Designation: E 666 – 9703

Standard Practice for
Calculating Absorbed Dose From Gamma or X Radiation[1]

This standard is issued under the fixed designation E 666; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the Department of Defense.

1. Scope

1.1 This practice presents a technique for calculating the absorbed dose in a material from knowledge of the radiation field, the

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[1] This practice is under the jurisdiction of ASTM Committee E-10 on Nuclear Technology and Applications and is the direct responsibility of Subcommittee E10.07 on Radiation Dosimetry for Radiation Effects on Materials and Devices.

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composition of the material, **(1-5)**[2,3] and a related measurement. The procedure is applicable for X and gamma radiation provided the energy of the photons fall within the range from 0.01 to 20 MeV.

1.2 A method is given for calculating the absorbed dose in a material from the knowledge of the absorbed dose in another material exposed to the same radiation field. The procedure is restricted to homogeneous materials composed of the elements for which absorption coefficients have been tabulated **(2).** It also requires some knowledge of the energy spectrum of the radiation field produced by the source under consideration. Generally, the accuracy of this method is limited by the accuracy to which the energy spectrum of the radiation field is known.

1.3 The results of this practice are only valid if charged particle equilibrium exists in the material and at the depth of interest. Thus, this practice is not applicable for determining absorbed dose in the immediate vicinity of boundaries between materials of widely differing atomic numbers. For more information on this topic, see Practice E 1249.

1.4 Energy transport computer codes[4] exist that are formulated to calculate absorbed dose in materials more precisely than this method. To use these codes, more effort, time, and expense are required. If the situation warrants, such calculations should be used rather than the method described here.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*
E 170 Terminology Relating to Radiation Measurements and Dosimetry[5]
E 380 Practice for Use of the International System of Units (SI) (the Modernized Metric System)[6]
E 665 Practice for Determining Absorbed Dose Versus Depth in Materials Exposed to the X-Ray Output of Flash X-Ray Machines[5]
E 668 Practice for Application of Thermoluminescence-Dosimetry (TLD) Systems for Determining Absorbed Dose in Radiation-Hardness Testing of Electronic Devices[5]
E 1249 Practice for Minimizing Dosimetry Errors in Radiation Hardness Testing of Silicon Electronic Devices Using Co-60 Sources[5]

2.2 *International Commission on Radiation Units and Measurements (ICRU) Reports:*
ICRU Report 14—Radiation Dosimetry: X Rays and Gamma Rays with Maximum Photon Energies Between 0.6 and 60 MeV[7]
ICRU Report 18—Specification of High Activity Gamma-Ray Sources[7]
ICRU Report 21—Radiation Dosimetry: Electrons with Initial Energies Between 1 and 50 MeV[7]
ICRU Report 33—Radiation Quantities and Units[7]
ICRU Report 34—The Dosimetry of Pulsed Radiation[7]

3. Significance and Use

3.1 The absorbed dose is a more meaningful parameter than exposure for use in relating the effects of radiation on materials. It expresses the energy absorbed by the irradiated material per unit mass, whereas exposure is related to the amount of charge produced in air per unit mass. Absorbed dose, as referred to here, implies that the measurement is made under conditions of charged particle (electron) equilibrium (see Appendix X1). In practice, such conditions are not rigorously achievable but, under some circumstances, can be approximated closely.

3.2 Different materials, when exposed to the same radiation field, absorb different amounts of energy. ~~In~~ Using the techniques of this standard, charged particle equilibrium must exist in order to relate the absorbed dose in one material to the absorbed dose in ~~another material, the condition of charged particle equilibrium must exist. On the other hand,~~ another. Also, if the radiation is attenuated by a significant thickness of an absorber, the energy spectrum of the radiation will be changed, and it will be necessary to correct for this.

NOTE 1—For comprehensive discussions of various dosimetry methods applicable to the radiation types and energies and absorbed dose rate ranges discussed in this method, see ICRU Reports 14, 21, and 34.

4. Calculation of Absorbed Dose

4.1 The absorbed dose, *D*, at a point may be expressed as:

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[2] The boldface numbers in parentheses refer to the list of references appended to this practice.

[3] For calculation of absorbed dose in biological materials such as tissue or bone, etc., ICRU Report 14 provides more information and procedures for a more accurate calculation than this practice.

[4] Information on and packages of computer codes can be obtained from The Radiation Safety Information Computational Center, Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, TN 37831-6362. This information center collects, organizes, evaluates, and disseminates shielding information related to radiation from reactors, weapons, and accelerators and to radiation occurring in space.

[5] *Annual Book of ASTM Standards*, Vol 12.02.

[6] *Annual Book of ASTM Standards*, Vol 14.02.

[7] Available from International Commission on Radiation Units and Measurements (ICRU), 7910 Woodmont Ave., Suite 800, Bethesda, MD 20814.

$$D = I \int_0^\infty \psi(E)[\ \mu_{en}(E)/\rho]\ dE \tag{1}$$

where $\psi(E)$ is the energy fluence per unit energy at the point of interest; $\mu_{en}(E)/\rho$ is the mass energy absorption coefficient **(2)**; and I is a normalizing factor. If all of the variables in Eq 1 are expressed in SI units, I = 1. In this case the units for D are Gy (J kg^{-1}), of $\psi(E)$, are m^{-2}, of μ_{en}/ρ are m^{2}kg^{-1}, and of E are J. For an alternative use of the normalizing factor I, see Appendix X2. For further information on the use of energy absorption coefficients to calculate absorbed dose see the discussion in Attix **(1)**. The energy fluence spectrum, $\psi(E)$, is that which is incident at the point where the dose is to be determined. In practice, the limits of integration are the limits of energy over which $\psi(E)$ is of a significant magnitude. If material intervenes between the source and the point of dose determination, then the spectrum used in the calculation must be the output spectrum of the source modified by the absorbing effects of the intervening material. The values of $\mu_{en}(E)/\rho$ are found in the tables of Ref 2.

NOTE 2—For units and terminology in reports of data, E 170 and ICRU Report 33 may be used as guides.

4.2 If the material in which the absorbed dose is to be calculated is a homogeneous combination of materials not listed in the tables of Ref 2, $\mu_{en}(E)/\rho$ is determined as follows:

4.2.1 From Ref 2, obtain values of $\mu_{en}^i(E)/\rho$ for each component, i.

4.2.2 Determine the atomic fraction, f_i, for each component.

4.2.3 Calculate $\mu_{en}(E)/\rho$ from the following equation:

$$\mu_{en}(E)/\rho = \sum_i f_i[\mu_{en}^i(E)/\rho] \tag{2}$$

4.2.4 Values of $\mu_{en}(E)/\rho$ must be determined for each value of E for which $\psi(E)$ is significant, where E is the photon energy.

4.3 The integral contained in Eq 1 is evaluated numerically. The values of $\mu_{en}(E)/\rho$ in Ref 2 are tabulated for specific energies. In evaluation of the integral referred to in actual practice, it is often desirable to choose energy intervals that would not correspond to the tabulated values in Ref 2. In such cases, the appropriate value of $\mu_{en}(E)/\rho$ for the chosen energies should be determined by an acceptable interpolation procedure. The range of energy over the total photon spectrum is divided into energy intervals or bins. The width of these bins is somewhat flexible but should be chosen small enough so as not to distort the shape of the spectrum. For the purpose of selecting appropriate values of $\mu_{en}(E)/\rho$, the energy value selected for each energy interval can be taken either as that energy at the beginning or midpoint of each energy interval over the entire spectrum.

4.4 The spectrum, $\psi(E)$, is commonly given in arbitrary units and may be normalized to some source parameter. If a standard or calibrated dosimeter is used, then the integral in Eq 1 must be calculated for the material from which this dosimeter is constructed. The value of I is then given by the observed dose, D, measured by the dosimeter divided by the value of the integral.

5. Estimating the Absorbed Dose in One Material from That Measured in Another Material

5.1 If the absorbed dose is known in one material, A, then the absorbed dose can be estimated in another material, B, using the method described in this section.

5.1.1 The absorbed dose observed in A occurs at some depth in the region of material A; similarly, it is desired to know the absorbed dose in material B at some depth in the region of material B. If it is presumed that we know the surface energy fluence spectrum $\psi_o(E)$ (the energy fluence spectrum incident on the surface of materials A and B) then the energy fluence spectrum $\psi(E)$ to be used in Eq 1 must be related to the known surface energy fluence spectrum $\psi_o(E)$. A good approximation to the ~~integral~~ attenuated energy fluence spectrum at mass-depth t is given by

$$\int_0^\infty \psi_t(E)dE = \int_0^\infty \psi_o(E)e^{-[\mu_{en}(E)/\rho]t}dE \tag{3}$$

$$\psi_t(E) = \psi_o(E)e^{-[\mu_{en}(E)/\rho]t} \tag{3}$$

where t is the mass-thickness (in kg·m^2) of material between the surface and the depth of interest, E is a particular energy represented in the spectrum, and $\psi_t(E)$ is the energy fluence per unit energy at mass-depth t. For a derivation of Eq 3 see Appendix X4. See also the qualifications of 5.1.3 and 5.1.4. For a demonstration of the experimental plausibility of Eq 3, see Appendix X5.

5.1.2 Using Eq 1 and 3, the relationship between the known dose D_A and the desired dose D_B can be expressed as

$$\frac{D_A}{D_B} = \frac{\int_0^\infty [\psi_o(E)e^{-[\mu_{en}^A(E)/\rho_A]t_A}][\mu_{en}^A(E)/\rho_A]dE}{\int_0^\infty [\psi_o(E)e^{-[\mu_{en}^B(E)/\rho_B]t_B}][\mu_{en}^B(E)/\rho_B]dE} \tag{4}$$

where μ_{en}^A, ρ_A, and t_A are the ~~mass~~ energy absorption coefficient, the density and the relevant mass-thickness for material A, and where similar notation is used for material B. For further details on the derivation of Eq 4, see Appendix X6. All the variables in Eq 4 are presumed to be known except the desired value for D_B. The integrals in Eq 4 must be performed numerically.

5.1.3 The use of Eq 3 is based on the existence of charged particle equilibrium (for further discussion see 1.3). This condition may be reasonably well met when the region of interest is at a sufficient distance from boundaries representing changes in atomic number or material density (see Appendix X1).

5.1.4 Wide Beam vs. Narrow Beam Approximation.

5.1.4.1 The use of the ~~mass~~ energy ~~absorption~~ coefficient, μ_{en} , in Eq 3 is based on the assumption that the irradiation approaches the "wide beam" as opposed to "narrow beam" condition. The wide beam and narrow beam conditions represent limiting cases which are only approximately realized for real experiments. In the narrow beam case, photons which are scattered out of the narrow beam are assumed to be lost from the beam, and are assumed to have no further importance to the experiment. In the broad beam case, photons which are scattered out of a given small region of the broad beam are presumed to be replaced by photons scattering in from adjacent regions of the beam. For the narrow beam limiting case, Eq 3 should be replaced by

$$\psi_t(D) = \psi_o(E)e^{-[\mu(E)/\rho]t} \tag{5}$$

$$\psi_t(E) = \psi_o(E)e^{-[\mu(E)/\rho]t} \tag{5}$$

where μ is the photon attenuation coefficient. Values of $\mu(E)/\rho$ are found in the tables of Ref 2. For most practical problems the results of photon attenuation lie between the results of Eq 3 and Eq 5.

5.1.4.2 It is possible to determine the magnitude of the change which would have resulted had Eq 1 and Eq 5 been used rather than using Eq 1 and Eq 3 in order to develop Eq 4. The resulting change in the ratio D_A/D_B calculated by Eq 4 is related to the factor

$$F(E) = \frac{e^{-[\mu_{en}^B(E)/\rho_B]t}}{e^{-[\mu^B(E)/\rho_B]t}} \frac{e^{-[\mu^A(E)/\rho_A]t}}{e^{-[\mu_{en}^A(E)/\rho_A]t}} \tag{6}$$

If, over the energy range of interest, *F(E)* differs from unity by a percentage which is greater than the acceptable dosimetry error, then the application of this practice may be inappropriate. In that case an appropriate transport calculation is recommended (see 1.5).

5.1.4.3 Depending of the scattering geometry, it is possible for the absorbed dose to be higher than calculated either by μ or μ_{en}. This buildup of dose is often due to backscatter of radiation within a thick sample. For an extensive discussion of this and similar effect, see Ref 1.

6. Accuracy

6.1 The accuracy of this practice depends primarily on the accuracy to which the incident energy spectrum is known. In general, even a poor estimate of a spectrum will give a better estimate of the absorbed dose at a given location than one would get by assuming some sort of single" effective photon energy." Although ^{60}Co and ^{137}Cs have well-defined primary gamma-ray energies, the radiation energy spectrum from most practical sources contains a significant Compton scattered component that could lead to significant errors if neglected (see ICRU Report 18).

6.2 As stated in 1.3, the results of this practice are not valid unless charged particle equilibrium conditions exist in the material at the depth of application. For depths less than that required for equilibrium, the absorbed dose could be higher or lower than this method would predict. At depths greater than required for equilibrium, the accuracy of the results depends primarily upon the accuracy of the attenuation correction applied in Eq 3 and the knowledge of the incident energy spectrum.

6.3 The procedures used in this method neglect the possible nonlocality of energy deposition by secondary electrons but do correct for production of bremsstrahlung by secondary electrons. For the energy range specified in this practice, these considerations contribute about 5 % or less to the overall uncertainty.

7. Keywords

7.1 calculation of absorbed dose; charged particle equilibrium; radiation dosimetry

APPENDIXES

(Nonmandatory Information)

X1. CHARGED PARTICLE EQUILIBRIUM THICKNESS

X1.1 Whenever a material is irradiated with X or gamma rays, there is initially an increase in energy absorption as the radiation penetrates the material. After some finite thickness, the radiation energy absorption reaches a maximum and then decreases. The thickness necessary to reach the maximum energy deposition is commonly called the "charged particle equilibrium" thickness and is a function of the radiation energy and the mass energy absorption coefficient of the material being irradiated.

X1.2 Fig. X1.1 is a typical plot of energy deposition as a function of depth in a material. The absorbed dose of 0.85 is used for illustration purposes only. Each source and absorbing material combination will have its own characteristic curve. Whenever a specimen is irradiated from all sides, it is necessary to surround the specimen with an ~~absorber~~ equilibrator in order to ensure charged particle equilibrium throughout the specimen. However, when a specimen is irradiated unidirectionally, approximate charged particle equilibrium can be achieved by placing ~~absorbers~~ equilibrators only in contact with the front and back surfaces
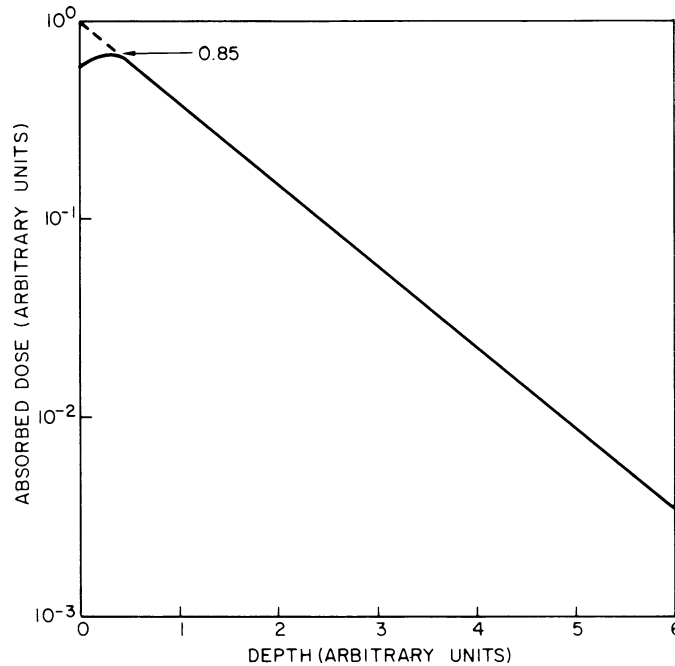
FIG. X1.1 Typical Absorbed Dose versus Depth in a Material

of the specimen. Frequently, the initial rise in Fig. X1.1 is not seen. This is due to electrons from the environment striking the target along with the incident photons.

X1.3   In some instances, the equilibrium thickness should be taken to be equal to the practical range, R_p, of the maximum energy secondary electrons. For aluminum, R_p, in g·cm^{-2} can be calculated from the following equation taken from ICRU Report 21:

$$R_p = 0.530\, E_o - 0.106 \tag{X1.1}$$

where E_o is the energy in MeV of the maximum energy secondary electrons generated by the photons from the source. The value of equilibrium thickness determined by this method is greater than that described in X1.1 and X1.2. (See Appendix X3 of Practice E 668 for a detailed discussion of the conditions for applying each method.)

X2. SAMPLE CALCULATION OF ABSORBED DOSE FOR BROAD PHOTON ENERGY SPECTRA

X2.1   To illustrate this method, a sample calculation is presented here. It is based on the following assumptions:

X2.1.1   A properly used silicon dosimeter has measured the output from a flash X-ray machine to be 15 Gy (1.5×10^3 rad) at a specific location.

Note X2.1—Gy symbolizes gray which is the SI unit for absorbed dose. One gray = one joule per kilogram. (See Practice E 380.) The old unit of absorbed dose was the rad. 1 Gy = 100 rad.

X2.1.2   The output X-ray spectrum from the machine varies as $1/E$ from 100 keV to 1 MeV as shown in Fig. X2.1, where E is the photon energy. The output photon fluence of the flash X-ray machine is shown in Fig. X2.1 to be normalized to the photon energy in keV at the point of measurement and the input energy to the machine in joules. Therefore, the units of the arbitrary scale of the abscissa are photons cm^{-2} · keV^{-1} ·J^{-1}.

X2.2   Since the quantity of interest is the absorbed dose, the normalized output energy fluence is required rather than photon fluence. The normalized energy fluence for each energy interval can be obtained by multiplying the normalized photon fluence in Fig. X2.1 by the corresponding value of the photon energy per energy interval. The results are shown in Fig. X2.2, where the normalized energy fluence has a constant value equal to 100 keV · cm^{-2}·keV^{-1}·J^{-1} over the entire energy range.

X2.3   At this point, consider the dosimeter that is calibrated in terms of absorbed dose in silicon and is contained in a layer of material that puts it at the peak of the absorbed dose versus depth curve. An illustration of such a curve is shown in Fig. X1.1 for the described spectrum. Although this curve is typical of some measured, it must always be determined experimentally for each irradiation source and relevant conditions.

X2.3.1   Fig. X1.1 indicates that the peak in the silicon-absorbed dose versus depth curve is 85 % of the extrapolated surface dose for the incident spectrum shown in X2.1. Therefore, the surface extrapolated dose in this example is obtained by dividing the
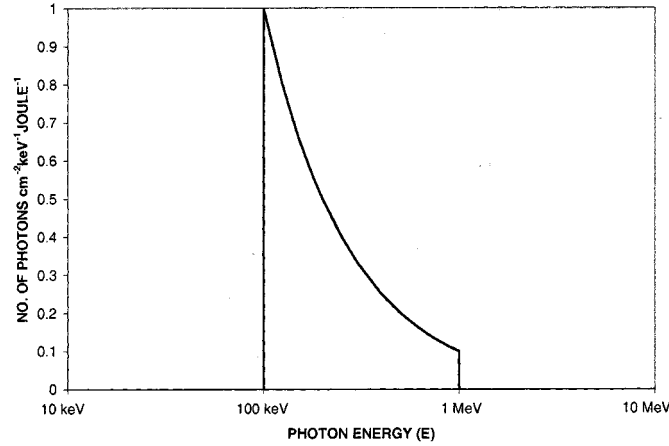
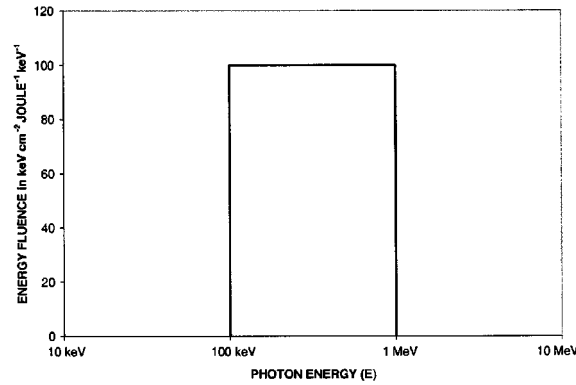**FIG. X2.1 Assumed Number-Energy Spectrum of X Rays from a Flash Machine**

FIG. X2.2 Assumed Energy Fluence Spectrum of X Rays from a Flash Machine

measured absorbed dose (15 Gy) by 0.85 and is equal to 17.6 Gy.

X2.3.2 This surface dose just calculated has the same functional relationship with $\psi(E)$ as the dose defined in Eq 1. The other parameters in that equation are those assumed for this example; that is, $\psi(E)$ is the incident energy fluence spectrum; $\mu_{en}(E)/\rho$ is the mass energy absorption coefficient for silicon; and I is the normalizing constant. The energy limits for the integral are from 0.1 to 1 MeV as shown in Fig. X2.2. The values for the mass energy absorption coefficients, $\mu_{en}(E)/\rho$, are given in Table X2.1 for the energy intervals used. As previously discussed in X2.2, $\psi(E)$ is normalized to the input energy to the machine and has units of keV $\cdot$ cm$^{-2}\cdot$keV$^{-1}\cdot$J^{-1}.

X2.3.3 The normalizing constant, I, is obtained by the use of Eq 1 as follows:

TABLE X2.1 Numerical Evaluation of the Integral in Eq X2.1 for the Number-Energy Spectrum Shown in Fig. X2.1 to Determine the Normalization Constant I

Photon Energy, keV	Energy Fluence Times Energy Interval, keV$\cdot$ cm$^{-2}\cdot$J^{-1}	μ_{en}/ρ (cm$^2\cdot$g^{-1})	Product, keV$\cdot$J$^{-1}\cdot$g^{-1}
100	50×10^2	0.0459	2.29×10^2
150	50×10^2	0.0312	1.56×10^2
200	100×10^2	0.0293	2.93×10^2
300	100×10^2	0.0294	2.94×10^2
400	100×10^2	0.0298	2.98×10^2
500	100×10^2	0.0298	2.98×10^2
600	200×10^2	0.0295	5.90×10^2
800	200×10^2	0.0288	5.76×10^2
Total			27.34×10^2

$$I = \frac{D}{\int_{0.1\ \text{MeV}}^{1\ \text{MeV}} \psi(E)\ [\mu_{en}(E)/\rho]dE}$$

$$= \frac{17.6}{27.34 \times 10^2}$$

$$= 6.4 \times 10^{-3} \text{Gy} \cdot \text{J} \cdot \text{g} \cdot \text{keV}^{-1} \qquad\qquad (X2.1)$$

where the surface dose, D, is from that calculated in X2.3.1, and the integral is evaluated numerically with the detailed results shown in Table X2.1.

X2.4 Now, consider in this sample calculation what absorbed dose would occur at some depth in an iron specimen. The depth of interest is taken to be 6.4 mm (about 0.25 in.) and is equivalent to 5 g·cm^{-2}.

X2.4.1 To make this calculation, Eq 1 is used with $\psi(E)$ that is now the energy fluence spectrum at a depth of 5 g·cm^{-2} in iron and is not the spectrum incident on its surface. The appropriate attenuated spectrum is calculated by use of Eq 3. Values of the mass energy absorption coefficients are obtained from Ref 2, and the incident spectrum is from Fig. X2.1. Details of this calculation are shown in Table X2.2, with the resulting energy fluence spectrum shown in Fig. X2.3. As can be seen, 5 g·cm^{-2} of iron significantly changes the shape of the energy fluence spectrum.

X2.4.2 The calculation of the integral in Eq 1 is now carried out as in X2.3.3, using values for $\mu_{en}(E)/\rho$ for iron from Ref 2 and this attenuated spectrum from Table X2.2. The result is shown in Table X2.3.

X2.4.3 The absorbed dose for this sample calculation for an iron specimen is then the product of the integral from Table X2.3 times the normalization factor, I, as determined in X2.3.3; that is:

$$D = 6.4 \times 10^{-3} \times 28.02 \times 10^2 = \qquad\qquad (X2.2)$$

$$18\ \text{Gy (Fe)} = 1.8$$

$$\times 10^{-3}\ \text{rad (Fe)}$$

TABLE X2.2 Conversion of the Incident X-Ray Spectrum Shown in Fig. X2.1 to the Attenuated Energy-Fluence Spectrum Shown in Fig. X2.3

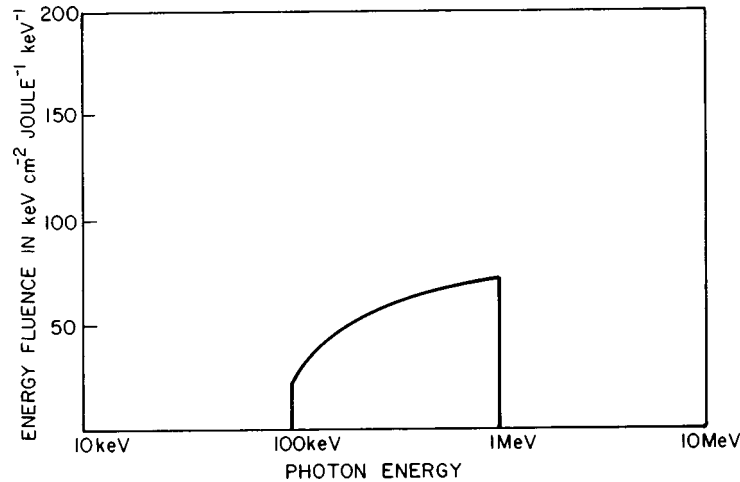| Photon Energy, keV	Incident Spectrum, cm^{-2}· keV^{-1}·J^{-1}	μ_{en}/ρ (cm^2·g^{-1})	Attenuated Spectrum, cm^{-2}· keV^{-1}·J^{-1}	Energy Fluence Spectrum, keV · cm^{-2}· keV^{-1}·J^{-1}
100	1.0	0.219	0.33	33
150	0.67	0.0814	0.45	67
200	0.50	0.0495	0.39	78
300	0.33	0.0335	0.28	84
400	0.25	0.0308	0.21	84
500	0.20	0.0295	0.17	85
600	0.17	0.0286	0.15	90
800	0.13	0.0273	0.11	88

FIG. X2.3 Attenuated Energy Fluence Spectrum Resulting At a
Depth of 5 g·cm⁻² in Iron

TABLE X2.3 Numerical Evaluation of the Integral in Eq X2.1 for
the Energy-Fluence Spectrum Shown in Fig. X2.3 to Determine
the Absorbed Dose in Iron at a Depth of 5 g·cm⁻²

Photon Energy, keV	Energy Fluence Times Energy Interval, keV · cm⁻²·J⁻¹	μ_{en}/ρ (cm²·g⁻¹)	Product, keV · g⁻¹·J⁻¹
100	16.5×10^2	0.219	3.61×10^2
150	33×10^2	0.0814	2.69×10^2
200	78×10^2	0.0495	3.86×10^2
300	84×10^2	0.0335	2.81×10^2
400	84×10^2	0.0308	2.59×10^2
500	85×10^2	0.0295	2.51×10^2
600	180×10^2	0.0286	5.15×10^2
800	176×10^2	0.0273	4.80×10^2
Total			28.02×10^2

X3. SAMPLE CALCULATION OF ABSORBED DOSE FOR NEARLY MONOENERGETIC PHOTON ENERGY SPECTRA

X3.1   Radioactive isotopes, such as ^{137}Cs and ^{60}Co, are examples of sources from which a nearly monoenergetic photon spectrum can be obtained. This sample calculation applies to such isotope sources. It is assumed that the scattered photons from such sources are a negligible component of the incident photon fluence. In actual practice, this assumption is often not valid, and, if this is the case and the actual photon energy spectrum is known, then the calculation procedure described in Appendix X2 is applicable.

X3.2   For example, suppose a ^{60}Co facility provides an exposure rate, $\dot{X}$, of 1.5×10^3 roentgens per hour at a given location. It is assumed that the incident photon energy spectrum has a negligible scattered component at the location of interest.

X3.2.1   For the same iron specimen used in Appendix X2, the absorbed dose rate at a depth of 5 g·cm⁻² is given by the equation:

$$D = 8.69 \times 10^{-3} \cdot \frac{(\mu_{en}/\rho)_{Fe}}{(\mu_{en}/\rho)_{air}} \cdot X \ \exp{-[(\mu_{en}(E)/\rho)_{Fe} \cdot t]} \tag{X3.1}$$

where the value of 8.69×10^{-3} converts roentgens to Gy (air); μ_{en}/ρ is the mass energy absorption coefficient for the photon energies (average 1.25 MeV) from the ^{60}Co source; $\dot{X}$ is the exposure rate; and t is the depth in iron. (For more details on such calculations, see ICRU Report 14.)

X3.2.2   The ratio of mass energy absorption coefficients of iron to air only varies from 0.942 at 1 MeV to 0.940 at 2 MeV. Therefore, the 1-MeV value is chosen for this calculation. Using the equation in X3.2.1, the result for this example is:

$$D_{Fe} = 8.69 \times 10^{-3} \cdot 0.942 \cdot 1.5 \times 10^3 \cdot 0.886 \tag{X3.2}$$

$$= 11 \text{ Gy(Fe)} \cdot \text{h}^{-1} = 1.1 \times 10^3 \text{ rad(Fe)} \cdot \text{h}^{-1}$$

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X4. BEAM ATTENUATION

X4.1 Eq 3 (see Section 5) is a useful approximation for calculating the attenuation of the energy fluence spectrum at some mass-depth t. This appendix provides a derivation of Eq 3, explicitly stating the approximations being made.

X4.2 The assumptions used in this derivation are as follows:

X4.2.1 Charged particle equilibrium is assumed. For further discussion see 5.1.3 and Appendix X1.

X4.2.2 Wide beam geometry is assumed. For further discussion see 5.1.4.

X4.2.3 It is assumed that the mass-depth t is small. For further discussion see X4.5. It is important to note that the use of Eq 6 also requires the use of small values of t.

X4.2.4 One dimensional plane geometry is assumed. This assumption is consistent with the assumption of wide beam geometry. For further discussion see 5.1.4.

X4.3 If the energy fluence spectrum $\psi_o(E)$ is incident on the slab, then $\psi_t(E)$ may be defined as the energy fluence spectrum at mass-depth t and

$$\psi(t) = \int_0^\infty \psi_t(E)dE \tag{X4.1}$$

as the energy fluence at mass-depth t. Further, the energy per unit area deposited in an infinitesimal slab of mass-thickness dt is

$$-\frac{d\psi(t)}{dt} \cdot dt \tag{X4.2}$$

X4.4 Eq 1 has defined absorbed dose as

$$D = \int_0^\infty \psi_t(E)[\mu_{en}(E)/\rho]dE \tag{X4.3}$$

and Eq X4.2 is a statement of energy deposited, so it follows that

$$\int_0^\infty \psi_t(E)\left[\mu_{en}(E)/\rho\right]dE = -\frac{d\psi(t)}{dt} . \tag{X4.4}$$

X4.5 Next, the case of a monochromatic incident spectrum with energy E_o will be considered. It will further be assumed that $\mu_{en}(E)$ does not vary much over the range of photon energies of interest. To elaborate, $\mu_{en}(E)$ is assumed to vary insignificantly over the range of energies including the energy of the primary (incident) photons and the energies of a majority of the scattered photons produced by that primary beam. This is a reasonable assumption if the primary beam passes through a sufficiently small mass-thickness. That is, if $\mu_{en}(E)\,t$ is sufficiently small, then the photon energy changes from the incident energy E_o will be sufficiently small. With these stipulations it is a reasonable approximation to remove the mass energy absorption coefficient from the integral giving

$$\left[\mu_{en}(E_o)/\rho\right] \int_0^\infty \psi_t(E)dE = -\frac{d\psi(t)}{dt} \tag{X4.5}$$

Or, using X4.1,

$$\left[\mu_{en}(E_o)/\rho\right]\psi(t) = -\frac{d\psi(t)}{dt} \tag{X4.6}$$

$$[\mu_{en}(E_o)/\rho]\psi(t) = -\frac{d\psi(t)}{dt} \tag{X4.6}$$

which has the solution

$$\psi(t) = \psi(0)e^{-[\mu_{en}(E_o)/\rho]t} \tag{X4.7}$$

which, using X4.1, is the same as

$$\int_0^\infty \psi_t(E)dE = \psi_o(E_o)e^{-[\mu_{en}(E_o)/\rho]t} . \tag{X4.8}$$

X4.6 This solution can be generalized to a polychromatic incident beam giving

$$\int_0^\infty \psi_t(E)dE = \int_0^\infty \psi_o(E)e^{-[\mu_{en}(E)/\rho]t} \, dE \tag{X4.9}$$

thus demonstrating Eq 3.

X5. EXPERIMENTAL JUSTIFICATION OF Eq 3

X5.1   Evans (6) shows the transmission of ^{60}Co gamma rays through cylindrical lead shields (see Fig. X5.1). The middle curve in this plot shows this experimental data and gives the transmission of all photons (primary plus scattered) transmitted as a function of wall thickness.

X5.2   A first approximation to the photon transmission is shown in the lower curve in ~~this figure.~~ Fig. X5.1. This curve represents a calculated transmission of primary (unscattered) gamma rays. That is, it is assumed that Compton scattered radiation is lost from the transmitted beam. This calculated transmission takes the form of a straight line on a semi-log plot with a slope given by the total attenuation coefficient, μ_0.

X5.3   A second approximation to the photon transmission is given by the upper curve in ~~this figure.~~ Fig. X5.1. For small values of wall thickness, the transmission as a function of thickness can be approximated by a straight line with a slope equal to $\mu_0 - \sigma_s$. ~~Physically this means that as primary to the beam (at the rate σ_s).~~ Note that this straight line is a good approximation to the experimental data for thicknesses less than one mean free path ("1 mfp"). Using the terminology of this standard, $\mu_0 - \sigma_s$ is identical to the energy absorption coefficient μ_{en}. Note that the use of a straight line implies the assumption that μ_{en} does not change significantly as a function of the thickness. See Section X4.5 for further discussion of this assumption.

X5.4   The experimental example of Fig. X5.1 is for the case of a lead shield. This should be close to a worst case example in that lead has a relatively high cross section for Compton scattering. Eq 3 is therefore shown to be plausible for sufficiently small values of the mass-depth, t.

X6. DOSE CALCULATION

X6.1   This appendix provides a derivation of dose as used in Eq 4 (See Section 5). This derivation will use the derivations of Appendix X4.

X6.2   Substituting Eq X4.1 into Eq X4.4 yields

$$\int_0^\infty \psi_t(E)\left[\mu_{en}(E)/\rho\right]dE = -\frac{d}{dt}\int_0^\infty \psi_t(E)dE . \tag{X6.1}$$

Substituting Eq 1 and Eq 3 gives

$$D = \int_0^\infty \psi_t(E)\left[\mu_{en}(E)/\rho\right]dE = -\frac{d}{dt}\int_0^\infty \psi_o(E)e^{-[\mu_{en}(E)/\rho]t}dE \tag{X6.2}$$
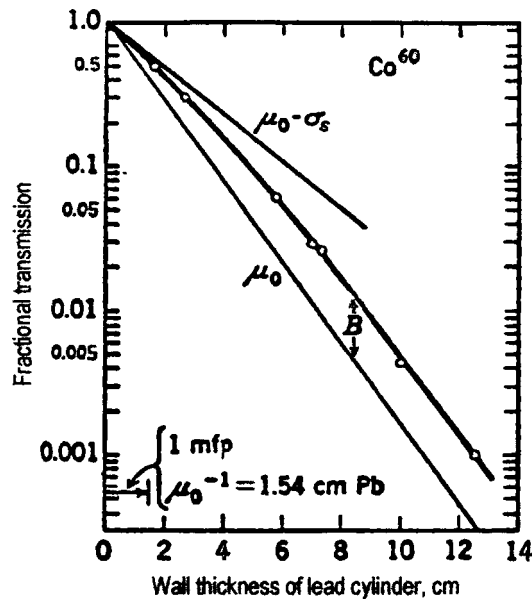

FIG. X5.1 Transmission of ^{60}Co γ rays through cylindrical lead shields. Theoretical lines for the transmission of primary photons (μ_0) and for the true absorption of primary energy ($\mu_0 - \sigma_s$) are shown for comparison. [Figure from Evans (6).]

or

$$D = \int_0^\infty \psi_o(E) e^{-[\mu_{en}(E)/\rho]t} [\mu_{en}(E)/\rho] dE . \tag{X6.3}$$

This equation permits us to derive Eq 4.

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