Designation: C 1557 – 03ε1

Standard Test Method for Tensile Strength and Young's Modulus of Fibers[1]

This standard is issued under the fixed designation C 1557; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

ε[1] NOTE—Fig. 3 was editorially corrected in June 2004.

1. Scope

1.1 This test method covers the preparation, mounting, and testing of single fibers (obtained either from a fiber bundle or a spool) for the determination of tensile strength and Young's modulus at ambient temperature. Advanced ceramic, glass, carbon and other fibers are covered by this test standard.

1.2 *This standard may involve hazardous materials, operations, and equipment. This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:* [2]

C 1239 Practice for Reporting Uniaxial Strength Data and Estimating Weibull Distribution Parameters for Advanced Ceramics

D 3878 Terminology of High-Modulus Reinforcing Fibers and their Composites

E 4 Practices for Load Verification of Testing Machines

E 6 Terminology Relating to Methods of Mechanical Testing

E 1382 Test Methods for Determining Average Grain Size Using Semiautomatic and Automatic Image Analysis

3. Terminology

3.1 *Definitions:*

3.1.1 *bundle*—a collection of parallel fibers. Synonym, *tow*.

3.1.2 *mounting tab*—a thin paper, cardboard, compliant metal, or plastic strip with a center hole or longitudinal slot of fixed gage length. The mounting tab should be appropriately designed to be self-aligning if possible, and as thin as practicable to minimize fiber misalignment.

3.1.3 *system compliance*—the contribution by the load train system and specimen-gripping system to the indicated cross-head displacement, by unit of force exerted in the load train.

3.2 For definitions of other terms used in this test method, refer to Terminologies D 3878 and E 6.

4. Summary of Test Method

4.1 A fiber is extracted randomly from a bundle or from a spool.

4.2 The fiber is mounted in the testing machine, and then stressed to failure at a constant cross-head displacement rate.

4.3 A valid test result is considered to be one in which fiber failure doesn't occur in the gripping region.

4.4 Tensile strength is calculated from the ratio of the peak force and the cross-sectional area of a plane perpendicular to the fiber axis, at the fracture location or in the vicinity of the fracture location, while Young's modulus is determined from the linear region of the tensile stress versus tensile strain curve.

5. Significance and Use

5.1 Properties determined by this test method are useful in the evaluation of new fibers at the research and development levels. Fibers with diameters up to 250×10^{-6} m are covered by this test method. Very short fibers (including whiskers) call for specialized test techniques **(1)**[3] and are not covered by this test method. This test method may also be useful in the initial screening of candidate fibers for applications in polymer, metal or ceramic matrix composites, and quality control purposes. Because of their nature, ceramic fibers do not have a unique strength, but rather, a distribution of strengths. In most cases when the strength of the fibers is controlled by one population of flaws, the distribution of fiber strengths can be described using a two-parameter Weibull distribution, although other distributions have also been suggested **(2,3)**. This test method constitutes a methodology to obtain the strength of a single fiber. For the purpose of determining the parameters of the distribution of fiber strengths it is recommended to follow this test method in conjunction with Practice C 1239.

[1] This test method is under the jurisdiction of ASTM Committee C28 on Advanced Ceramics and is the direct responsibility of Subcommittee C28.07 on Ceramic Matrix Composites .

Current edition approved April 10, 2003. Published August 2003.

[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] The boldface numbers in parentheses refer to the list of references at the end of this standard.

6. Interferences

6.1 The test environment may have an influence on the measured tensile strength of fibers. In particular, the behavior of fibers susceptible to slow crack growth fracture will be strongly influenced by test environment and testing rate **(4)**. Testing to evaluate the maximum strength potential of a fiber should be conducted in inert environments or at sufficiently rapid testing rates, or both, so as to minimize slow crack growth effects. Conversely, testing can be conducted in environments and testing modes and rates representative of service conditions to evaluate the strength of fibers under those conditions.

6.2 Fractures that initiate outside the gage section of a fiber may be due to factors such as stress concentrations, extraneous stresses introduced by gripping, or strength-limiting features in the microstructure of the specimen. Such non-gage section fractures constitute invalid tests. When using active gripping systems, insufficient pressure can lead to slippage, while too much pressure can cause local fracture in the gripping area.

6.3 Torsional strains may reduce the magnitude of the tensile strength **(5)**. Caution must be exercised when mounting the fibers to avoid twisting the fibers.

6.4 Many fibers are very sensitive to surface damage. Therefore, any contact with the fiber in the gage length should be avoided **(4,6)**.

7. Apparatus

7.1 The apparatus described herein consists of a tensile testing machine with one actuator (cross-head) that operates in a controllable manner, a gripping system and a load cell. Fig. 1 and Fig. 2 show a picture and schematic of such a system.

7.1.1 *Testing Machine*—The testing machine shall be in conformance with Practice E 4. The failure forces shall be accurate within ±1 % at any force within the selected force range of the testing machine as defined in Practice E 4. To determine the appropriate capacity of the load cell, the following table lists the range of strength and diameter values of representative glass, graphite, organic and ceramic fibers.

7.1.2 *Grips*—The gripping system shall be of such design that axial alignment of the fiber along the line of action of the machine shall be easily accomplished without damaging the test specimen. Although studies of the effect of fiber misalignment on the tensile strength of fibers have not been reported, the axis of the fiber shall be coaxial with the line of action of the testing machine within δ, to prevent spurious bending strains and/or stress concentrations:

$$\delta \leq \frac{l_o}{50} \tag{1}$$

where:

δ = the tolerance, m, and
l_o = the fiber gage length, m.

7.2 *Mounting Tabs*—Typical mounting tabs for test specimens are shown in Fig. 3. Alternative methods of specimen mounting may be used, or none at all (that is, the fiber may be directly mounted into the grips). A simple but effective approach for making mounting tabs with repeatable dimensions consists in printing the mounting tab pattern onto cardboard file folders using a laser printer. As illustrated in Fig. 3, holes can be obtained using a three-hole punch. Fig. 3 shows a typical specimen mounting method. The mounting tabs are gripped or connected to the load train (for example, by pin and clevis) so that the test specimen is aligned axially along the line of action of the test machine.

7.2.1 When gripping large diameter fibers using an active set of grips without tabs, the grip facing material in contact with the test specimen must be of appropriate compliance to allow for a firm, non-slipping grip on the fiber. At the same time, the grip facing material must prevent crushing, scoring or other damage to the test specimen that would lead to inaccurate results. Large diameter fibers (diameter > 50×10^{-6} m) can also be mounted inside hypodermic needles filled with an adhesive **(7)**. This is a good alternative to avoid crushing the fiber if pneumatic/hydraulic/mechanical grips were to be used. The adhesive must be sufficiently strong to withstand the gripping process, and prevent fiber "pull-out" during testing.

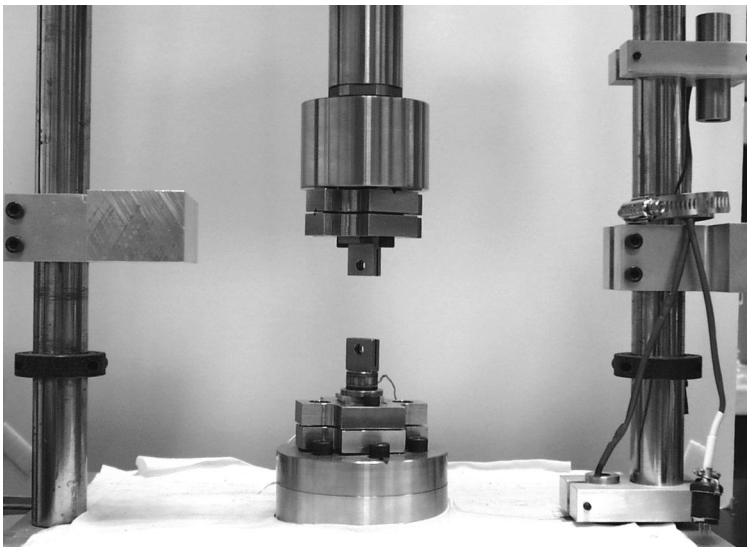


FIG. 1 Typical Fiber Tester

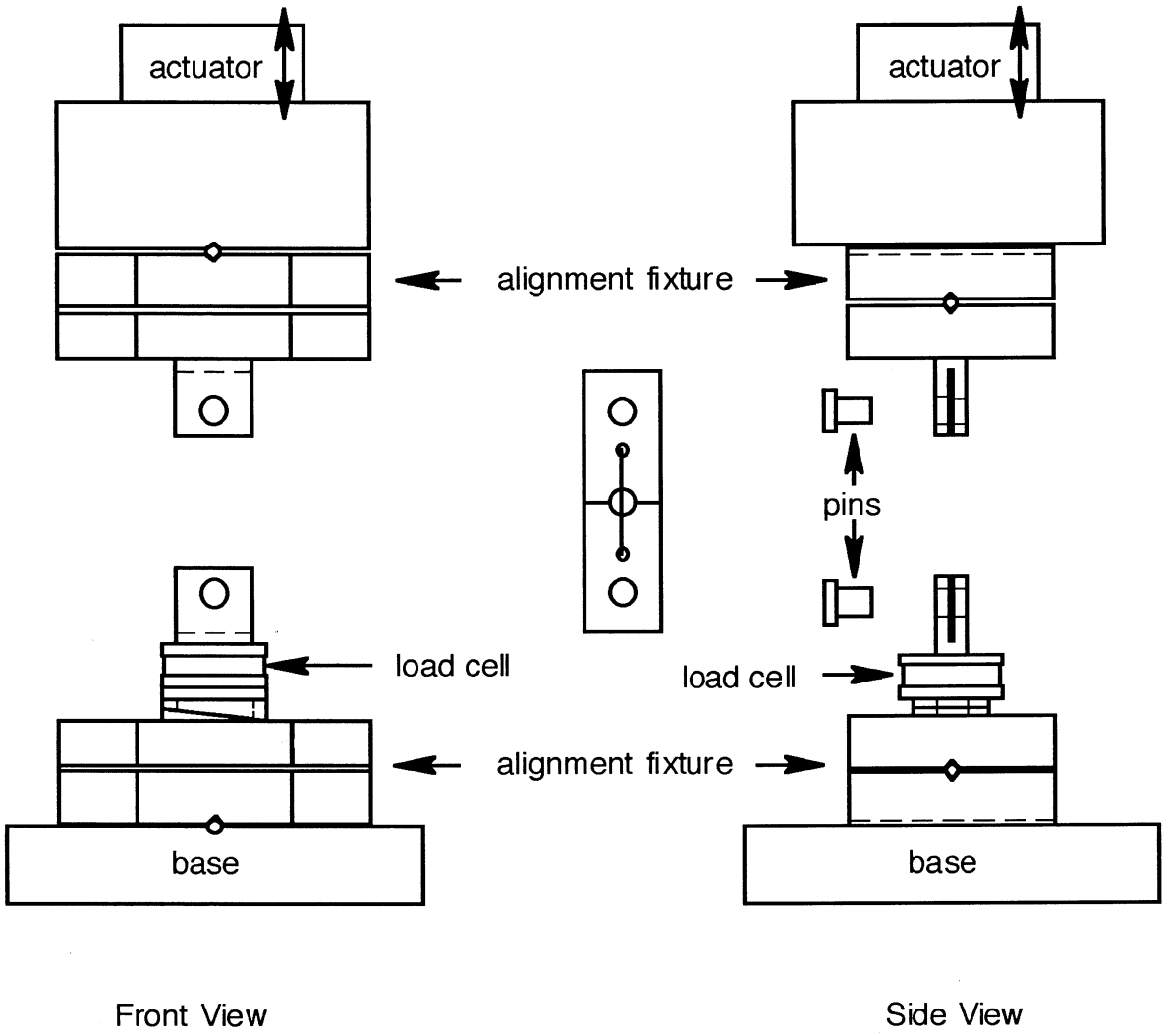


FIG. 2

TABLE 1 Room Temperature Tensile Strength of Fibers (25 × 10^{-3} m Gage Length)

Fiber	Diameter, m	Strength, Pa
CVD-SiC	50-150 × 10^{-6}	2-3.5 × 10^{9}
polymer-derived SiC	10-18 × 10^{-6}	2-3.5 × 10^{9}
sol-gel derived oxide	1-20 × 10^{-6}	1-3 × 10^{9}
single-crystal oxide	70-250 × 10^{-6}	1.5-3.5 × 10^{9}
graphite	1-15 × 10^{-6}	1-6 × 10^{9}
glass	1-250 x× 10^{-6}	1-4 × 10^{9}
aramid	12-20 × 10^{-6}	2-4 × 10^{9}

7.3 *Data Acquisition*—At a minimum, autographic records of applied force and cross-head displacement versus time shall be obtained. Either analog chart recorders or digital data acquisition systems may be used for this purpose although a digital record is recommended for ease of later data analysis. Ideally, an analog chart recorder or plotter shall be used in conjunction with the digital data acquisition system to provide an immediate record of the test as a supplement to the digital record. Recording devices must be accurate to ± 1 % of full scale and shall have a minimum data acquisition rate of 10 Hz with a response of 50 Hz deemed more than sufficient.

8. Precautionary Statement

8.1 During the conduct of this test method, the possibility of flying fragments of broken fibers may be high. Means for containing these fragments for later fractographic reconstruction and analysis is highly recommended. For example, vacuum grease has been used successfully to dampen the fiber during failure and capture the fragments. In this case, vacuum grease is applied in the gage section of the fiber so that the former does not bear any force. An appropriate solvent can be used afterwards to remove the vacuum grease.

9. Procedure

9.1 *Test Specimen Mounting*:

9.1.1 Randomly choose, and carefully separate, a suitable single-fiber specimen from the bundle or fiber spool. The total length of the specimen should be sufficiently long (at least 1.5 times longer than the gage length) to allow for convenient handling and gripping. Handle the test specimen at its ends and avoid touching it in the test gage length.

NOTE 1—Because the strength of fibers is statistical in nature, the magnitude of the strength will depend on the dimensions of the fiber being

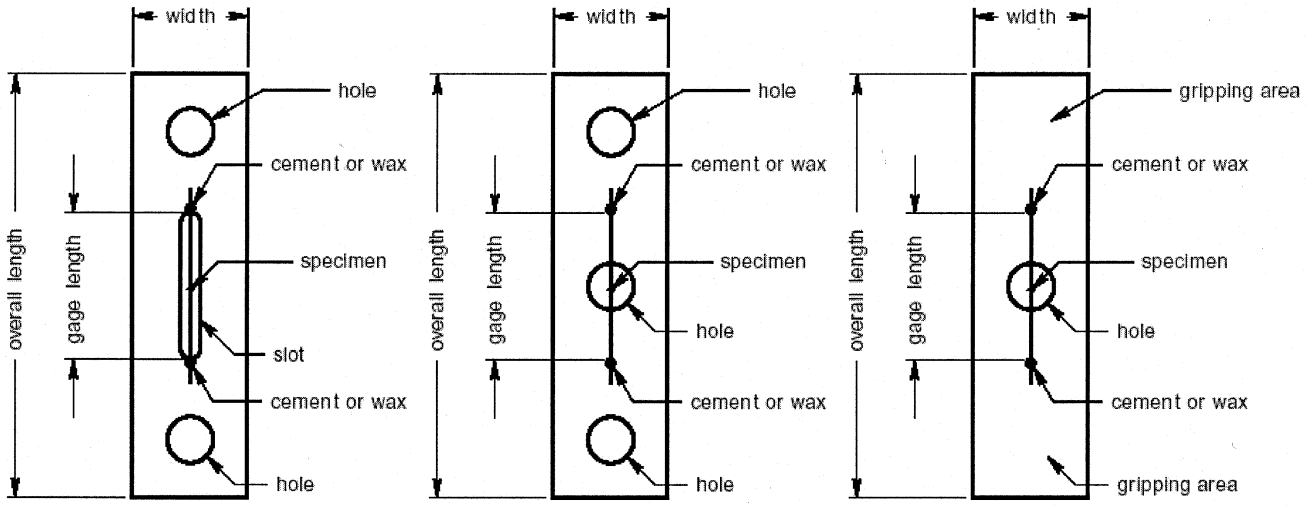


FIG. 3 Mounting Tab

evaluated. In composite material applications, the gage length of the fiber is usually of the order of several fiber diameters, but it has been customary to test fibers with a gage length of 25.4×10^{-3} m. However, other gage lengths can be used as long as they are practical, and in either case, the value of the gage length must be reported.

9.1.2 *When Using Tabs*:

9.1.2.1 A mounting tab (Fig. 3) may be used for specimen mounting. Center the test specimen over the tab using the printed pattern with one end taped to the tab.

9.1.2.2 Tape the opposite end of the test specimen to the tab exercising care to prevent fiber twisting. It has been found that the tensile strength of fibers decreases significantly with increasing torsional strain **(5)**.

9.1.2.3 Carefully place a small amount of suitable adhesive (for example, epoxy, red sealing wax) at the marks on the mounting tab that define the gage length, and bond the fiber to the mounting tab.

9.1.2.4 Determine the gage length to the nearest $\pm 5 \times 10^{-4}$ m or ±1 % of the gage length, whichever is smaller.

9.2 *Optical Strain Flags*—If optical flags are to be used for strain measurement, they may be attached directly to the fibers at this time, using a suitable adhesive or other attachment method. Note that this may not be possible with small-diameter fibers ($\delta < 5 \times 10^{-6}$ m).

9.3 *Test Modes and Rates*—The test shall be conducted under a constant cross-head displacement rate. Rates of testing must be sufficiently rapid to obtain the maximum possible strength at fracture within 30 s. The user may try as an initial value a test rate of 8×10^{-6} m/s. However, rates other than those recommended here may be used to evaluate rate effects. In all cases the test mode and rate must be reported.

9.4 Ensure that the machine is calibrated and in equilibrium (no drift).

9.5 Set the cross-head and data recorder speeds to provide a test time to specimen fracture within 30 s.

9.6 Grasp a mounted test specimen in one of the two tab grip areas (or pin load one end of the mounting tab). Zero the load cell.

9.7 Position the cross-head so that the other tab grip area may be grasped as in 9.6. Check the axial specimen alignment using whatever methods have been established, as described in 7.1.2.

9.8 If using tabs, with the mounting tab un-strained, cut both sides of the tab very carefully at mid-gage as shown in Fig. 4. Alternatively, the sides of the tab can be burned using a soldering iron, for example. If the fiber is damaged, then it must be discarded.

9.9 Initiate the data recording followed by the operation of the test machine until fiber failure. Record both the cross-head displacement and force, and strain if applicable.

9.10 Recover the fracture surfaces and measure the cross-sectional area of a plane normal to the axis of the fiber at the fracture location or in the vicinity of the fracture location. Determine the fiber cross-sectional area using with a linear spatial resolution of 1.0 % of the fiber diameter or better, using laser diffraction techniques **(8-11)**, or an image analysis system in combination with a reflected light microscope or a scanning electron microscope **(12)** (see Test Methods E 1382). Note that in practice, a reflected white light microscope can provide a maximum resolution of 0.5×10^{-6} m and therefore its use may be impractical when measuring the cross-sectional area of small diameter fibers. Because stiff fibers tend to shatter upon failure, it is recommended to capture the fiber fragments using vacuum grease, because vacuum grease is an effective medium to dampen the energy released by the fiber upon fracture. The user of this standard should be aware that the need to recover the fracture surfaces of the fiber to determine the fiber cross-sectional area is consistent with the need to do fractography to identify the strength-limiting flaws for the proper estimation of the parameters of the distribution of fiber strengths.

NOTE 2—The user of this standard test method must be aware that the diameter of many ceramic fibers varies not only among fibers in a bundle, but also along the length of each fiber **(13-16)**. It has been customary to

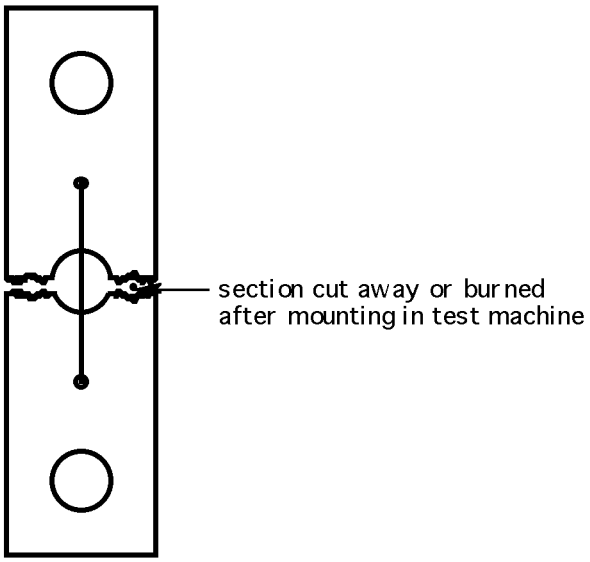


FIG. 4 Cutting Sides of Tab

determine individual fiber strength values using the average cross-sectional area of a group of fibers. However, it has been demonstrated that this procedure leads to significant errors both in the determination of the actual fiber strength and in the estimates of the parameters of the distribution of fiber strengths **(17-19)**.

NOTE 3—When the fiber diameter varies along its length and the diameter of a fiber is measured before the fiber is tested, there is a risk that the measurement will be obtained at a location of the fiber that doesn't coincide with the failure location. Monte Carlo simulations have been carried out to estimate the magnitude of the error in the determination of the tensile strength of a fiber when the value of the cross-sectional area used for the calculation of strength doesn't correspond to that of the fracture plane. The results of these simulations have shown that the magnitude of the error increases with the degree of variability of the fiber diameter along its length **(20)** (see Appendix X1).

NOTE 4—Therefore, it is necessary to determine the cross-sectional area of the fiber on a plane perpendicular to the axis of the fiber at the location of failure, or in the vicinity of the failure location, after performing the mechanical test, and use that value of the cross-sectional area for the determination of the fiber strength.

10. Calculations

10.1 *Tensile Strength*—Calculate the tensile strength of the fiber as follows:

$$T = \frac{F}{A} \tag{2}$$

where:
T = tensile strength, Pa,
F = force to failure, N, and
A = fiber cross-sectional area at fracture plane (normal to fiber axis), m^2.

10.2 *Strain*—Calculate the tensile strain of the fiber as follows:

$$\epsilon = \frac{\Delta l}{l_o} \tag{3}$$

where:
Δl = the elongation of the gage length, m, and
l = the gage length, m.

10.2.1 *Direct Measurement of Elongation*—Direct measurement of the specimen elongation (in the gage section) is achieved by monitoring the displacement of the flags attached to the fiber.

10.2.2 *Indirect Measurement of Elongation*—In the absence of a direct measurement of specimen elongation, the actual specimen elongation in the gage length can be determined by subtracting the displacement associated with the system compliance from the total cross-head displacement **(21)**.

10.2.2.1 *System Compliance*—The system compliance must be determined experimentally for a given test machine, gripping system and fiber type. The system compliance is determined as follows:

10.2.2.2 Perform tensile tests according to the procedures given in 9.1-9.10 on single fiber specimens with various different gage lengths. Test specimens with at least three different gage lengths, and perform at least, three tests for each value of the gage length.

10.2.2.3 For each test, obtain the force versus cross-head displacement curve, and determine the inverse of the slope of the initial linear region of the force versus cross-head displacement curve in m/N. (See Fig. 5.)

(1) Note that the recorded cross-head displacement is:

$$\Delta L = \Delta l + C_S F \tag{4}$$

where:
ΔL = recorded cross-head displacement, m,
C_S = system compliance, m/N, and
Δl = elongation of the specimen gage length, m.

(2) For the fiber, using Eq 3:

$$\epsilon = \frac{\sigma}{E} = \frac{F}{EA} = \frac{\Delta l}{l_o} \tag{5}$$

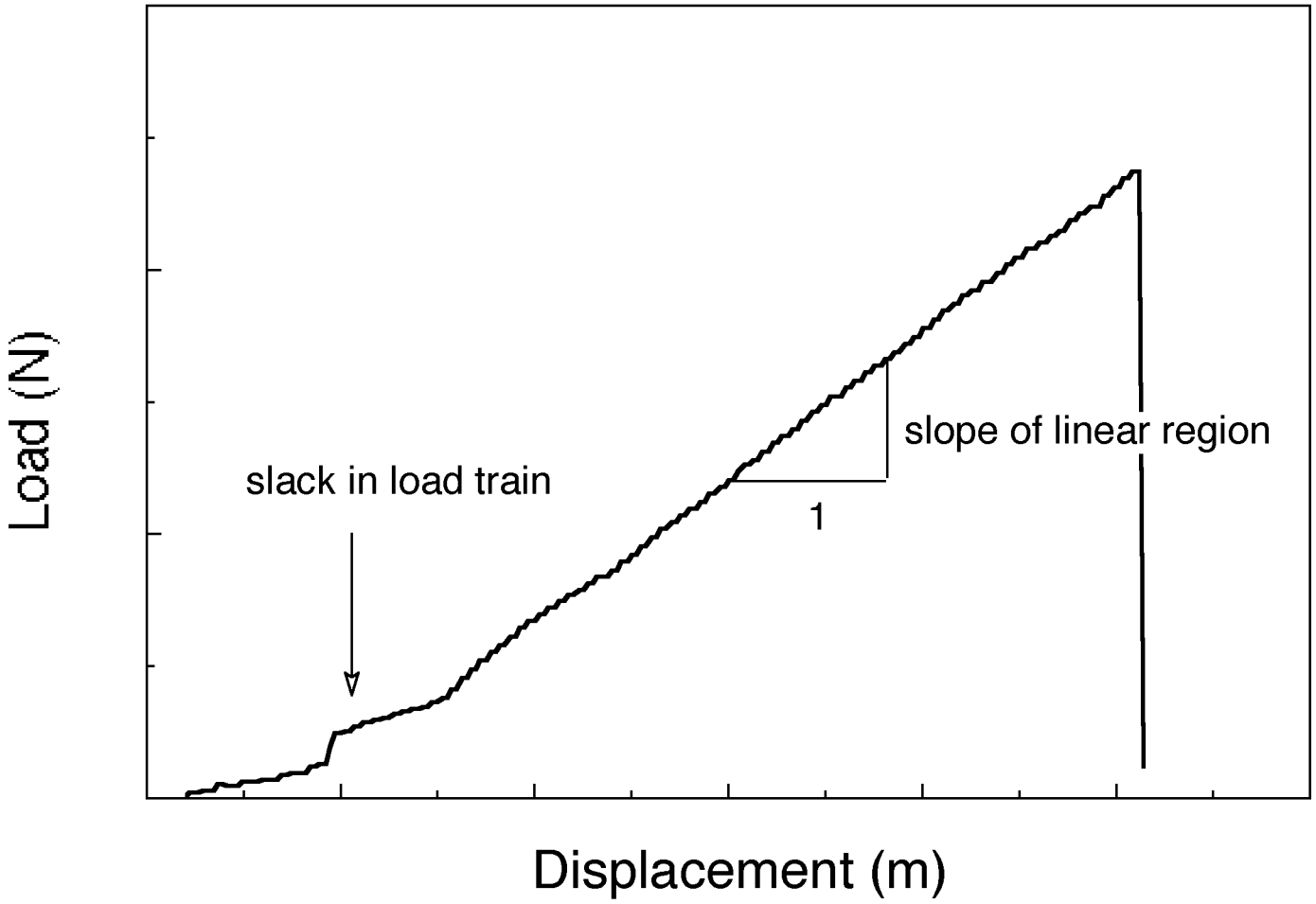


FIG. 5 Typical Load versus Cross-head Displacement Curve

where:
σ = tensile stress, Pa, and
E = fiber Young's modulus, Pa.

(3) The combination of Eq 4 and 5 yields:

$$\frac{\Delta L}{F} = \frac{\Delta l}{F} + C_S = \frac{l_o}{EA} + C_S \quad (6)$$

(4) Therefore, a plot of ($\Delta L/F$) (that is, the inverse of the slope of the force versus cross-head displacement curves) versus (l_o/A), will yield a straight line with constant slope of ($1/E$) and intercept C_S which is the value of the system compliance. (See Fig. 6.)

10.2.2.4 From Eq 4, determine the actual elongation of the gage section of the specimen as follows:

$$\Delta l = \Delta L - C_S F \quad (7)$$

10.3 *Young's Modulus*—Determine the fiber Young's modulus from the slope of the linear region of the stress-strain curve. If strain was not measured directly, the fiber Young's modulus can be obtained from the process used to calculate the system's compliance according to 10.2.2.1-10.2.2.3.

11. Report

11.1 The report shall include the following:

11.1.1 Complete identification of the test specimen, including material type, source, manufacturer's name and code or batch number, previous history, etc.,

11.1.2 Method of selecting specimen,

11.1.3 Method of mounting the test specimen,

11.1.4 Fiber cross-sectional area including method of determination,

11.1.5 Specimen gage length,

11.1.6 System compliance, if required,

11.1.7 Test machine parameters; cross-head speed, speed of data collection, load cell used, gripping method,

11.1.8 Method used for determination of gage length elongation,

11.1.9 The force to failure,

11.1.10 Tensile strength,

11.1.11 Young's modulus, if measured,

11.1.12 Ambient conditions of test room (temperature and relative humidity), and

11.1.13 Date of test.

12. Precision and Bias

12.1 Because of the lack of a wide database, no definitive statement can be made at this time concerning the precision and bias of this test method.

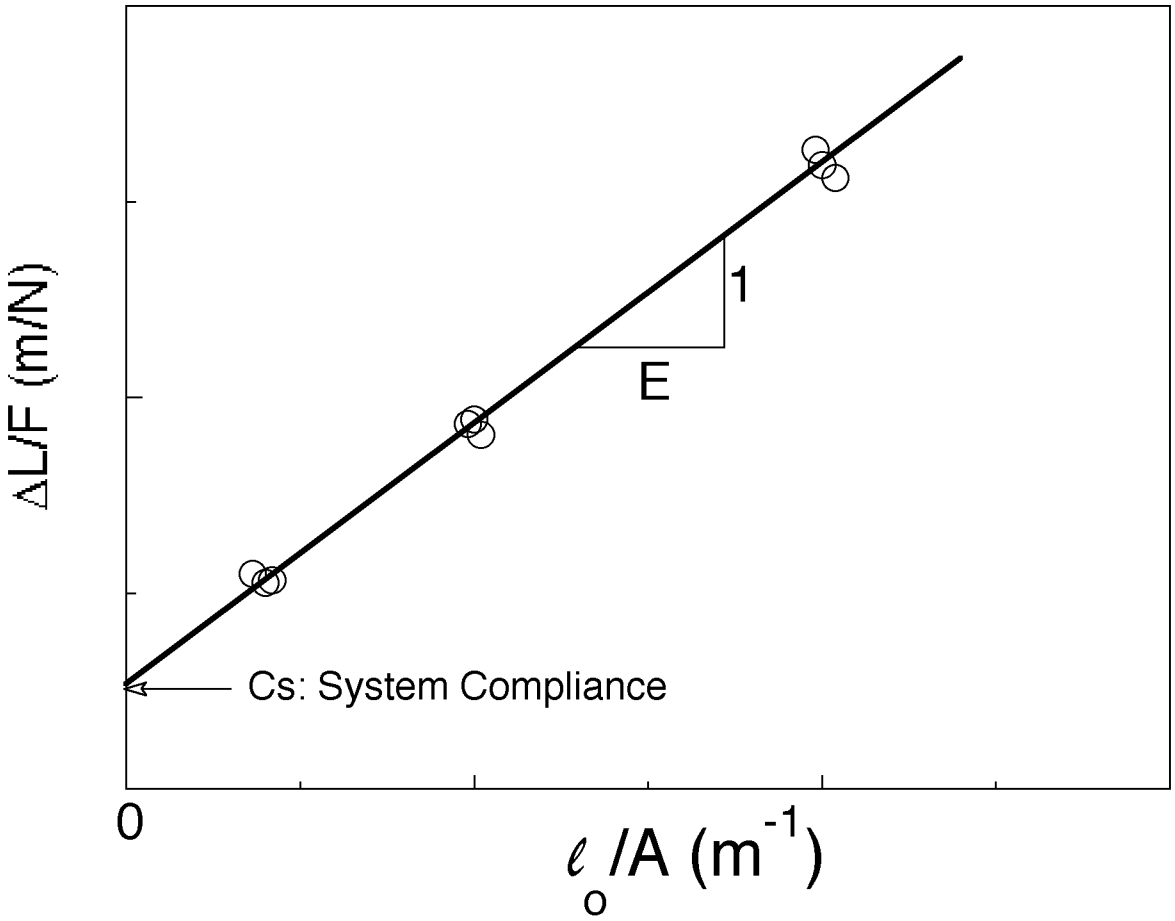


FIG. 6 Method for Determining the System Compliance

APPENDIX

(Nonmandatory Information)

X1.

X1.1 To estimate the magnitude of the error in the determination of fiber strength when using a value of the fiber diameter different from that at the fracture plane, a Monte Carlo simulation was performed. The diagram shown in Fig. X1.1 summarizes the process.

X1.2 Let us consider a fiber of length l that belongs to a collection of fibers with strengths that are distributed according to a two-parameter Weibull distribution. Let us assume that the diameter of this fiber varies randomly along its length and that the diameter profile can be described by a mean value (ϕ_m)and standard deviation (ϕ_{sd}). Fig. X1.2 shows profiles of such fibers. Let us subdivide the fiber into n segments each of length lo, diameter fi and strength si. The strength of each segment is calculated according to Eq X1.1.

$$\sigma_i = \sigma_o \left(\frac{l_o}{l} \left(\frac{\phi_o}{\phi_i} \right)^2 ln \frac{1}{(1 - P_{fi})} \right)^{\frac{1}{m}} \quad \text{(X1.1)}$$

where:

σ_o = the characteristic strength associated with a volume of the fiber of magnitude,

$$\frac{(\pi \phi_o^2 l_o)}{4} \quad \text{(X1.2)}$$

and:

P_{fi} = the probability of failure associated with the tensile stress σ_i.

X1.3 Let us find the force needed to break the fiber by taking the product of the minimum of the n strengths and the cross-sectional area of the fracture plane (that is, the cross-sectional area of the section with the smallest strength). Let us define the "modified strength" as the ratio of the breaking force and the cross-sectional area of the fiber at a location that is selected randomly among the n segments, and that may or may not coincide with the fracture plane. Following this procedure

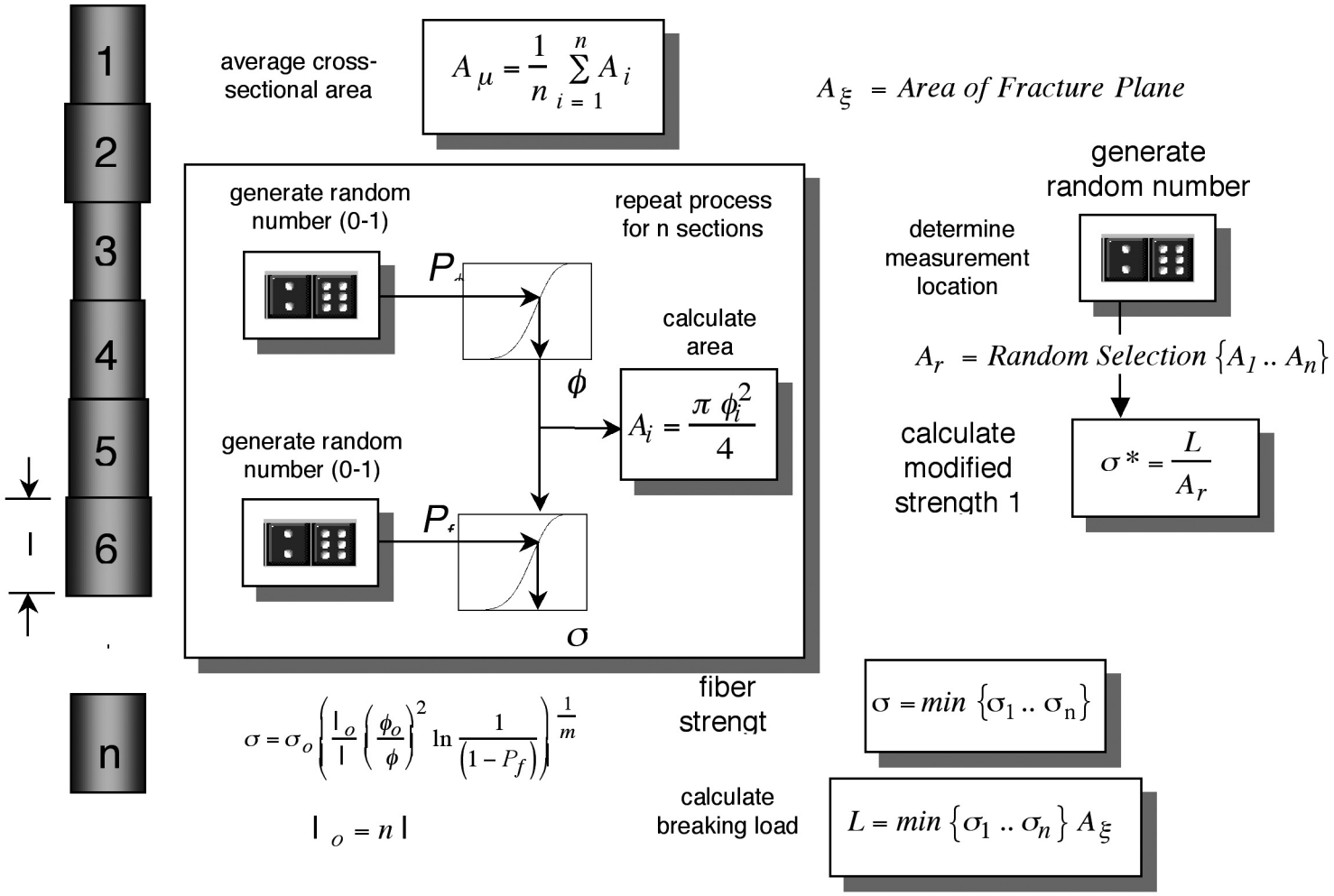
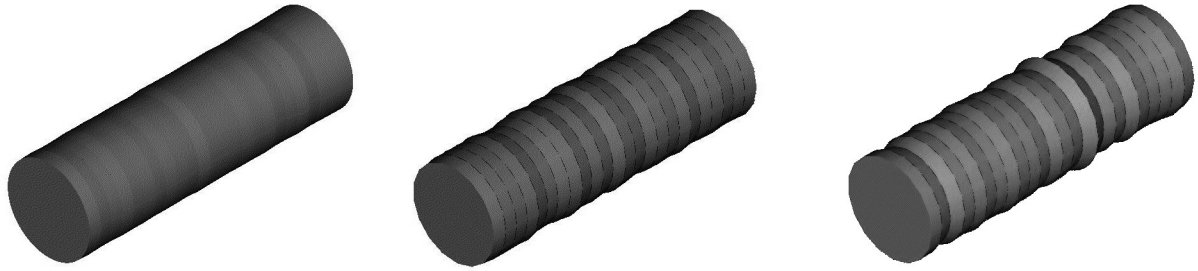


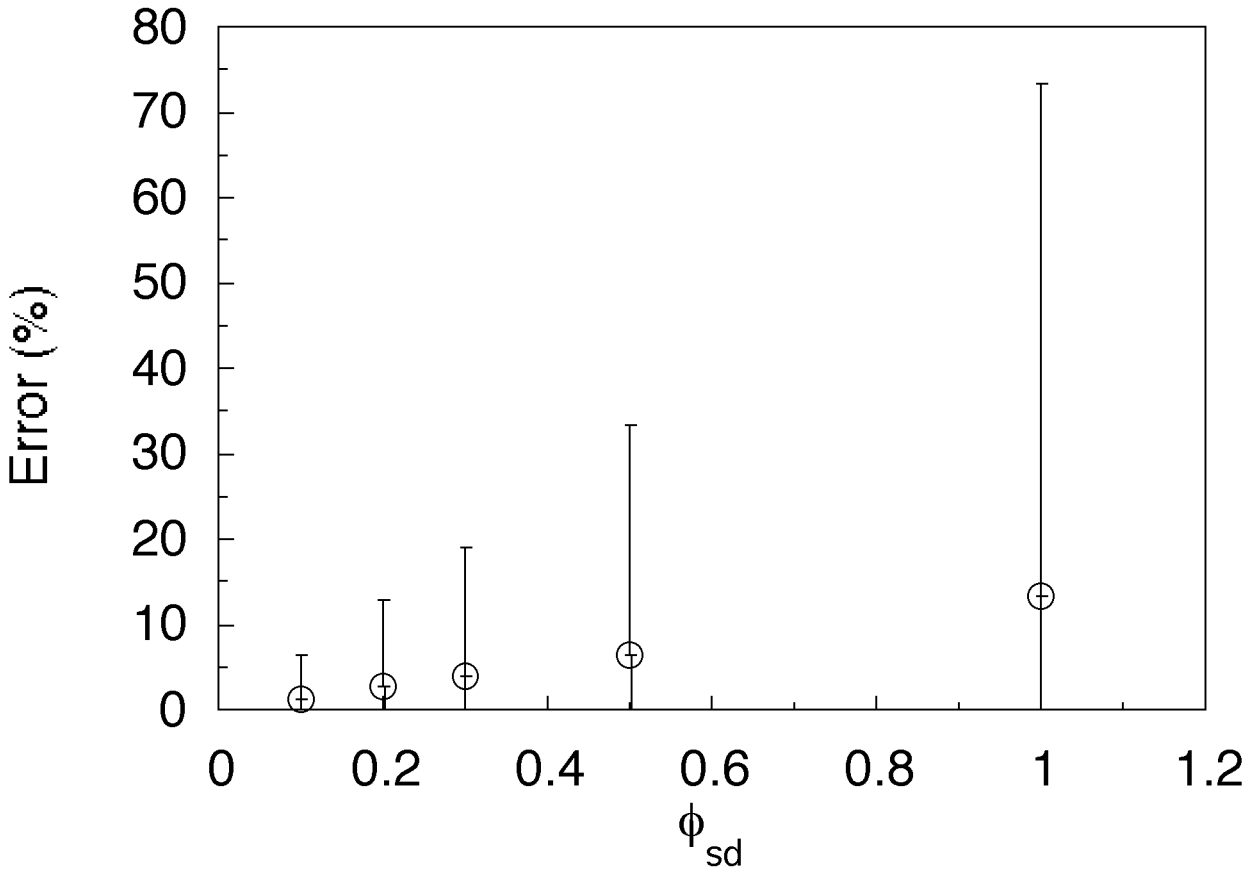
FIG. X1.1 Schematic of Monte Carlo Simulation



NOTE—Calculations were carried out for a fiber that is 25×10^{-3} m long and 15×10^{-6} m in diameter. Radial and axial scales are not the same in these diagrams.

FIG. X1.2 Fiber Profiles Illustrating Variability of Fiber Diameter Along its Length

results are presented in Fig. X1.3 for the magnitude of the error in the determination of the fiber strength, for various values of the variability of the diameter along the fiber length. Values of 15×10^{-6} m for the average fiber diameter and 5 for the Weibull modulus (which are typical of many ceramic fibers) were used in the simulations. The error bars correspond to the largest and smallest values of the errors resulting from 1000 repetitions.



NOTE—Diameter variability values are in 1×10^{-6} m for a fiber with 15×10^{-6} m diameter.

FIG. X1.3 Error in Determination of Fiber Strength as a Function of Variability in Fiber Diameter (ϕ_{sd}) Along the Length of the Fiber

REFERENCES

(**1**) Petrovic, J. J., Milewski, J. V., Rohr, D. L., and Gac, F. D., "Tensile Mechanical-Properties of SiC Whiskers," *Journal of Materials Science*, 20, 4, 1985, pp. 1167-1177.

(**2**) Phani, K. K., "A New Modified Weibull Distribution Function for the Evaluation of the Strength of Silicon Carbide and Alumina Fibers," *Journal of Materials Science*, 23, 1988, pp. 2424-2428.

(**3**) Gurvich, M. R., DiBenedetto, A. T., and Ranade, S. V., "A New Statistical Distribution for Characterizing the Random Strength of Brittle Materials," *Journal of Materials Science*, 32, 1997, pp. 2559-2564.

(**4**) Kotchick, D. M., and Tressler, R. E., "Surface Damage and Environmental Effects on the Strain Rate Sensitivity of the Strength of Sapphire and Silicon Carbide Filaments," *Journal of Materials Science*, 10, 22, 1975, pp. 608-612.

(**5**) Fukunaga, H., and Goda, K., "Tensile Strength of Nicalon™ SiC Fibers Subjected to Torsional Strain," *Journal of Materials Science Letters*, 10, 1991, pp. 179-180.

(**6**) Kotchick, M., Hink, R. C., and Tressler, R. E., "Gage Length and Surface Damage Effects on the Strength Distributions of Silicon carbide and Sapphire Filaments," *Journal of Composite Materials*, Vol 9, 1975, pp. 327-336.

(**7**) McHenry, K. D., and Tressler, R. E., "Elevated Temperature Strength of Silicon Carbide-on-Carbon Filaments," *Journal of Composite Materials*, Vol 9, 1975, pp. 73-76.

(**8**) Li, C-T, Tietz, J., "Improved Accuracy of the Laser Diffraction Technique for Diameter Measurement of Small Fibers," *Journal of Materials Science*, 25, 1990, pp. 4694-4698.

(**9**) Krucinska, I., and Stypka, T., "Direct Measurement of the Axial Poisson's Ratio of Single Carbon Fibers," *Composites Sci. Tech*., Vol 41, 1991, pp. 1-12.

(**10**) Giannelli, D., "Single Filament Diameter Measurement System Based on Laser Diffraction," *Proceedings of the 22nd International SAMPE Technical Conference*, November 6-8, 1990, pp. 546-553.

(**11**) Tzeng, S. S., "Structure and Property Relationships in Carbon Fibers," Ph.D. Thesis, Rensselaer Polytechnic Institute, Troy, NY, December 1992.

(**12**) Wilson, D. M., "Statistical Tensile Strength of Nexterl ™610 and Nextel™ 720 Fibers," *Journal of Materials Science*, 32, 1997, pp. 2535-2542.

(**13**) Lissart, N., and Lamon, J., "Statistical Analysis of Failure of SiC Fibers in the Presence of Bimodal Flaw Populations," *Journal of Materials Science*, 32, 22, 1997, pp. 6107-6117.

(**14**) Morimoto, T., Goering, J., and Schneider, H., "A New Method for Measuring Diameter Distribution Along Ceramic Fibers," *Ceram. Eng. Sci. Proc*., 19, 3, 1998, pp. 47-54.

(**15**) Youngblood, G. E., Eiholzer, C. R., Lewinsohn, C. A., Jones, R. H., Hasegawa, A., and Kohyama, A., "Fiber Diameter Variation-Sample Preparation and Analysis," *Ceram. Eng. Sci. Proc*., 20, 3, 1999, pp. 481-486.

(**16**) Tanaka, T., Nakayama, H., Sakaida, A., and Horikawa, N., "Estimation of Tensile Strength Distribution for Carbon Fiber with Diameter

Variation Along Fiber," *Materials Science Research International*, Vol. 5, No. 2, 1999, pp. 90-97.

(**17**) Lara-Curzio, E., and Russ, C., "On the Relationship Between the Parameters of the Distributions of Fiber Diameters, Breaking Loads and Fiber Strengths," *J. Materials Science Letters*, 18, 24, 1999, pp. 2041-2044.

(**18**) Lara-Curzio, E., and Russ, C. M., "Why it is Necessary to Determine Each Fiber Diameter When Estimating the Parameters of the Distribution of Fiber Strengths," *Ceram. Eng. and Sci. Proc*., 20, 1999, pp. 681-688.

(**19**) Parthasarathy, T. A., "Extraction of Weibull Parameters of Fiber Strength from Means and Standard Deviations of Failure Loads and Fiber Diameters," *J. Am. Ceram. Society*, 83, 3, 2001, pp. 588-592.

(**20**) Lara-Curzio, E., and Garcia, D., "Strength Statistics of Fiber Bundles: The Effect of Fiber Diameter Variation Along and Among Fibers, on the Determination of the Parameters of Distribution of Fiber Strengths," *Ceram. Eng. and Sci. Proc*., 22, 3, 2001, pp. 363-370.

(**21**) Li, C-T, and Tietz, J., "Improvement in Fiber Testing of High-Modulus Single-Filament Materials," *J. Am. Ceram. Soc*., Vol 68, No. 8, 1985, pp. C202-C204.