2.3 The preferred form for expression of complete results of radial resistivity variation measurements is obtained by combining calculated radial resistivity variation with its attendant uncertainty at the 95 % confidence, or 2σ, level, and expressing this combination as a percent:

$$\{[y \pm 2\,\sigma(y)] \times 100\}\ \% \tag{X2.7}$$

X2.4 As an example of the calculation of results in this

[7] Volk, W., Applied Statistics for Engineers, McGraw-Hill Book Co., New York, N.Y., 1969, p. 154.

form, assume a typical set of test results obtained by a single laboratory using either Sampling Plan A or B on a wafer of any resistivity with a 25 % measured difference between ρ_1 and ρ_2 and a relative standard deviation, percent, of individual resistivity measurements, (ρ), of 0.5 %:

y = 0.25,
r = 1.25,
$\Sigma(\rho)$ = $\pm$0.5 %,
J = 4, and
K = 2.

X2.4.1 Using these values, and substituting into Eq X2.6,

$$\sigma^2(y) = [(0.5/100)^2]\{(1/4) + [1.25)^2/2]\}$$

$$\sigma(y) = \pm 0.00508$$

$$2\sigma(y) = \pm 0.0102$$

where $\sigma(y)$ = estimated standard deviation of a radial resistivity variation measurement.

X2.4.2 The final expression for radial resistivity variation with its uncertainty is then:

$$\{[y \pm 2\sigma(y)] \times 100\}\,\% = (25 \pm 1.02)\,\%$$

X2.5 As a second example consider a specimen with a relative radial resistivity variation, y, of 0.01. Again substituting into Eq X2.5, using the same values of $\Sigma(\rho)$, J, and K:

$$\sigma^2(y) = [(0.5/100)^2][1/4 + (1.0)^2/2]$$

$$\sigma(y) = \pm 0.00436$$

$$2\sigma(y) = \pm 0.00872$$

X2.5.1 The final expression for radial resistivity variation with its uncertainty is then:

$$\{[y \pm 2\sigma(y)] \times 100\}\,\% = (1 \pm 0.87)\,\%$$

X2.6 It is common to write the uncertainty, or standard deviation of a measured quantity, as a separate parameter and to express it as a percent of that measured quantity, as is done for individual resistivity measurements. From Table X2.1 it can be seen that for a fixed standard deviation of individual resistivity measurements, the absolute standard deviation of radial variation is nearly constant, regardless of the size of the radial resistivity variation while the relative standard deviation, expressed as a percentage of the radial variation, has the appearance of degrading the quality of the measurement if the relative resistivity variation, y, is small. Therefore it is not appropriate to express the uncertainty in radial resistivity variation as a percentage of the radial variation.