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Translation

Track/bridge Interaction

Recommendations for calculations

Interaction voie/ouvrages d'art
Recommandations pour les calculs
Interaktion Gleis/Brücke
Empfehlungen für die Berechnungen

Union Internationale des Chemins de fer
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Warning

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Summary

Interaction between track and bridge, i.e. the consequences of the behaviour of one on the behaviour of the other, occurs because they are interlinked, regardless of whether the track is directly fastened or has a ballast bed.

This interaction takes the form of forces in the rails and in the deck and its bearings, as well as displacement of the various elements of the bridge and track.

If the interaction is under control, then the bridge will continue to fulfil its function, i.e. supporting the track without the track being subject to anomalies.

There are two types of anomaly: rail fractures, or disruption of the link between track and bridge such that track stability is no longer guaranteed (one notable example of instability is when the ballast loosens following considerable displacement of the bridge deck such that sufficient resistance to lateral buckling at the ends of the bridge is no longer guaranteed).

Therefore interaction - strictly speaking - must be taken into account as a serviceability limit state (SLS) as regards the bridge, as well as being an ultimate limit state (ULS) as regards the rail. Forces and displacements must therefore be calculated using the partial safety factors for the loads concerned. The relevant factors are applied to the forces according to the checks required at ultimate limit state as regards the strength of the bearings and the substructure.

This leaflet, which is a result of the work of ERRI Specialists' Committee D 213, gives methods for calculating the forces and displacements that are linked to interaction phenomena.

The acceptable limit states for the track depend on its design and state of maintenance. The permissible limit values given in the leaflet, whether for displacements or additional stress in the rail due to interaction phenomena, are the values that are widely permitted for standard track components in a good state of maintenance. If a railway for its own reasons operates outside the foreseen scope of application, that railway will still be able to use the calculation methods in this leaflet by replacing the criteria given here with new criteria based on its own experience and observations (this mainly concerns 72 N/mm^2 for permissible additional compressive stress in the rail and 5 mm displacement of the deck under braking forces).

Similarly, the track strength taken into account and the temperature increase envisaged are drawn from the knowledge of the various railways. It is perfectly possible to use this leaflet but with different values, if the need were to arise.

It should also be noted that the displacements (or rotations) to be checked only concern what has to be checked to guarantee that the behaviour of the bridge cannot damage the track and alter its behaviour.

There are other checks to be made as regards displacement (and rotation) of the structure, these being concerned with problems of comfort, dynamic behaviour or simply strength. The checks with regard to interaction do not cover the other checks that are necessary.

1 - Basic assumptions, criteria and computer calculations

1.1 - Description of the phenomenon

A large percentage of the track on all railways now consists of continuous welded rails (CWR). This is particularly true for high and very high speed track, where this type of track is now used in a dedicated way.

Numerous studies have investigated the evaluation and limitation of rail stresses and the stability of the track on embankment (i.e. not on bridges). Such studies have looked both at the effects of thermal stresses alone and at the combined effects of thermal stresses and traffic forces.

The use of an expandable deck, capable of moving relative to the CWR track, introduces a discontinuity into the characteristics of the track bed. This discontinuity is responsible for relative movements between the track bed and the track as the deck expands and contracts, causing forces to be applied to the rails and the structure, as well as changes in the additional stresses due to forces induced by traffic loads.

The need for a review of the issues surrounding the use of CWR has become apparent. The following sections will therefore describe the phenomena on a bridge carrying CWR in some detail.

1.1.1 - Review of the principles governing continuous welded rail on an embankment

In general, the rail is fixed to the sleeper by elastic fastenings, which apply a predefined clamping force to secure the rail to the sleeper. This clamping force is normally such that all the longitudinal movement of the rail is transmitted to the sleepers, the resistance to rail/sleeper sliding being greater than the resistance to longitudinal movement offered by the ballast. As the free movement of the rails under the influence of thermal and traffic forces is opposed by the ballast, the rails are subject to longitudinal forces.

Continuous welded rail includes a "central" zone where expansion and contraction are completely prevented and two "breather" zones at each end, some 150 m in length. Expansion devices at the ends of the CWR have a variation of opening of 50 mm and permit the free movement of the ends of the CWR.

The thermal effects generated are hence as shown in Fig. 1 on the following page.

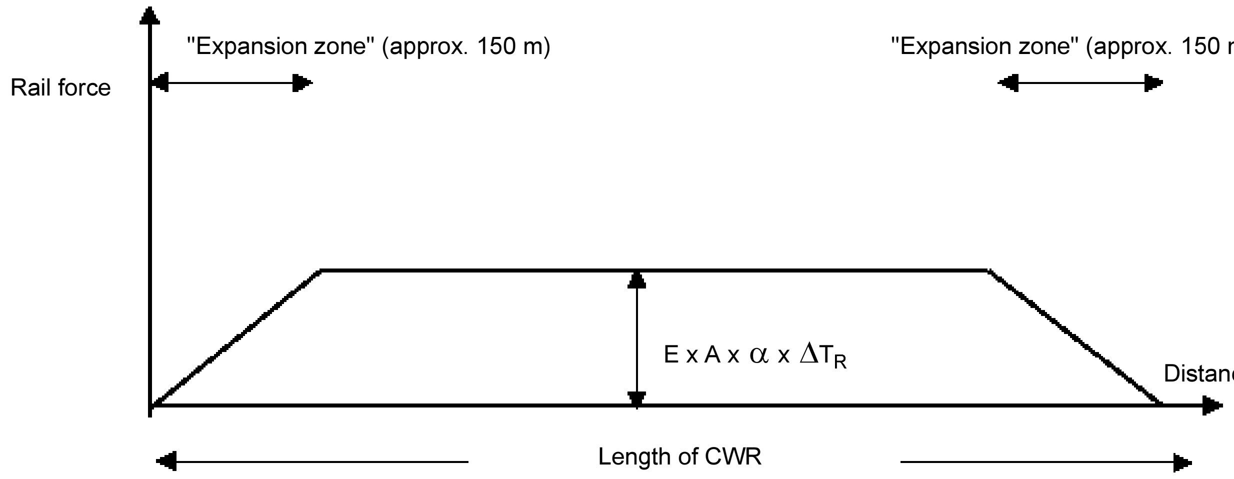


Fig. 1 - Behaviour of CWR under the effects of temperature changes

where:

α = is the coefficient of thermal expansion

ΔT_R = is the change in rail temperature relative to the reference or laying temperature

E = is Young's Modulus for steel (210 000 N/mm^2)

A = is the combined cross-section of two rails

F = is the force in the track

For instance: ΔT_R = + 50°C and UIC 60 rails, Force in track (F) $\Rightarrow E \times A \times \alpha \times \Delta T_R \approx$ 1600 KN (when α is taken equal to 1,0 x 10^{-5}).

1.1.2 - Effect of the presence of a bridge in the track

Introducing a bridge under a CWR track means, effectively, that the CWR track is resting on a surface subject to deformation and movements, hence causing displacement of the track.

Given that both track and bridge are able to move, any force or displacement that acts on one of them will induce forces in the other.

Interaction therefore takes place between the track and the bridge as follows:

- Forces applied to a CWR track induce additional forces into the track and/or into the bearings supporting the deck and movements of the track and of the deck.
- Any movement of the deck induces a movement of the track and an additional force in the track and, indirectly, in the bridge bearings.

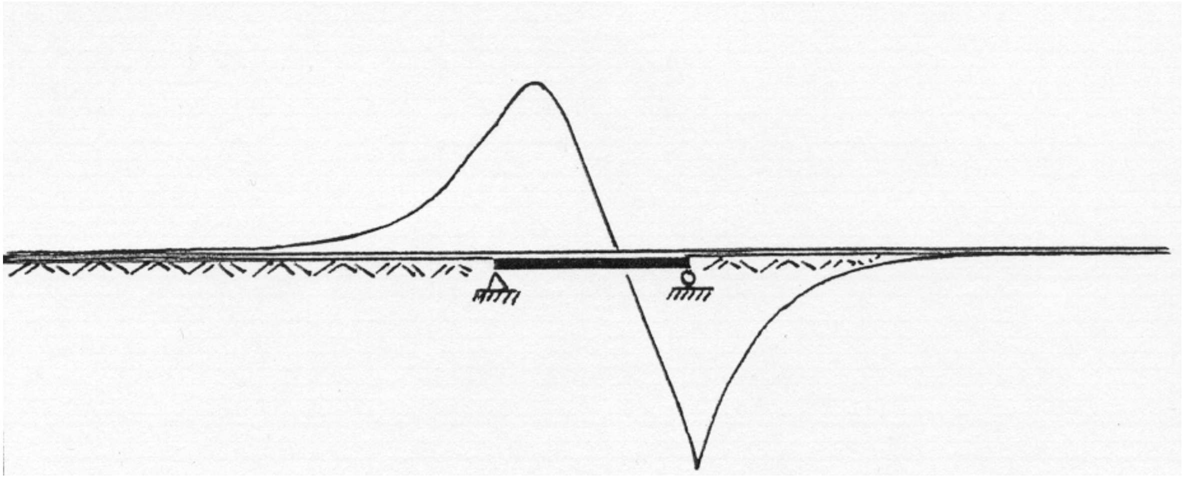


Fig. 2 - Example of a curve showing rail stresses due to temperature variations in the bridge deck

1.1.3 - Parameters affecting the phenomenon

A distinction can be made between bridge parameters and track parameters.

1.1.3.1 - Bridge parameters

1.1.3.1.1 - Expansion length

In case of a single deck bridge with a simply supported deck or a continuous deck with the fixed elastic support at one end, the expansion length is the span length or the total length of the deck. If the fixed elastic support is located at some intermediate point, the deck is considered to have two expansion lengths, on either side of the fixed elastic support (see Point 1.3.2 - page 9).

More generally, the expansion length L is the distance (in metres) between the thermal centre-point and the opposite end of the deck. The position of the thermal centre-point depends on the position and type of supports.

Four types are possible: fixed elastic, moving, moving friction and elastic. Although no clear distinction can be made between fixed elastic and elastic support in terms of theoretical behaviour, it is useful to state that fixed elastic supports consist of a fixed bearing on a flexible substructure (pier or abutment) while elastic supports consist of a flexible bearing on a flexible substructure.

In case of a succession of decks the total expansion length at a certain joint is the sum of the expansion lengths of the two nearest decks.

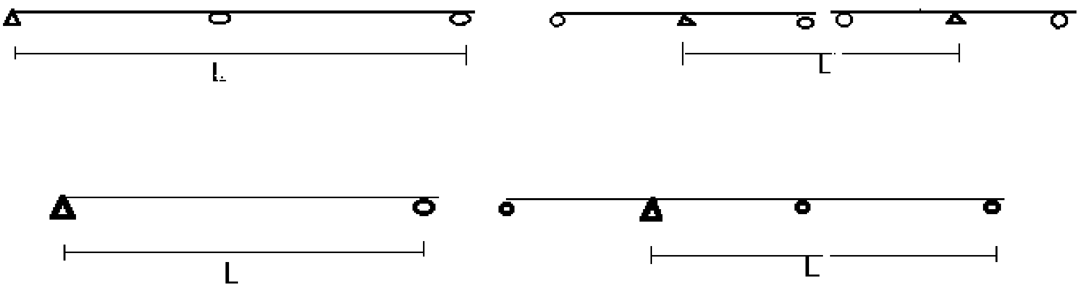


Fig. 3 - Expansion length

1.1.3.1.2 - Span length

Because of the fact that the vertical loading on the deck causes a longitudinal displacement of the deck end, the span length influences the track/bridge interaction.

1.1.3.1.3 - Support stiffness

The resistance of the deck to horizontal displacement is a fundamental parameter as it affects all the interaction phenomena. This factor is determined primarily by the total stiffness of the supports. See point 1.3.2 - page 9 for a development of this point.

The total support stiffness is composed of the stiffness of each support. In turn, the stiffness of each support is composed of the stiffness of the bearing and of the various components of the support (the pier, the base, the foundations and the soil in which they are embedded).

1.1.3.1.4 - Bending stiffness of the deck

The bending stiffness of the deck is a parameter, inasmuch as the vertical deformation of the deck displaces the upper edge of the deck in the horizontal direction. This deformation also generates interaction forces.

1.1.3.1.5 - Height of the deck

The distance of the upper surface of the deck slab from the neutral axis of the deck and the distance of the neutral axis from the centre of rotation of the bearing affect the interaction phenomena due to the bending of the deck.

1.1.3.2 - Track parameters

1.1.3.2.1 - Track resistance

The resistance k of the track per unit length to longitudinal displacement u is an important parameter. This parameter, in turn, depends on a large number of other factors (whether the track is loaded or unloaded, whether the track is ballasted or not, manner in which the track is laid, the standard to which the track is maintained etc...). It is essential that full information concerning the relationship between the rail and the deck and / or the embankment is available.

1.1.3.2.2 - Cross sectional area of the rail

The cross-sectional area of the rail A is also a track parameter.

1.2 - Track behaviour

This chapter describes the behaviour of the track on its supporting structure, taking into account the type of track. The relationship between track displacement and force applied depends on the track structure adopted, the standard of maintenance, any defects that may be present, the vertical load applied to the rail and the frequency of the forces applied.

1.2.1 - Ballasted track

The resistance to longitudinal displacement depends on the following:

- The resistance of the rail to displacement relative to the sleeper. This resistance is provided by the rail fastening, and its magnitude depends on the efficiency of the clamping action.

- The resistance of the sleeper/rail assembly to displacement relative to the deck. This resistance is provided by the tendency of the ballast to resist any movement of the sleeper and by the friction between ballast and deck.

For unloaded ballasted track the second of these factors is far less significant than the former.

1.2.1.1 - General principles governing track behaviour

The resistance of the track to longitudinal displacements is a function of the displacement of the rail relative to its supporting structure. This resistance increases rapidly while the displacement remains low, but remains virtually constant once the displacement has reached a certain magnitude.

Fig. 4 shows the resistance of the track to longitudinal displacement as a function of the above displacement.

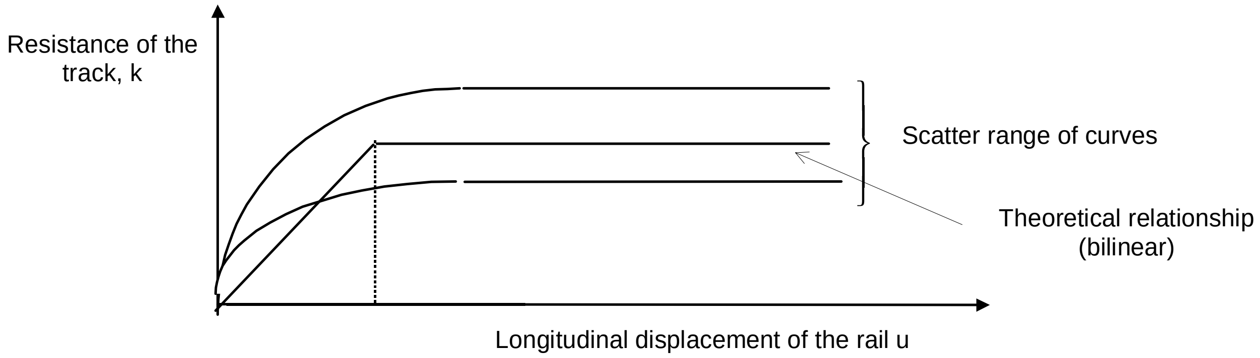


Fig. 4 - Longitudinal resistance of the track as a function of longitudinal displacement

The resistance to longitudinal displacement is higher on loaded track than on unloaded.

The resistance of the rails to longitudinal displacement relative to the sleepers is the determining factor when the ballast is frozen and in the case of unballasted track (see Point 1.2.2 - page 8).

1.2.1.2 - Bilinear behaviour of the track

In order to simplify calculations, the curves in Fig. 4 (see page 6) can be replaced by bilinear functions as shown in Fig. 5, where the magnitude of the resistance k is expressed as a function of the displacement of the rail u relative to its supporting structure.

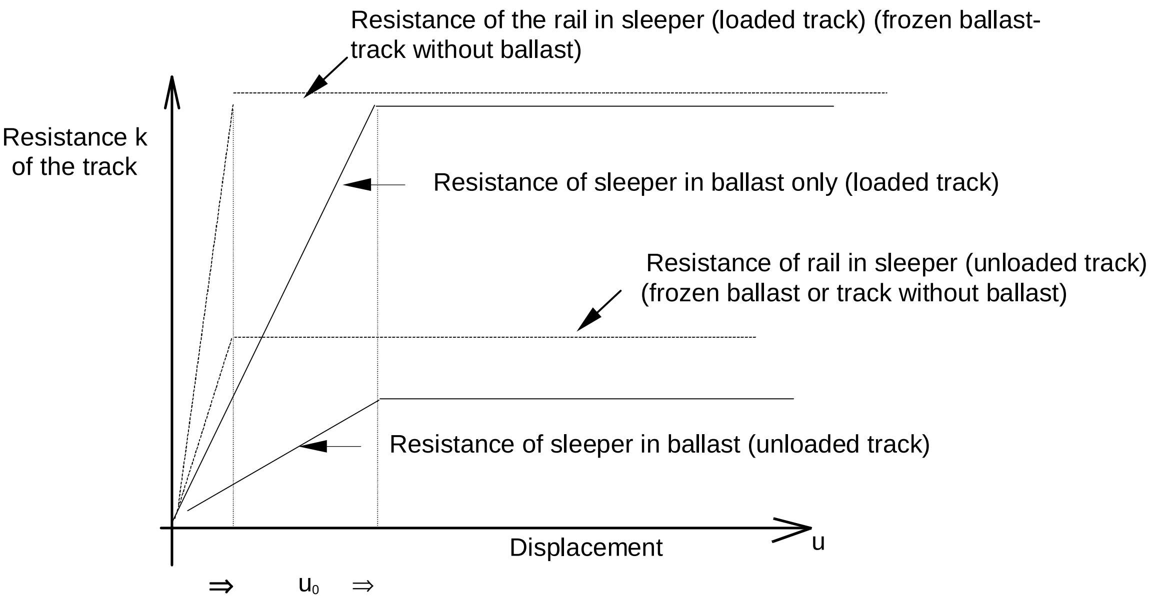


Fig. 5 - Resistance k of the track per unit length as a function of the longitudinal displacement u of the rails

The bilinear function allows the resistance of the track to be represented to a level of precision sufficient for calculation purposes.

The relationship between resistance and displacement varies according to the type of track structure and maintenance procedures adopted.

The conventional values assumed for ballasted track, with reference to Fig. 5, are as follows:

- Displacement u_0 between elastic and plastic zones:

u_0 = 0,5 mm	for the resistance of the rail to sliding relative to the sleeper,
u_0 = 2 mm	for the resistance of the sleeper in the ballast.

NB : Some railways adopt other values for u_0 for the ballast alone, e.g. NS uses 2,8 mm.

- Current values of resistance k in the plastic zone:

k = 12 kN/m	Resistance of sleeper in ballast (unloaded track), moderate maintenance,
k = 20 kN/m	Resistance of sleeper in ballast (unloaded track), good maintenance,
k = 60 kN/m	Resistance of loaded track or track with frozen ballast.

NB : Some railways use only k = 20 kN/m for unloaded track, and for loaded track 40 kN/m (NS) or 60 kN/m (DBAG).

1.2.2 - Unballasted track

The bilinear function also applies in the case of rail fastened directly using rail fastenings. In this case, the displacement u_0 at the beginning of the plastic zone is 0,5 mm, and the resistance k is 40 kN/m for unloaded track and 60 kN/m for loaded track.

The special case of rail embedded in resin may be dealt with by adopting a linear relation without a plastic zone, with the following values for resistance k:

Unloaded track:	k = 13 kN/mm per linear metre track for a maximum displacement u_0 = 7 mm;
Loaded track:	k = 19 kN/mm per linear metre track for a maximum displacement u_0 = 7 mm.

1.3 - Behaviour of the bridge

In order to study the track/bridge interaction on the bridge side, the following aspects have to be taken into consideration:

- The static arrangement of the bridge.

- The behaviour of the bearings.

- The behaviour of the supports.

- The total support stiffness.

- The bending behaviour of the deck.

1.3.1 - Static arrangement

The static arrangement of a bridge is defined by the number of decks, the number of supports per deck (including any shared supports), the positions of fixed and movable supports, the span lengths, the expansion length(s) and the positions of the rail expansion devices, if any. The most usual static arrangements are shown in Fig. 6 (see page 9).

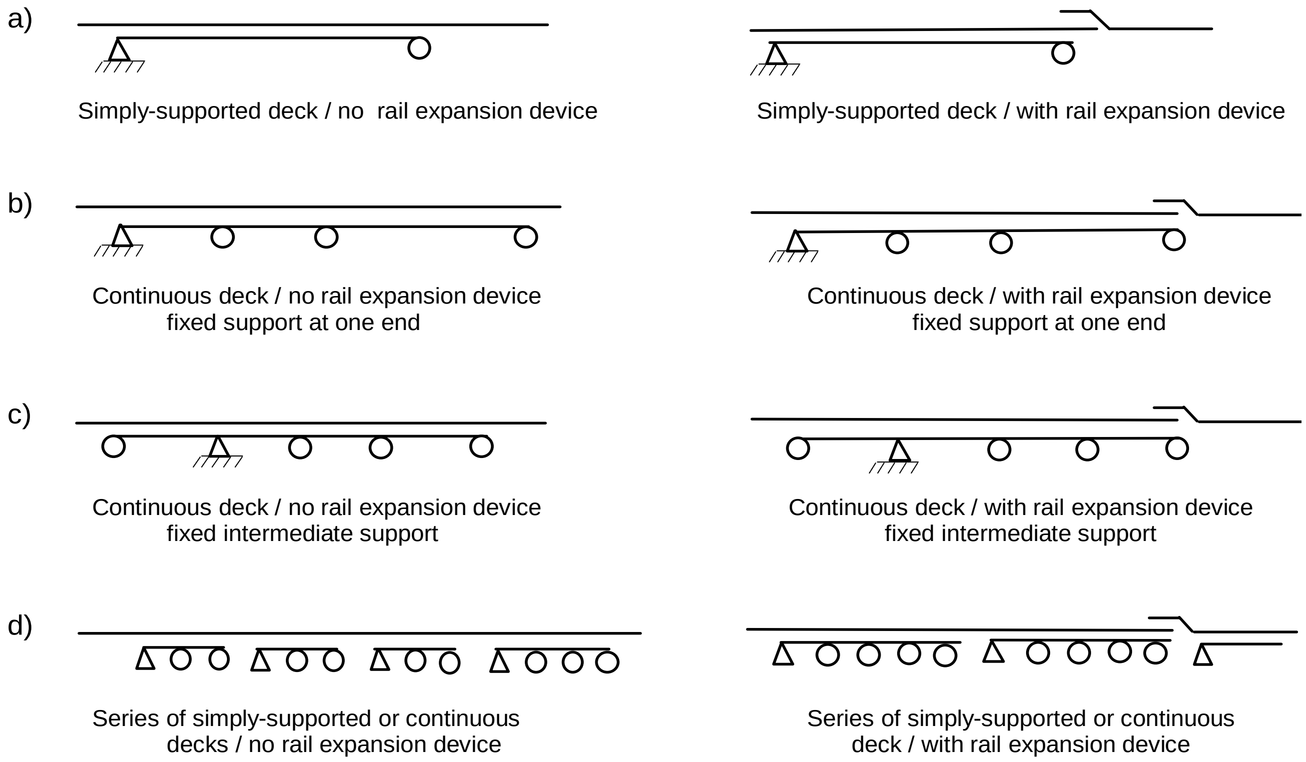


Fig. 6 - Outlines of the most usual static arrangements

1.3.2 - Behaviour of the supports

1.3.2.1 - Behaviour of the bearings

The types of bearings used and their characteristics have a major effect on the resistance of the deck to displacement. In general the stiffness of a moving bearing is usually ignored. For more accurate calculations and in the case of a moving bearing with a certain degree of elastic stiffness (e.g. an elastomer bearing), the stiffness of the bearing should be taken into account (see Point 2.1.3.5 - page 27).

The value of the coefficient of friction may vary from 0 % to 5 %.

1.3.2.2 - Resistance of the supports to horizontal displacement

The stiffness K of the support, including its foundation, to displacement along the longitudinal axis of the bridge is given by:

$$K = \frac{H[kN]}{\Sigma\delta_i[cm]}$$

With $\delta_i = \delta_p + \delta_\phi + \delta_h + \delta_a$

Where δ_p = the displacement at the head of the support due to elastic deformation;

δ_ϕ = the displacement at the head of the support due to rotation of the foundation or foundation slab;

δ_h = the displacement of the support due to the horizontal movement of the foundation;

δ_a = the relative displacement between the upper and lower parts of the bearing.

The value of the displacement component is determined at the level of the bearing or at the level at which other structural assemblies are connected to the supports, as shown in Fig. 7 below.

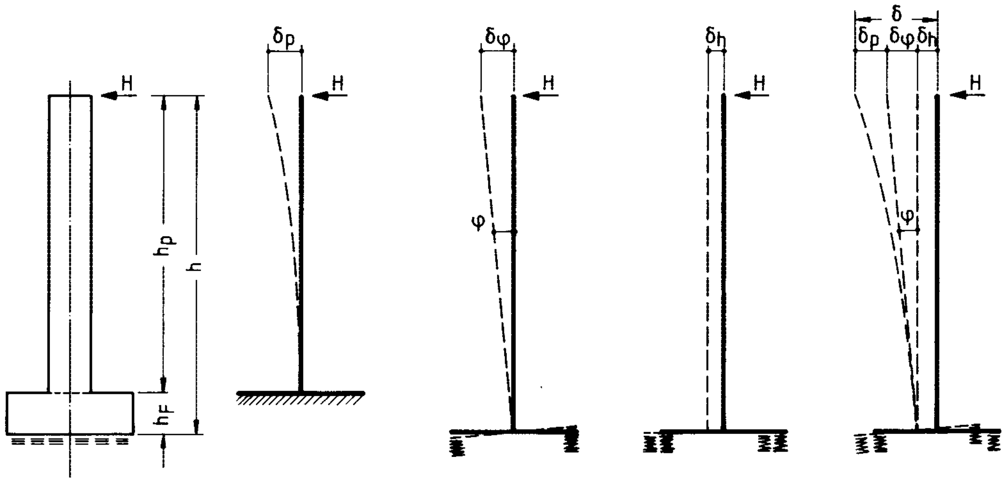


Fig. 7 - Factors influencing support stiffness K

1.3.2.3 - Foundation stiffness

When calculating the stiffness of the foundations, it is necessary to select the modulus of elasticity appropriate to the various load cases (temperature variations, braking/acceleration), taking into account the characteristics and geotechnical parameters of the site.

The static modulus of elasticity is used when calculating temperature variations, and the dynamic modulus when calculating the effects of braking and acceleration.

1.3.2.4 - Total support stiffness

Usually, for braking actions, calculations of total support stiffness should take account of the contributions of all supports that resist the longitudinal displacement.

In most cases, one single fixed support is considered. When there are more than one supports contributing to the total stiffness, the sum of their contributions should be considered. In the case of sliding bearings with friction, point 2.1.3.5 - page 27 should be applied. In case of elastic bearings, the total support stiffness is the sum of the stiffness of all the supports, and the location of the effective resulting fixed point should be determined.

The total stiffness K_{tot} is thus given by $\sum K_i + 2 \sum F_{fj}$ kN/cm, where K_i is the stiffness of the supports with fixed or elastomer bearings and F_{fj} is the friction resistance of the movable bearings.

1.3.3 - Bending behaviour of the deck

Vertical traffic loads on the bridge generate large track/bridge interaction forces as a result of deck bending, which causes longitudinal displacement of the upper edge of the deck end. The interaction effects depend primarily on the flexibility of the deck and on the position of its neutral axis, but are also influenced by the stiffness of the fixed elastic support and by the height of the deck.

Horizontal displacement of the deck due to traffic loads remains constant when considered along the neutral axis but varies when measured at the upper part of the slab supporting the track. The flexibility of the fixed support reduces the displacements measured above by a constant amount equal to the backward displacement of the support.

These displacements, which result in interaction between the deck and the track, generate large forces in the track and the supports. Fig. 8 below gives a detailed diagram of the situation.

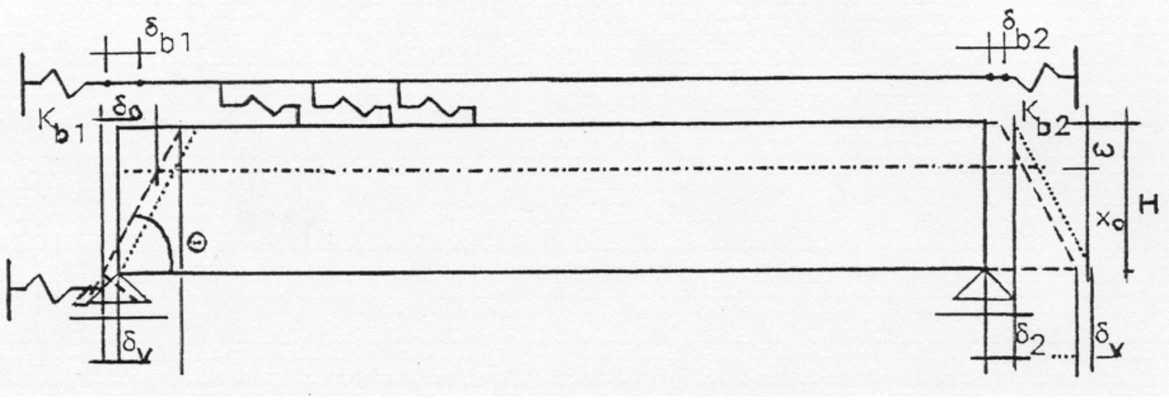


Fig. 8 - Effect of deck bending on the end sections

1.4 - Actions to be taken into account

1.4.1 - Introduction

The cases that could lead to interaction effects are those that cause relative displacement between the track and the deck.

The cases concerned are as follows:

1. The thermal expansion of the deck only, in the case of CWR, or the thermal expansion of the deck and of the rail, whenever a rail expansion device is present.
2. Horizontal braking and acceleration forces.
3. Rotation of the deck on its supports as a result of the deck bending under vertical traffic loads.
4. Deformation of the concrete structure due to creep and shrinkage.

5. Longitudinal displacement of the supports under the influence of the thermal gradient.

6. Deformation of the structure due to the vertical temperature gradient.

In most cases, the first three effects are of major importance for bridge design. The effects are added in accordance with the rules set out below. Each load case is also discussed in a later paragraph.

1.4.2 - Actions due to changes in temperature

The following aspects of temperature variation should be considered:

- Changes in the uniform component of the temperature which causes a change in length in a free-moving structure.

- Differences in temperature between the deck and the rails, in the case of track with an expansion device.

The reference temperature for a bridge is the temperature of the deck when the rail is fixed. The temperature of the bridge does not deviate from the reference temperature by more than ± 35°C, and the temperature of the rail does not deviate by more than ± 50°C. The difference in temperature between deck and track does not exceed ± 20°C (in the case of track with an expansion device).

In the case of CWR a variation in the temperature of the track does not cause a displacement of the track and thus there is no interaction effect due to the variation in the temperature of the track.

1.4.3 - Actions due to braking and acceleration

The braking and acceleration forces (F_R) applied at the top of the rail are assumed to be distributed evenly over the length under consideration L_f with the following standard values:

- Acceleration:	q_{lak} = 33 kN/m per track, with L x $q_{lak} \leq$ 1 000 kN for design based on load model 71 and on load model SW/0
- Braking:	q_{lbk} = 20 kN/m per track, with L x $q_{lak} \leq$ 6 000 kN for design based on load model 71 and on load model SW/0
	q_{lbk} = 35 kN/m per track for design based on load model SW/2

When using load model SW, only the length of the loads on the structure is taken into account.

These standard values are used for all types of track - CWR or jointed, with and without an expansion device.

For lines restricted to specific types of traffic, it is possible to use braking and acceleration forces equivalent to 1/4 of the axle load imposed by the vehicles concerned, still using the maximum values mentioned above.

The braking and acceleration forces are to be combined with the corresponding vertical loads.

In the case of a bridge carrying two or more tracks, the acceleration forces on one track are to be added to the braking forces on the other. Only two tracks need to be considered.

1.4.4 - Actions due to bending of the deck

Vertical traffic loads cause the deck to bend, which in turn causes rotation of the end sections and displacement of the upper edge of the deck end, as explained in point 1.3.3 - page 11. The loads may be applied to the deck, to the embankment at both ends of the deck or to the embankment at one end only, and the rotation and displacement phenomena apply both to theoretical load trains and actual, specific trains.

These phenomena need to be considered for both deck ends. The effects in the case of a through-girder bridge differ from those observed on a deck slab bridge.

1.5 - Consequences for the bridge and for the track

The phenomena described in the previous articles need verifications to be made concerning the bridge and the track. The different criteria to be satisfied are detailed in the following points.

1.5.1 - Combining the load case effects

For the calculation of the total support reaction, and in order to compare the global stress in the rail with the permissible value set by each railway, the global effect $\sum R$ is calculated as follows:

$\sum R = \alpha R\ (\Delta T) + \beta R\ (\text{braking}) + \gamma R\ (\text{bending})$

The values of the coefficients for the support reactions, α, β and γ, are combination factors.

For the calculation of the global values of rail stresses and displacements, α, β and γ all have the value 1 for continuous or simply-supported decks.

1.5.2 - Permissible additional stresses in continuous welded rail on the bridge

Theoretical stability calculations, on UIC 60 CWR, of a steel grade giving at least 900 N/mm^2 strength, minimum curve radius 1 500 m, laid on ballasted track with concrete sleepers and consolidated > 30 cm deep ballast, well consolidated ballast, give a total possible value for the increase of rail stresses due to the track/bridge interaction.

The maximum permissible additional compressive rail stress is 72 N/mm^2.

The maximum permissible additional tensile rail stress is 92 N/mm^2.

In case of other rails than UIC 60 the permissible additional compression and tensile rail stresses should be specified by the relevant authority.

1.5.3 - Absolute and relative displacement

Limits have to be placed on the displacement of the deck and track in order to prevent excessive deconsolidation of the ballast because, if this were to occur, the conditions mentioned in the previous section might no longer be met. The displacement limits also play a role in limiting indirectly the additional longitudinal stress in the rails.

These limits are as follows:

- The maximum permissible displacement between rail and deck or embankment under braking and/or acceleration forces is 4 mm;
- For the same braking and/or acceleration forces, the maximum absolute horizontal displacement of the deck δ_{abs} is ± 5 mm if the rails run across one or both ends of the bridge/embankment transition;
- In the case of CWR on ballasted track with expansion devices, the maximum permissible absolute horizontal displacement of the deck under the same loads is 30 mm.

1.5.4 - End rotations of the deck

The end rotation of a bridge deck due to traffic loads is an important factor for determining satisfactory track/bridge interaction behaviour. In order to determine an appropriate limit to the end rotation of a bridge deck it is necessary to consider also other criteria such as dynamic effects (ballast maintenance) and passenger comfort.

- Under vertical load, the displacement of the upper edge of the deck end must also be limited, in order to maintain ballast stability. Obviously, the effects of this displacement must be added to the effects of temperature variation and of braking/acceleration. This limit results in a maximum permissible value for deck end rotation:

  - In the case of CWR on ballasted track, the permissible displacement between the top of the deck end and the embankment or between the top of two consecutive deck ends due to vertical bending is:

    $\delta_{(\theta H)}$ = 8 mm in the three cases

    For a single-track deck, this rotation is determined under the effect of a LM 71 multiplied by the corresponding dynamic factor. For a multiple-track deck, a maximum of two tracks are loaded.

- The maximum vertical displacement of the upper surface of the end of a deck relative to the adjacent construction has to be limited. The permissible value should be specified by the relevant authority.

1.5.5 - Support reactions

The interaction as described in this part A results in horizontal support reactions at the fixed elastic supports, and these must be taken into account along with conventional support reactions when calculating the structure and supports.

1.5.6 - Rail expansion devices

It is preferable to avoid expansion devices in the track, but one should always be inserted at the free end of the deck if the total additional rail stress or the above mentioned displacements exceed the permissible values.

Using the possibility of locating the fixed support at the middle of the deck, it is possible to increase the length of a single deck carrying CWR (see hereinafter).

Generally speaking, this proof will lead to the following conclusion (which should be verified): the maximum expansion length of a single deck carrying CWR without expansion device will be:

- 60 m for steel structures carrying ballasted track (maximum length of deck with fixed bearing in the middle:120 m),

- 90 m for structures in concrete or steel with concrete slab carrying ballasted track (maximum length of deck with fixed bearing in the middle: 180 m).

In the case of unballasted track, a specific evaluation should be done.

Even when the calculated stresses and displacements do not exceed the permissible values, it may be necessary to fit an expansion device in the track. This is the case when the daily variation of the length of the deck exceeds the permissible values taking into account the track maintenance conditions (permissible ΔL to be defined by each railway; generally between 10 and 15 mm).

The calculations are made using a precise track arrangement (CWR, expansion devices, joints). When for any reason (for example maintenance works consisting of severing CWR) this track arrangement is modified, the service conditions on the bridge should be reviewed (prohibition of braking, for example); the bearings could also be modified.

A new analysis of the interaction effects should be made, when the functioning of the bearings (and/or of the supports) is changed.

1.6 - Calculations using the leaflet

For dimensioning the structure from the point of view of track/bridge interaction, three different steps of calculations can be used:

The predimensioning method described in point 1.6.1 - page 15, a method using the tools given in points 2 - page 23 and 3 - page 32, and the computer calculations described in point 1.7 - page 17.

1.6.1 - Predimensioning method

When calculating the interaction effects, the stiffness of the support is one of the key parameters. The stiffness of the support depends upon the support design and this design depends, among other considerations, on the horizontal force on the support. In order to obtain an initial estimate of the horizontal force in the support due to braking/acceleration, the following method can be used:

For bridges carrying ballasted track, with fixed support at one end and with or without expansion device at the free end, the characteristic braking/acceleration force transferred by the deck to the bearings is the total braking/traction force applied to the rail over the bridge multiplied by the reduction coefficient β given on the next page and the minimum stiffness of the support must be $K > 20.\beta.L / 5$ in kN/mm.

Table 1 : Reduction coefficients β for obtaining the part of the horizontal force on the rail transferred to the supports

Overall length of structure	Continuous track	Expansion device at one end
[m]	Single or double track	Single or double track
30	0,5	
60	0,5	0,60
90	0,6	0,65
120	0,7	0,70
150	0,75	0,75
180		0,80
210		0,85
240		0,90
270		0,90
300		0,90

The table refers to continuous track only. If for any reason the continuity of the track were to be interrupted (modification of the track form or rail breakage), new arrangements must be made. The resistance of the supports and of the bearings under the new forces transmitted by the track should be verified in particular.

For bridges carrying track with an expansion device at both ends of the deck, the coefficient β is equal to 1.

1.6.2 - Design method using Points 2 and 3 of the leaflet

1.6.2.1 - Point 2 (Calculations without interaction)

For structures consisting of one deck the values of the interaction effects can be calculated by using the design graphs in Appendices A - page 36 and B - page 42 and the formulae given in point 2 - page 23.

The design graphs and formulae refer to single track structures consisting of one simply-supported bridge deck with a fixed bearing at one end.

Point 2 also contains rules to evaluate interaction effects in single deck bridges, which differ from the standard (two tracks, continuous decks, etc.)

The criteria to be satisfied for the application of point 2 - page 23 are the following:

- In the case of continuous welded rail:
  The maximum permissible additional compressive rail stress due to temperature variation of the deck, braking/acceleration and deck-end rotation evaluated from the diagrams and with reference to point 2.1.4 - page 27 is 72 N/mm^2 ($\sigma_{rail} \leq 72$ N/mm^2).
  The maximum permissible absolute horizontal displacement of the deck due to braking/acceleration evaluated with $F_{support}$ issued from the diagrams and $K_{support}$ is 5 mm ($\delta_{abs} \leq 5$ mm).
  The maximum permissible absolute horizontal displacement of the top of the deck-ends due to vertical bending (including the dynamic factor) and calculated without considering any interaction is 10 mm ($\delta_{(\theta H)} \leq 10$ mm).

- In the case of a deck carrying CWR with an expansion device at one end:
  The maximum permissible absolute horizontal displacement of the deck due to braking/acceleration is 5 mm ($\delta_{abs} \leq 5$ mm).
  The maximum permissible absolute horizontal displacement of the top of the deck-end without the expansion device due to vertical bending is 10 mm ($\delta_{(\theta H)} \leq 10$ mm).
  The maximum vertical displacement of the upper surface of the end of a deck relative to the adjacent construction has to be limited. The permissible value should be specified by the relevant authority.

- In the case of a deck carrying an expansion device at both ends or with jointed track:
  The maximum permissible absolute displacement of the deck due to braking/acceleration is 30 mm ($\delta_{abs} \leq 30$ mm).
  The maximum lifting of the top of the deck end has to be limited. The permissible value should be specified by the relevant authority.

1.6.2.2 - Point 3 (Calculations with interaction)

The criteria to be satisfied are the same as mentioned in point 1.7.2 - page 18 for the permissible additional tensile and compressive stresses, as well as for the different displacements.

The permissible displacement of 5 mm due to braking/acceleration is also to be considered as a permissible relative displacement between two consecutive decks.

1.7 - Calculations with a computer program

1.7.1 - Validation of the computer program

Computer programs for track-bridge interaction analyses should be validated before use, by analysing the test-cases reported in Appendix D - page 57. For the first group of test cases the solution is also given in this leaflet, as it is useful to calibrate some solution parameters. For the second group of test-cases the solution can be obtained either from the National Railway Authority or from ERRI.

As indicated in point 1.7.3 - page 19 different types of analyses can be made at different levels of accuracy. The different types can be grouped into the following two categories:

1. Simplified separate analyses for thermal variations, braking/acceleration forces, vertical deflections;

2. Complete analyses of the simultaneous effects of thermal variations, braking/acceleration forces, vertical deflections.

According to the capabilities of the computer program, either the simplified or the complete analysis can be used. For this reason both types of analysis have been carried out in solving the two groups of test-case, which are presented in Appendix D - page 57.

A program is considered as validated when the error on the single effects as well as on the overall effect is less than 10% with respect to the corresponding type of analysis (sum of the effects or global effect). Larger tolerances, up to 20%, can be accepted if the error is on the safe side.

1.7.2 - Criteria to be met

The criteria which have to be met in the calculations done with a computer program are the following:

- In the case of continuous welded rail:
  The maximum permissible additional compressive rail stress due to temperature variation of the deck, braking/acceleration and deck-end rotation is 72 N/mm^2 ($\sigma_{rail} \leq 72$ N/mm^2).
  The maximum permissible additional tensile rail stress due to temperature variation of the deck, braking/acceleration and deck-end rotation is 92 N/mm^2 ($\sigma_{rail} \leq 92$ N/mm^2).
  The maximum permissible relative horizontal displacement between the deck and the rail due to braking/acceleration is 4 mm ($\delta_{rel} \leq 4$ mm).
  The maximum permissible absolute horizontal displacement of the deck due to braking/acceleration is 5 mm ($\delta_{abs} \leq 5$ mm).
  The maximum permissible displacement between the top of the deck-end and the embankment or between the tops of two consecutive deck-ends due to vertical bending (including the dynamic factor) is 8 mm ($\delta_{(\theta H)} \leq 8$ mm) (with interaction).

- In the case of a deck carrying CWR with an expansion device at one end:
  The maximum permissible relative horizontal displacement between the deck and the rail due to braking/acceleration is 4 mm ($\delta_{rel} \leq 4$ mm).
  The maximum permissible absolute horizontal displacement of the deck due to braking/acceleration is 5 mm ($\delta_{abs} \leq 5$ mm).
  Where there is no expansion device the maximum permissible displacement between the top of the deck-end and the embankment or between the tops of two consecutive deck-ends due to vertical bending (including the dynamic factor) is 8 mm ($\delta_{(\theta H)} \leq 8$ mm).
  Where an expansion device is fitted, the maximum lifting of the top of the deck end has to be limited. The permissible value should be specified by the relevant authority.

- In the case of a deck carrying track with an expansion device at both ends or with track with joints:
  The maximum permissible absolute horizontal displacement of the deck due to braking/acceleration is 30 mm ($\delta_{abs} \leq 30$ mm).
  The maximum permissible vertical displacement of the upper surface of the end of a deck relative to the adjacent construction has to be limited. The permissible value should be specified by the relevant authority.

1.7.3 - General recommendations for computer-assisted interaction analysis

The track-structure interaction effects should be evaluated in terms of longitudinal actions carried by the supports (reactions at the fixed supports), additional rail stresses, as well as absolute and relative displacements of the rails and deck, in order to determine the quantities to be verified according to point 1.7.2 - page 18.

The interaction effects should be evaluated by a series of non-linear simulation analyses to study the behaviour of the track-bridge system, subjected to the thermal actions of the bridge deck and, where on rail expansion device is fitted, of the rail, as well as to the vertical loads and the longitudinal braking or acceleration forces.

The track/structure interaction should be calculated with a structural arrangement of the type shown in Fig. 9 taking into account:

- the bridge parameters mentioned in points 1.1.3.1 - page 4 and 1.3 - page 8 such as: static arrangement of the bridge, geometry of the decks and supports, deck-lengths, supports (pier, base, foundations and soil), bending stiffnesses and height of the decks,
- the track parameters mentioned in points 1.1.3.2 - page 5 and 1.2 - page 6 such as: cross sectional area of the rail and track stiffnesses taking account of bilinear behaviour,
- the loads and imposed displacements mentioned in point 1.4 - page 11 such as: temperature variation of the decks, temperature variation of the rails in case of track with expansion device, braking and acceleration, vertical bending of the decks due to train loads.

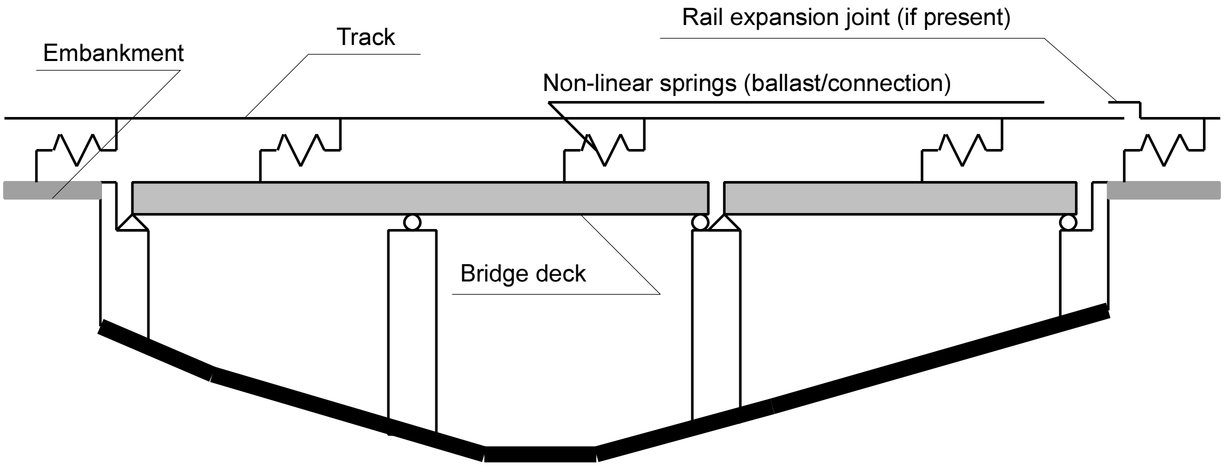


Fig. 9 - Structural diagram for the evaluation of track-bridge interaction effects

In general the centre-lines of the track and of the deck should be located in the model exactly in their actual position as shown in Fig. 10 (see page 20).

Similarly the position of the constraints should be those of the bearing devices (specifically of their rotation centre) in case of fixed bearings.

The connection between track elements and deck elements, as well as between deck elements and supports, may be modelled using rigid elements. For simplicity, the height of the track may be taken as equal to the height of the top of the deck. For the temperature variation and braking/acceleration load cases, the decks may be modelled without taking account of the difference of height between the track, the top of the deck and the top of the bearings.

It is not necessary to consider a detailed model of the substructure (bearing-pier-foundation system and bearing-abutment-foundation systems), when standard bridges are considered.

The horizontal flexibility of the substructure elements (piers, abutments) may then be evaluated separately and taken into account when determining the stiffness of the constraints (see Fig. 11 - page 21), according to point 1.3.2.2 - page 9. On the contrary the substructure should be accurately modelled for special bridge types, such as arch bridges, cable-stayed bridges or truss bridges.

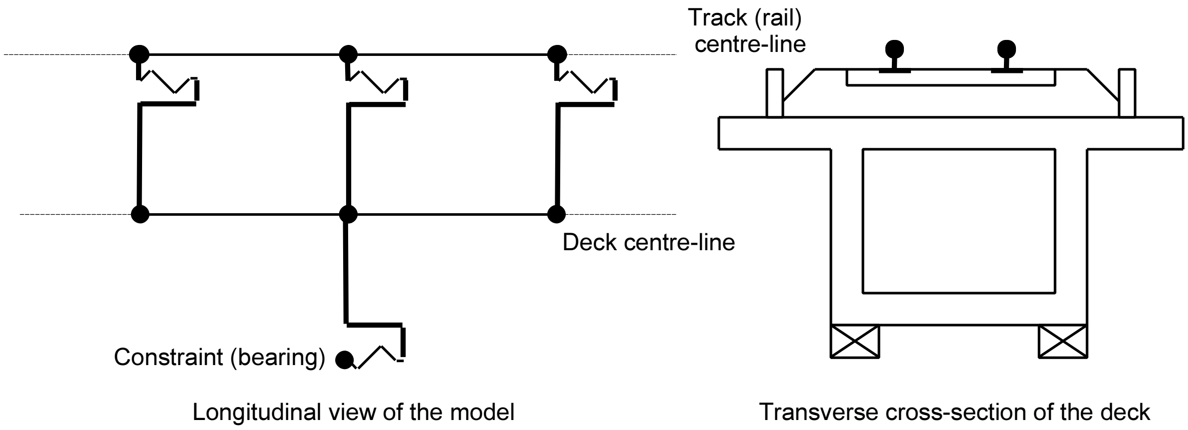


Fig. 10 - Typical model of the track-deck-bearing system

In any case, for the deck elements, the rail elements, the constraints modelling fixed supports and the substructure elements, if any, a linear elastic behaviour should be assumed, while for the elements connecting rail and deck elements a non-linear law reproducing the actual behaviour of the ballast - rail fastening system or of the direct fastening should be assumed, taking into account the actual vertical load (see Point 1.2 - page 6). An elasto-plastic law whose plastic threshold depends on the vertical load, such as the ones shown in Fig. 5 - page 7, may be assumed for simplicity. For the moving bearings made of sliding devices, the need to take account of friction should be considered. In such cases, an element whose longitudinal resistance depends on the vertical reaction should be used.

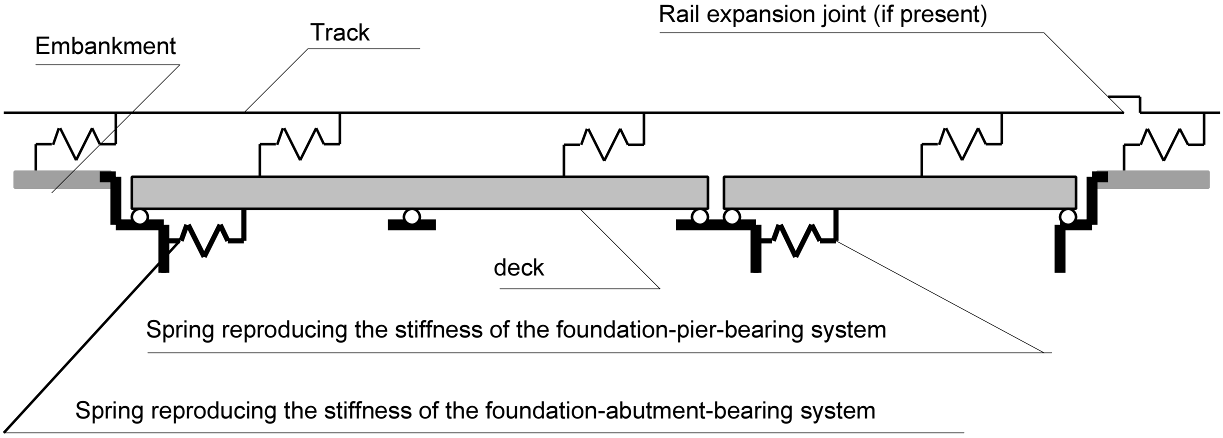


Fig. 11 - Simplified structural model for interaction analyses

Depending of the duration of the applied action (long for thermal variations, short for train loads and horizontal forces) and the type of soil, different foundation stiffnesses can be considered for the different load cases, always assuming however the most conservative conditions. When the stiffness of the substructure elements (piers, abutments) is uncertain, e.g. as a result of concrete cracking or soil characteristics, the two extreme values of the interval within which the actual values are included should be taken and two analyses carried out for each load case. In usual situations, a conservative evaluation of the support reactions and of the rail stresses will be obtained by considering respectively the maximum and the minimum values of the support stiffness.

An upper limit for the stiffness of a pier with a fixed bearing may be obtained by assuming the foundations to be perfectly rigid and all the tolerances in the bearing device to be zero.

Modelling the track and decks as discrete elements should guarantee an accurate evaluation of the items of major interest (support reactions, absolute and relative displacements of track and decks, rail stresses). For this purpose a finite-element model may be usefully adopted, where the track and the deck elements are modelled discretely over with a maximum element length of 2,00 m. The model should also include a part of the track on the adjacent embankments over at least 100 m.

According to the importance and the structural scheme of the bridge, different types of analyses may be made at different levels of accuracy. Two major categories of analyses may be considered:

1. Simplified separate analyses for thermal variations, braking/acceleration forces, vertical deflections;

2. Complete analyses of the joint effects of thermal variations, braking/acceleration forces, vertical deflections, simulating the train travelling the full length of the bridge with a step-by-step algorithm.

Both types of analyses should be carried out considering the nonlinear behaviour of the track-deck connection (ballast and fastenings) and, if friction is to be considered, of the sliding bearings. The effects of the train forces should be evaluated by considering different longitudinal resistances for the track in the different parts which are either loaded or not by the train.

When simplified separate analyses are carried out, the temperature effects should be evaluated using the minimum value of the longitudinal resistance derived from the typical elasto-plastic law of Fig. 5 - page 7. However train effects (vertical deflection and longitudinal forces) should be evaluated by considering the lowest resistance in the parts of track where there is no train load and the highest resistance where there is a train load.

When analysing the effects of vertical deflection in single-span bridges, train loads should be applied to the bridge and, if the train is long enough, to the embankment on one side only. Considering both embankments loaded would lead to a considerable overestimation of the effects (when a train moves from the bridge to the embankment, the increase in longitudinal resistance of the ballast on the embankment occurs when the deck is already fully deflected and has no influence on the interaction effects).

When analysing multispan bridges, either with simply supported spans or with a continuous deck, several positions of the train should be checked in order to find the most unfavourable conditions.

When making complete analyses, the assumed law for the ballast should take into account the dependence of its longitudinal resistance to vertical ballast stresses, i.e. on the position of the vertical train loads.

Generally speaking a complete analysis should cover the application of the thermal actions followed by the forces exerted by the moving train (vertical loads and braking/acceleration forces at the same time) in the same analysis. In any case the train should start outside the bridge or far from the point where the maximum effects are sought.

In the case of CWR, the temperature variation in the track may be assumed to be zero, as it does not affect the interaction effects (support reactions, additional rail stresses, absolute and relative displacements of track and deck), while the maximum and minimum values relevant to the deck should be considered. However, when there are expansion devices, the temperature variation in the track should be considered, and the most unfavourable conditions for interaction effects should be sought.

The vertical loads and braking/acceleration forces may be applied statically, with the values specified in point 1.4.3 - page 12. Where a complete analysis is carried out, the train loads should travel all along the entire bridge, and the maximum values of the interaction effects specified above should be recorded during the analysis.

In order to evaluate the maximum interaction effects, the most unfavourable situations should be considered, taking into account positive or negative temperature variations and different positions of the trains travelling along the bridge. Different temperature-sign and travelling-direction conditions will generally give maximum values for the different effects. In the case of double track structures, two moving trains should be combined so that the maximum effects are obtained. Several analyses are needed in some cases to identify the most unfavourable situations.

2 - Structures consisting of one bridge deck

2.1 - Structures consisting of one simply-supported, single-track bridge deck with a fixed bearing at one end

2.1.1 - General method

The interaction effects due to different actions depend upon a large number of parameters and should be checked against the required criteria.

Interaction between track and structure results in displacements, rail stresses and support reactions. The effects to be considered are: the support reaction, the rail stress at the end near the fixed bearing, the rail stress at the end near the movable bearing, the absolute deck displacement at the end near the fixed bearing and the relative displacement between the track and the deck or the embankment. The resulting values of displacements and rail stresses should be checked against the criteria given in point 1 - page 2.

The effects of the interaction between track and deck should be determined for the following load cases: temperature changes, braking/acceleration forces and vertical bending of the deck. The method used for combining the individual load cases is given in point 2.1.4 - page 27.

The magnitude of the interaction effects depends not only on the actions but also upon several of the track/bridge system parameters. When determining the interaction effects the following parameters should be considered: the deck length (L), the horizontal stiffness of the support $K_{support}$, the horizontal resistance of the connection between the track and the structure or the embankment k_{rail} and the track/bridge system parameters relevant to the bending of the deck, i.e. the positions of the track, the bearings and the neutral axis, the moment of inertia (I) and the modulus of elasticity (E).

The values of the interaction effects should be determined using the design curves and formulae given in this chapter. The design curves and formulae are based on certain assumptions regarding, for instance, the magnitude of the temperature variations and the magnitude and the position of braking forces. The use of other assumptions gives different results; point 2.1.3 - page 25 shows how to deal with changes in the basic assumptions.

The design curves and formulae are valid for single-track bridges carrying continuous-welded rails (CWR) or with an expansion device in the track. Use of the design curves and formulae for double-track bridges is covered in point 2.2 - page 28.

The design curves are limited to span length L = 110 m. For bridges with span length greater than 110 m, specific calculations should be made.

2.1.2 - Basic graphs and formulae

The interaction effects due to the load cases temperature variation, braking (as specified in Load Model 71) and vertical bending are determined from graphs.

The following assumptions were made in drawing the basic graphs:

- Single track,

- UIC 60 rail,

- No friction in the movable bearings.

2.1.2.1 - Design curves for the interaction due to temperature variation and braking as specified in Load Model 71

The design curves are given in Appendix A - page 36. They are divided into four groups, according to the load cases and whether or not an expansion device is used.

The expansion length of the deck is shown on the x-axis and the value of the interaction effects on the y-axis.

The following additional assumptions were made:

- Braking force 20 kN/m with a maximum total force of 6000 kN;
- For CWR track: temperature variation of the deck ΔT_{deck} = +35°C and $\alpha = 1{,}0 \times 10^{-5}$;
- Expansion length ≤ 110 m.

The values of $F_{support}$, and σ_{rail} for a given value of $K_{support}$ and k_{rail} are obtained by performing a linear interpolation (or extrapolation) with regard to the value of k_{rail}, and then performing a linear interpolation with regard to the value of $K_{support}$ using the closest two values of $K_{support}$ (K_2 and K_5 or K_5 and K_{20}).

There is no graph for relative rail displacement, as relative rail displacement is not needed for verifying the effects of temperature variation and always lies within the limit value for the effects due to braking, as long as the absolute displacement of the deck stays within the limit value of 5 mm.

There is no graph for absolute displacement of the deck. The value of the absolute displacement of the deck [δ_{deck}(fixed)] may be obtained by dividing the support reaction $F_{support}$ by the support stiffness $K_{support}$.

2.1.2.2 - Design curves for the interaction due to vertical bending of the bridge deck

The design curves for the evaluation of the interaction due to vertical bending of the bridge deck have been evaluated with reference to a standard longitudinal plastic shear resistance equal to 20 kN/m and 60 kN/m for unloaded and loaded track respectively.

The design charts are given for the following two different situations:

- deck bridge: the track lies on the top of the bridge deck (deck neutral axis below track axis),
- through girder bridge: the track lies between the girders supporting the slab (deck neutral axis above track axis).

Additional symbols are used in the design charts.

Compression stresses are considered to be negative and tensile stresses positive.

The charts to evaluate σ_{rail} at the fixed and movable ends and $F_{support}$ are given in Appendix B. They are drawn for ΘH = 8 mm for deck bridges and ΘH = 1mm for through girder bridges. For ΘH values other than 8 and 1 mm the following formulae apply:

Deck bridges:

$F_{support}[\Theta H] = F_{support}[8] \times (\Theta H/8)^{0,86}$

$\sigma_{rail}[\Theta H] = \sigma_{rail}[8] \times (\Theta H/8)^{0,86}$

Through bridges:

$F_{support}[\Theta H] = F_{support}[1] \times (\Theta H)$

$\sigma_{rail}[\Theta H] = \sigma_{rail}[1] \times (\Theta H)$

If a rail expansion device is located at the movable end of the deck, the rail stress at that position should be assumed zero, while the rail stress at the fixed end and the support reaction should be evaluated with the diagrams of Appendix B - page 42.

2.1.3 - Changes in the basic assumptions

2.1.3.1 - Cross-sectional area of the rail

The basic graphs of the interaction effects due to temperature variation and braking relate to the cross-sectional area of the UIC 60 rail, i.e. 15,372 x 10^{-3} m^2/ track.

The interaction effects for different track types may be calculated using the formulae below, in which A_{60} is the cross-sectional area of a track with UIC 60 rails and A_{rail} is the cross-sectional area of the track to be considered (two rails) [m^2].

For temperature variation:

$$F_{support}\,[A_{rail}] = F_{support}\,[A_{60}] \times \{1 - 0,24\,(1 - A_{rail}/A_{60})\}$$

$$\sigma_{rail}\,[A_{rail}] = \sigma_{rail}\,[A_{60}] \times \{1 + 1,05\,(1 - A_{rail}/A_{60})\}$$

For braking:

$$F_{support}\,[A_{rail}] = F_{support}\,[A_{60}] \times \{1 + 0,36\,(1 - A_{rail}/A_{60})\}$$

$$\sigma_{rail}\,[A_{rail}] = \sigma_{rail}\,[A_{60}] \times \{1 + 0,9\,(1 - A_{rail}/A_{60})\}$$

For vertical bending:

$$F_{support}\,[A_{rail}] = F_{support}\,[A_{60}] \times (A_{rail}/A_{60})^{0,1}$$

$$\sigma_{rail}\,[A_{rail}] = \sigma_{rail}\,[A_{60}] \times (A_{60}/A_{rail})^{0,9}$$

2.1.3.2 - Braking force

The basic graphs of the interaction effects due to braking are based on a braking force of 20 kN/m, and give the absolute values of the rail stresses and support reactions.

The interaction effects for an SW/2 braking force or an LM 71 acceleration force may be calculated by using the formulae below, where Q_L is the total horizontal load on the deck, due to braking and acceleration, expressed in kN/m.

Fixed end:	$F_{support} = F_{support}$ [20kN/m] x QL / (20 x L)
	$\sigma_{rail} = \sigma_{rail}$ [20 kN/m] x QL / (20 x L) + 10 N/mm^2
Movable end:	$\sigma_{rail} = \sigma_{rail}$ [20 kN/m] x QL / (20 x L) + 5 N/mm^2
Fixed end and movable end:	$\delta_{rail} = 1{,}3 \times \delta_{rail}$ [20 kN/m] x QL/ (20 x L)

The signs of the rail stresses and reactions depend on the sign of the braking/acceleration forces. Normally both signs shall be considered in order to find the most unfavourable combinations with the other effects, according to point 2.1.4 - page 27.

2.1.3.3 - Temperature variation for CWR

The basic graphs of interaction effects due to temperature variation are based on a deck temperature variation of +35°C and an α-value = 1,0 x 10^{-5}. The interaction effects for other positive values of temperature variation and other α-values can be calculated using the formulae below, where ΔT is the temperature variation to be considered.

$$F_{support} = F_{support}[1,0 \cdot 10^{-5}, 35°C] \times \left[\frac{\alpha \times \Delta T}{1,0 \cdot 10^{-5} \times 35°C}\right]^{1/2}$$

$$\sigma_{rail} = \sigma_{rail}[1{,}0 \cdot 10^{-5}, 35°C] \times \left\{\frac{\alpha \times (\Delta T + 10)}{1,0 \times 10^{-5} \times 45°}\right\}^{1/2}$$

The effects of a negative temperature variation may be calculated by multiplying the results obtained for the positive temperature variation by -1.

In the case of CWR track, it is not necessary to consider the temperature variation in the rails. In fact the temperature variation in the rails has no influence on $F_{support}$ and the verification of the value of σ_{rail} is relevant only to the additional stress in the rail due to the presence of the deck.

2.1.3.4 - Temperature variation in structures carrying track with an expansion device

The basic graphs for interaction effects due to temperature variation are based on a deck temperature variation of +30°C and a rail temperature variation of +50°C and an α-value of 1,0 x 10^{-5}. The interaction effects for other positive values of temperature variation may be calculated using the formulae below. In these formulae, the temperature difference between the deck and the rail has a constant value of 20°C.

$$F_{support}[\Delta T_{deck}, \Delta\Delta T = 20°,\ 1,0 \times 10^{-5}][kN] =$$

$$\frac{\alpha}{1,0 \times 10^{-5}}[F_{support}[\alpha T_{deck} = 30°,\ \Delta\Delta T = 20°] + 16 \times [\Delta T_{deck} - 30]]$$

The value of the additional rail stress due to the temperature variation may be taken as 0.

The effects of negative temperature variations may be calculated by multiplying the results obtained from the positive temperature variations by -1.

Normally both signs should be considered for temperature variations in order to find the most unfavourable combinations with the other effects, according to point 2.1.4.

2.1.3.5 - Friction in the movable supports

The effects of friction cannot be examined separately for each load case (temperature, braking, vertical bending). Rather, the effect of friction on the overall effects calculated in point 2.1.4 should be considered.

- The effects of friction on rail stresses and displacements are always favourable, especially when support stiffness is low, so that ignoring friction is in general conservative for safety.

- Friction produces a reaction force in movable supports and increases that of fixed supports under the influence of temperature variations and vertical bending.

- The longitudinal reaction of the movable support $F_{movable}$ is given by:

  $$F_{movable}(f) = F_{vertical}$$

  where f is the coefficient of friction and $F_{vertical}$ is the vertical support reaction.

- The longitudinal reaction of the fixed support $F_{support}(K, f)$ is given by:

  $$F_{support}(K, f) = \left[\frac{1 + \Sigma F_{movable}(f)}{F_{support}(\infty, 0)}\right] \times F_{support}(K, 0)$$

Use of this formula requires two values to have been calculated beforehand: the fixed support reaction for infinite stiffness and zero friction $F_{support}(\infty, 0)$ and the fixed support reaction for the actual stiffness K and zero friction.

$\Sigma F_{movable}(f)$ is the algebraic sum of the reactions of the movable supports, assuming opposite signs for movable bearings on opposite sides of the fixed support.

2.1.4 - Signs of the effects of load cases when combining the load-case effects

The values of the support reactions and rail stresses due to temperature variations and braking/ acceleration forces obtained from the graphs should be taken with both the positive and negative signs to find the most unfavourable condition, while the signs of the same quantities for the effects of vertical bending are given directly by the graphs.

When the value of the stress due to rotation has an opposite sign (positive or negative) to the values of the stresses due to temperature variation and braking, the value of the stress due to vertical bending should be taken equal to 0.

2.2 - Double-track bridges

2.2.1 - Temperature and braking effects

All the basic interaction effect graphs were plotted for single-track bridges.

In the case of a double-track bridge, braking forces on one track and acceleration forces on the other should be considered.

The interaction effects for a double-track bridge may be calculated using the graphs for single-track bridges [K/2] and the formulae below, where Q_L (in kN) is the sum of the braking and acceleration forces acting on the two tracks and K (in kN/m) is the support stiffness for double track.

Temperature effects:

$$(F_{\text{support}}[K])_{\text{double track}} = 2\left(F_{\text{support}}\left[\frac{K}{2}\right]\right)_{\text{single track}}$$

$$(\sigma_{\text{rail}}[K])_{\text{double track}} = \left(\sigma_{\text{rail}}\left[\frac{K}{2}\right]\right)_{\text{single track}}$$

Braking effects:

$$(F_{\text{support}}\ [K])_{\text{double track}} = \frac{Q_L}{20 \times L} \times \left(F_{\text{support}}\left[\frac{K}{2}\right]\right)_{\text{single track}}$$

Fixed end:

$$(\sigma_{\text{rail}}[K])_{\text{double track}} = \frac{Q_L}{2 \times 20 \times L} \times \left(\sigma_{\text{rail}}\left[\frac{K}{2}\right]\right)_{\text{single track}} + 10\ \text{N/mm}^2$$

Movable end:

$$(\sigma_{\text{rail}}[K])_{\text{double track}} = \frac{Q_L}{2 \times 20 \times L} \times \left(\sigma_{\text{rail}}\left[\frac{K}{2}\right]\right)_{\text{single track}} + 5\ \text{N/mm}^2$$

$$(\delta_{\text{deck}}[K])_{\text{double track}} = \left(\frac{F_{\text{support}}[K]}{K}\right)_{\text{double track}}$$

2.2.2 - Vertical bending effects

In the case of double-track bridges, the stiffness value of the fixed support should be halved before being used in the diagrams. The maximum displacement $\delta(\Theta H)$ due to end rotation should be calculated on the basis of the actual deformed shape of the deck under the loads acting on both tracks at the same time. If important three-dimensional effects can occur due to the asymmetry of loads, giving rise to important difference between the values of $\delta(\Theta H)$ for each of the two tracks, different values of $\delta(\Theta H)$ should be considered for each. The reaction of the fixed support should be calculated by adding the contributions of the two tracks obtained separately.

2.3 - Multiple-span continuous deck

2.3.1 - Multiple-span continuous deck with fixed support at one end

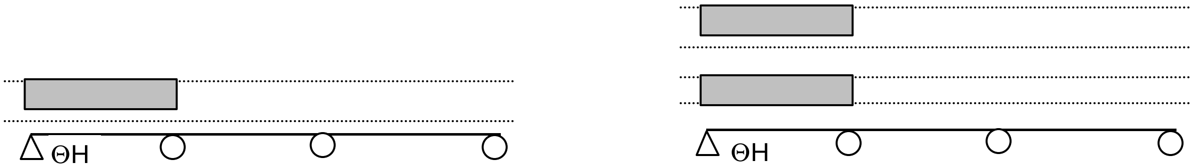
2.3.1.1 - Temperature and braking effects

For the temperature-variation and braking-load cases a multiple-span continuous deck with fixed support at one end may be treated as a simply-supported deck with fixed support at one end. The effect of the friction in the movable bearings may be taken into account by applying the recommendations of point 1.3.2 - page 9.

2.3.1.2 - Vertical bending effects

The end rotation displacement δ(ΘH) should be calculated at the fixed end on the basis of the actual deformed shape of the deck under the train loads. The load condition to be considered is the one in which conservatively only the first span (near the fixed support) is loaded, as shown in Fig. 12. In the case of double-track bridges, both tracks on the first span should be loaded, while the stiffness value of the fixed support should be halved before being used in the diagrams for single-track bridges.

If important three-dimensional effects are liable to occur due to the asymmetry of loads, giving rise to important differences between the values of δ(ΘH) corresponding to the two tracks, different values of δ(ΘH) should be considered for each. The reaction of the fixed support should be calculated by adding the contributions of the two tracks obtained separately.



Single-track bridge

Double-track bridge

Fig. 12 - Load conditions to evaluate interaction effects due to vertical loads for continuous bridges with fixed end support

2.3.2 - Multiple-span continuous decks with a fixed support located at an intermediate position

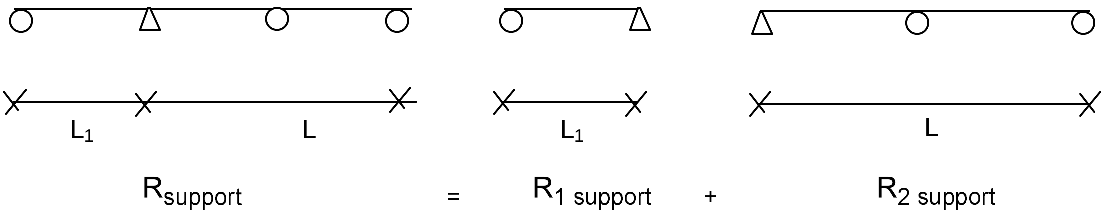
2.3.2.1 - Temperature and braking effects

Braking load-case:

For the braking load-case, a multiple-span continuous deck with a fixed support located at an intermediate position may be treated as a simply-supported deck with fixed support at one end.

Temperature-variation load-case:

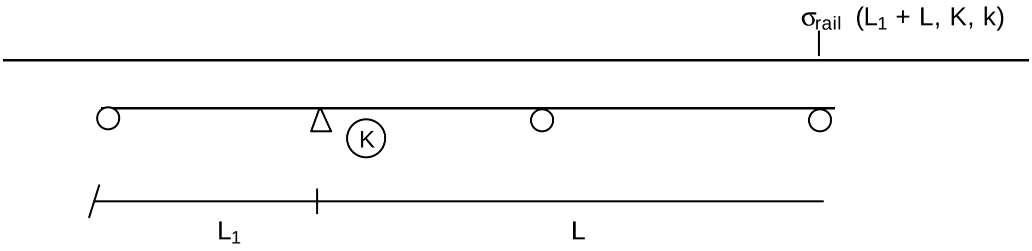
The support reaction at the fixed bearing located at an intermediate position of the deck is obtained as the algebraic sum of the support reactions determined for the simple cases obtained by dividing the static arrangement with the fixed support located at an intermediate position into two static arrangements where the fixed support is located at the end of the notional deck.



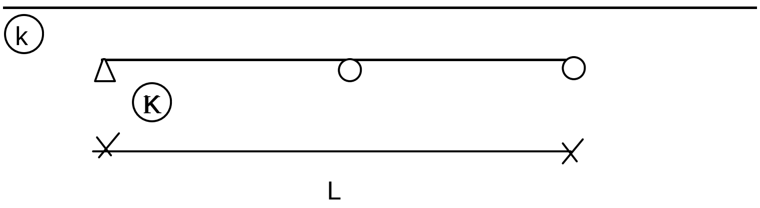
The stress in the rail at the movable support is derived from the value obtained from the curves for simply-supported deck given in Appendix A - page 36 with the help of the following analytical formula:

$$\sigma_{rail}(L_1 + L, K, k) = \sigma_{rail}(L, K, k) + \frac{\sigma_{rail}(L, \infty, k) - \sigma_{rail}(L, K, k)}{L} \times L_1$$

where σ_{rail} $(L_1 + L, K, k)$ is the stress in the rail at the movable support as represented below.



and σ_{rail} (L, K, k) is the stress in the rail obtained with the help of the design curves given in Appendix A - page 36 at the movable support for the static scheme as follows:



The value σ_{rail} (L, ∞, k) is obtained by assuming a very high value of the stiffness of the fixed support.

2.3.2.2 - Vertical bending effects (single- or double-track bridges)

The end rotation displacement $\delta(\Theta H)$ at the fixed support should be calculated on the basis of the actual deformed shape of the deck under the train loads. Generally the load conditions to be considered are those shown in Fig. 13. In the case of double-track bridges, both tracks at the same time should be loaded, while the stiffness value of the fixed support shall be halved before being used in the diagrams. If important three-dimensional effects are liable to occur due to the asymmetry of loads, giving rise to important differences between the values of $\delta(\Theta H)$ corresponding to the two tracks, different values of $\delta(\Theta H)$ should be considered for each. The reaction of the fixed support is calculated by adding the contributions of the two tracks, obtained separately using the charts in Appendix B (see page 42).

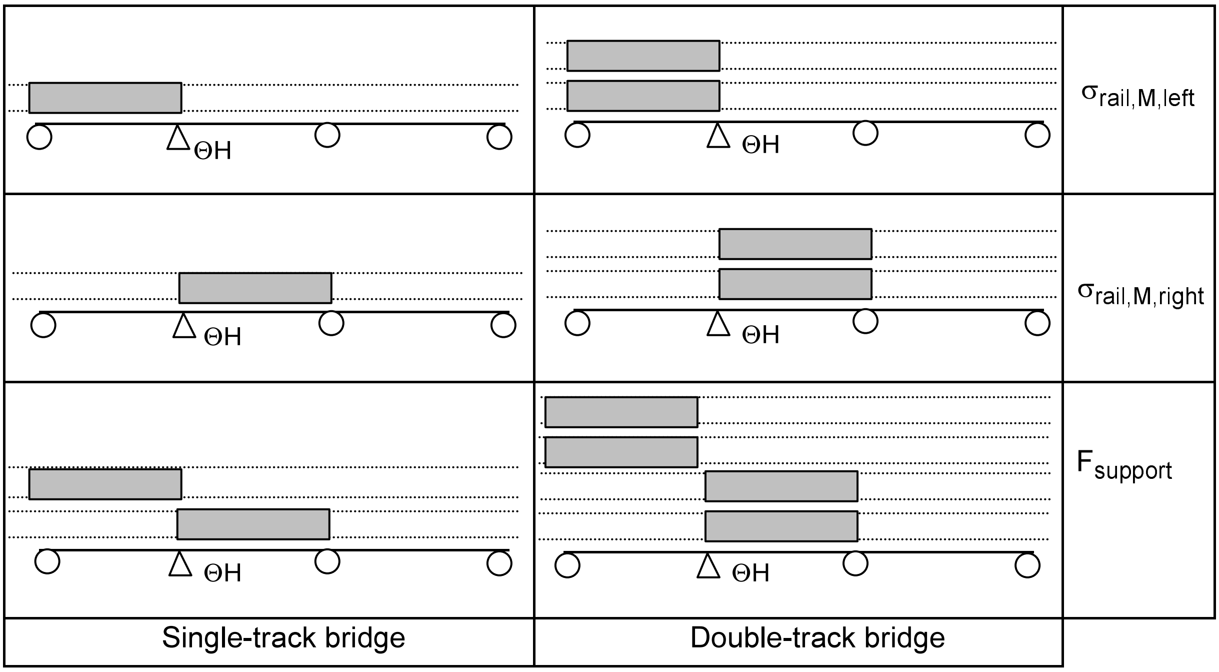


Fig. 13 - Load conditions to evaluate interaction effects due to vertical loads for continuous bridges with intermediate fixed support

2.3.3 - Multiple-span continuous deck with no fixed support

For the calculation of the temperature variation effects it is necessary to determine the fictitious fixed point or gravity centre of the stiffnesses.

The same is applicable for the case with more than one fixed bearing and for the case of simply-supported deck without any fixed bearing.

3 - Succession of decks

3.1 - Introduction

For reasons of life cycle costs and comfort, a railway bridge should be designed in such a way that CWR is possible without rail expansion devices.

One way of avoiding rail expansion devices on long bridges is to divide these bridges into separate decks.

In the case of a succession of simple-deck bridges the effects of actions can be reduced by shortening the length of the first and last decks.

When in a design of a particular bridge rail expansion devices cannot be avoided, the designer should contact the relevant railway authority.

3.2 - Verification and criteria

In the case of a succession of decks, the verification should be generally carried out by computer analysis

The validations of the computer program, the criteria to be met and the general rules for computer-assisted interaction analysis are given in points 1.7.1 - page 17, 1.7.2 - page 18 and 1.7.3 - page 19.

For simple configurations, i.e. characterised by regular conditions (regarding stiffness of piers, span lengths, static arrangement, etc.), simplified rules may be used to evaluate interaction effects according to point 3.3 - page 32.

The situations where simplified rules can be applied are selected so that rail stresses and displacements are well within the acceptable values, thus assuring good safety and maintenance conditions for track. Verifications of rail stresses and displacements are thus avoided and simplified evaluations can then be carried out only for support reactions.

3.3 - Simplified rules

3.3.1 - Deck-configuration conditions

The simplified rules may be applied if the following conditions of the deck-configuration are satisfied, in addition to the conditions already stated for simple deck bridges in points 2.1.2 - page 23 and 2.1.2.1 - page 24:

- The track on the bridge and at least on 100 m of the embankment at both sides of the bridge consists of CWR without a rail expansion device;

- All the decks have the same static arrangement (fixed support at the same position);

- The number of decks is more than 4, even though the simplified rules stated here can still be applied for bridges with 4 decks or less, as they can give an overconservative estimation of support reactions;

- In the case of 35°C maximum temperature variation of decks, their length is less than 30 m;

- In the case of 20°C maximum temperature variation of decks and no possibility of frozen ballast, their length is less than 60 m, (in case of a maximum temperature variation of decks between 35°C and 20°C and no possibility of frozen ballast the value of the maximum permissible length of the decks can be interpolated between 30 m and 60 m);

- The length of each deck does not differ by more than 20% with respect to the average value of the deck lengths;

- In the case of 35°C maximum temperature variation of decks and 30m maximum length of the decks, the stiffness of the fixed supports is greater than 2L[m] [kN/mm/track] multiplied by the number of tracks;

- In the case of 20°C maximum temperature variation of decks and 60m maximum length of the decks, the stiffness of the fixed supports is greater than 3L[m] [kN/mm/track] multiplied by the number of tracks;

- For deck lengths between 30 and 60 m, interpolation is permissible;

- The stiffness of each fixed support (except for a fixed support at the abutment) does not differ by more than 40% with respect to the average value of the support stiffnesses;

- The stiffness of the fixed support on the abutment should not be less than the average value of the support stiffnesses;

- The maximum displacement of the deck-end with reference to the adjacent abutment, due to vertical bending at the top of the slab supporting the track, evaluated without taking into account the interaction effect, is less than 10 mm;

- The sum of the absolute displacements of two consecutive deck-ends, due to vertical bending at the top of the slab supporting the track, evaluated without taking into account the interaction effect, is less than 15 mm.

3.3.2 - Verification with the simplified rules

When the conditions of point 3.3.1 - page 32 are satisfied, no verification of rail stresses and displacements due to braking/acceleration is needed.

3.3.3 - Calculation of the support reactions

When the conditions of point 3.3.1 - page 32 are satisfied, the support reactions due to temperature variations, braking/acceleration and vertical bending may be calculated using the simplified rules in the following table (the piers mentioned are those with a fixed support).

Support	**Temperature variation**	**Braking/traction**	**Vertical bending**
	$F_{supp}(\Delta T)$	F_{supp}(Braking)	$F_{supp}(\theta H)$
Abutment with first fixed support	$F_{supp\ (abutment)}(\Delta T)$	νQ_L	$F_{supp\ (abutment)}(\theta H)$
First pier	$0{,}2 \times F_{supp\ (abutment)}(\Delta T)$	Q_L	0
Intermediate piers	0	Q_L	0
(n-1)th pierr	$0{,}1 \times F_{supp\ (abutment)}(\Delta T)$	Q_L	0
(n)th pier	$0{,}5 \times F_{supp\ (abutment)}(\Delta T)$	Q_L	$0{,}5 \times F_{supp\ (abutment)}(\theta H)$

Assuming that the first deck is the one with the fixed support on the abutment, the following notes apply:

- Support reaction due to temperature variation $F_{support}(\Delta T)$:
  $F_{support(abutment)}(\Delta T)$ should be calculated using the graphs of Appendix A - page 36, where L is the length of the first deck.

- Support reaction due to braking/acceleration $F_{support}$(Braking):
  Q = 20 [kN/m/track]; for double track, the reaction is equal to double the single-track case (the case of two LM71 braking trains is assumed for simplicity for double-track bridges, with negligible errors).

  L is the length of the deck connected to the fixed support

  $\nu = 1$ when the stiffness of the abutment is the same as that of the piers.

  $\nu = 1{,}5$ when the stiffness of the abutment is five times greater than that of the piers.

  ν may be interpolated for intermediate stiffnesses.

- Support reaction due to vertical bending $F_{support}(\theta H)$:
  $F_{support(abutment)}(\theta H)$ should be calculated with the graphs in Appendix B - page 42.

The support reactions at the intermediate piers due to vertical bending are assumed to be zero, although their maximum values are not. This is because the load conditions which produce the maximum value of the support reactions due to braking give rise to negligible values for the support reactions due to vertical bending.

3.4 - Considerations for computer analysis

The interaction effects due to braking/acceleration forces and vertical bending may be calculated separately. In such a case the structural models are different for the two types of analyses, as infinite deck stiffness is to be assumed when braking/acceleration effects are analysed, while the real stiffness shall be assumed when vertical bending effects are analysed. For both load cases, as many analyses are required as there are different positions of the train(s) on the bridge to be considered. Normally only the conditions for which the train(s) completely occupies(y) either one, two, up to all the deck spans, lead to the maximum effects for rail stresses, displacements and support reactions. In case of braking, these load conditions correspond to the train stopping with its head on each support. However when combining the effects of braking/acceleration and vertical bending, consistent load conditions should be considered. In fact combining the maximum (envelope) values of the two effects can lead to considerable overestimation of the total effect.

When evaluating the displacement at the end(s) of the deck, the effects of braking/acceleration and of vertical bending should be evaluated separately, even when a complete analysis is carried out, since the verifications must be carried out separately for the two effects.

Appendix A - Diagrams of interaction effects due to braking and temperature variations for simply-supported, single-track decks with a fixed support at one end

A.1 - Continuous welded rails

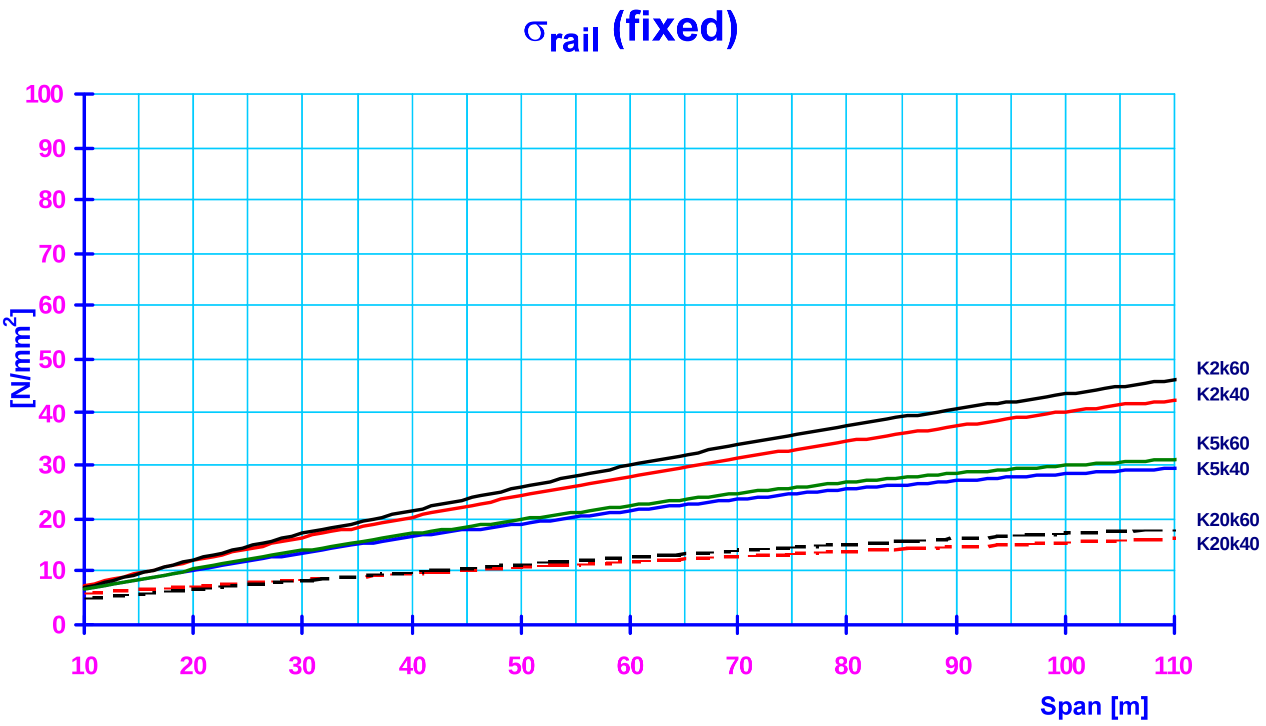


Fig. 1 - Braking load-case (fixed support) (20kN/m')

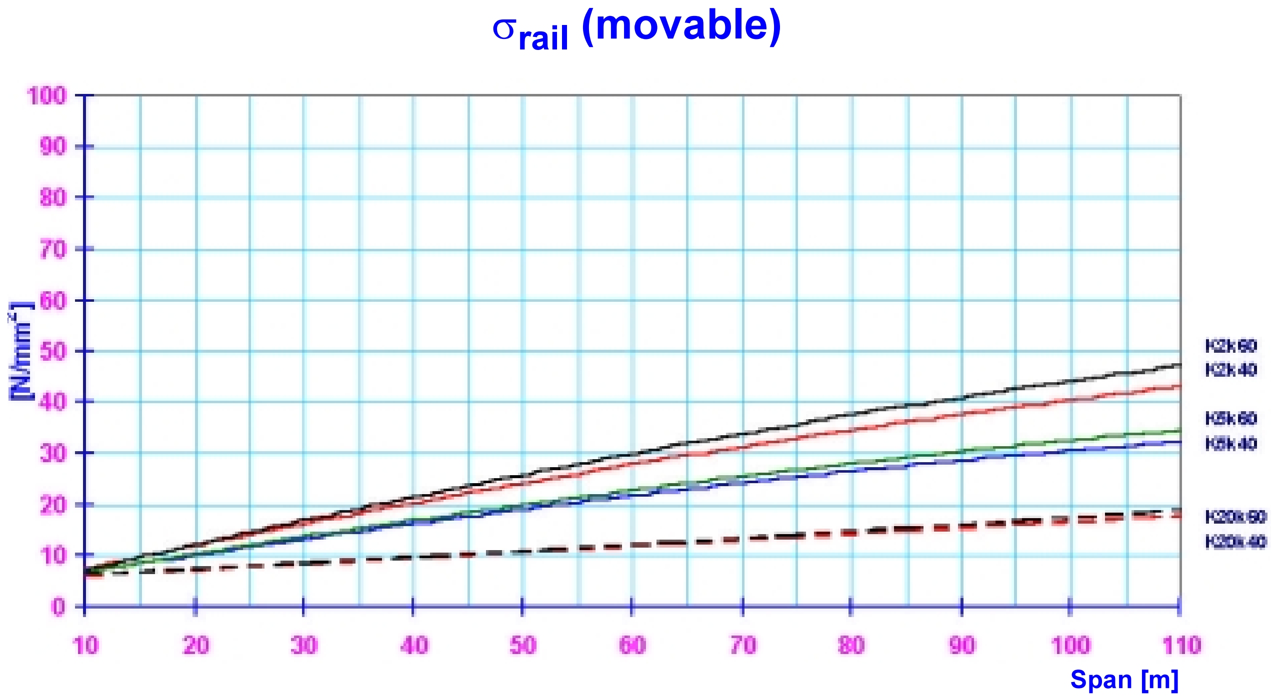


Fig. 2 - Braking load-case (movable support) (20kN/m')

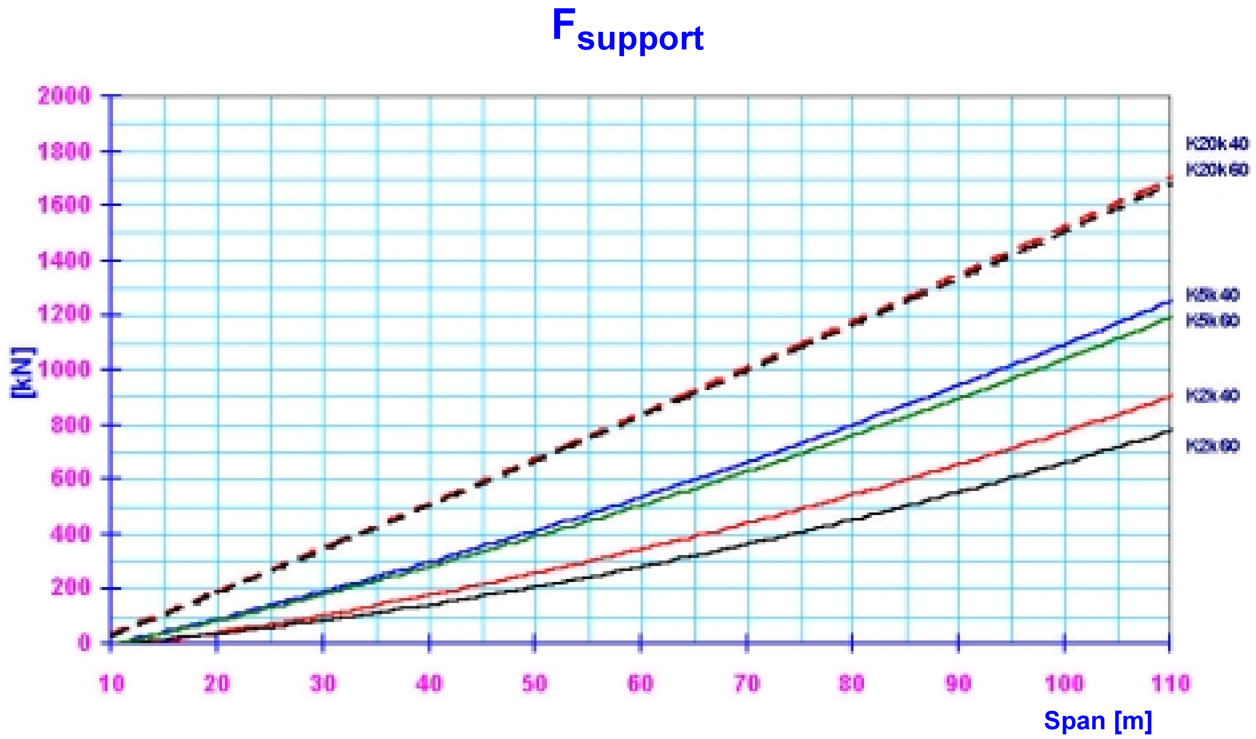


Fig. 3 - Braking load-case (20kN/m') (support reaction)

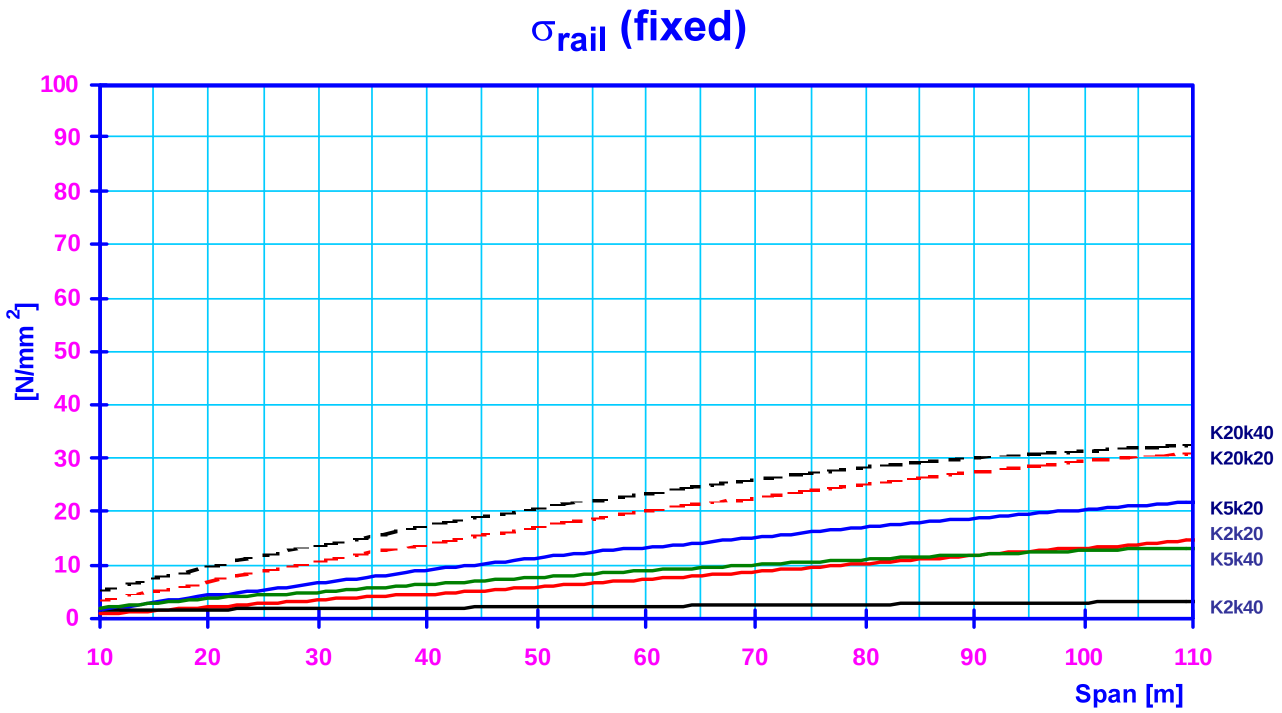


Fig. 4 - Temperature-variation load-case (ΔT_{deck} = 35°C) (fixed support)

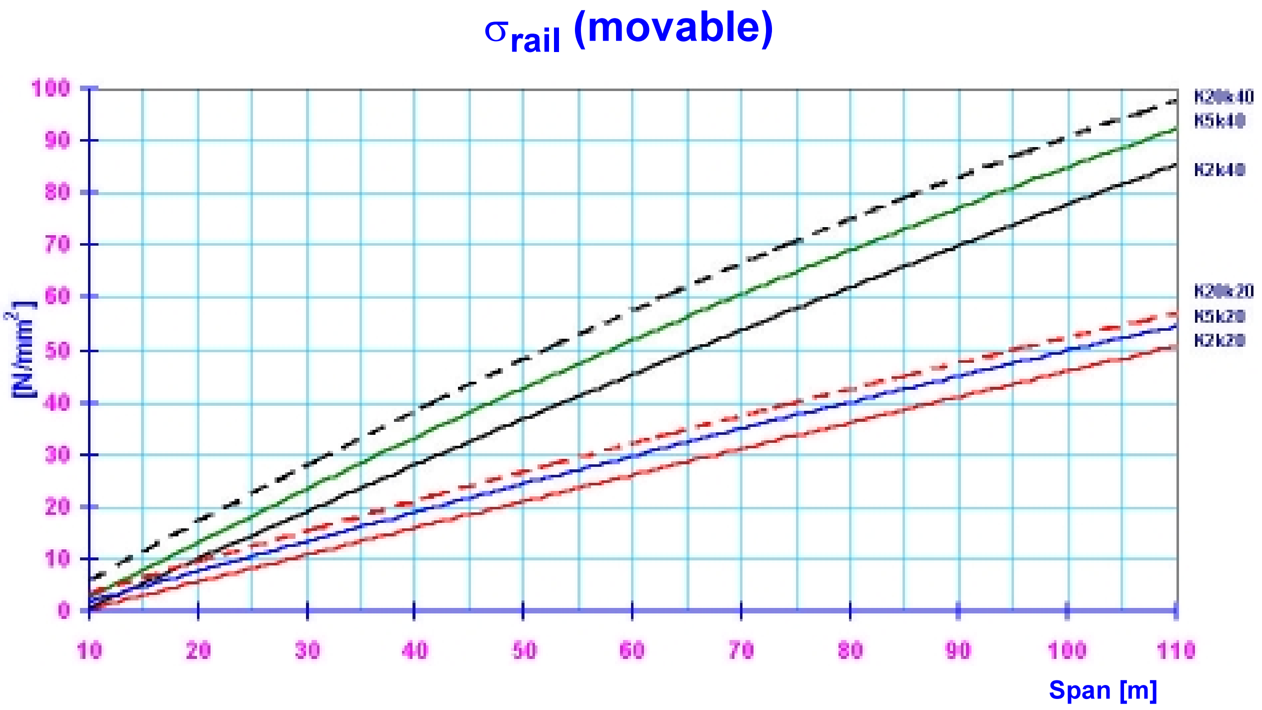


Fig. 5 - Temperature-variation load-case (ΔT_{deck} = 35°C) (movable support)

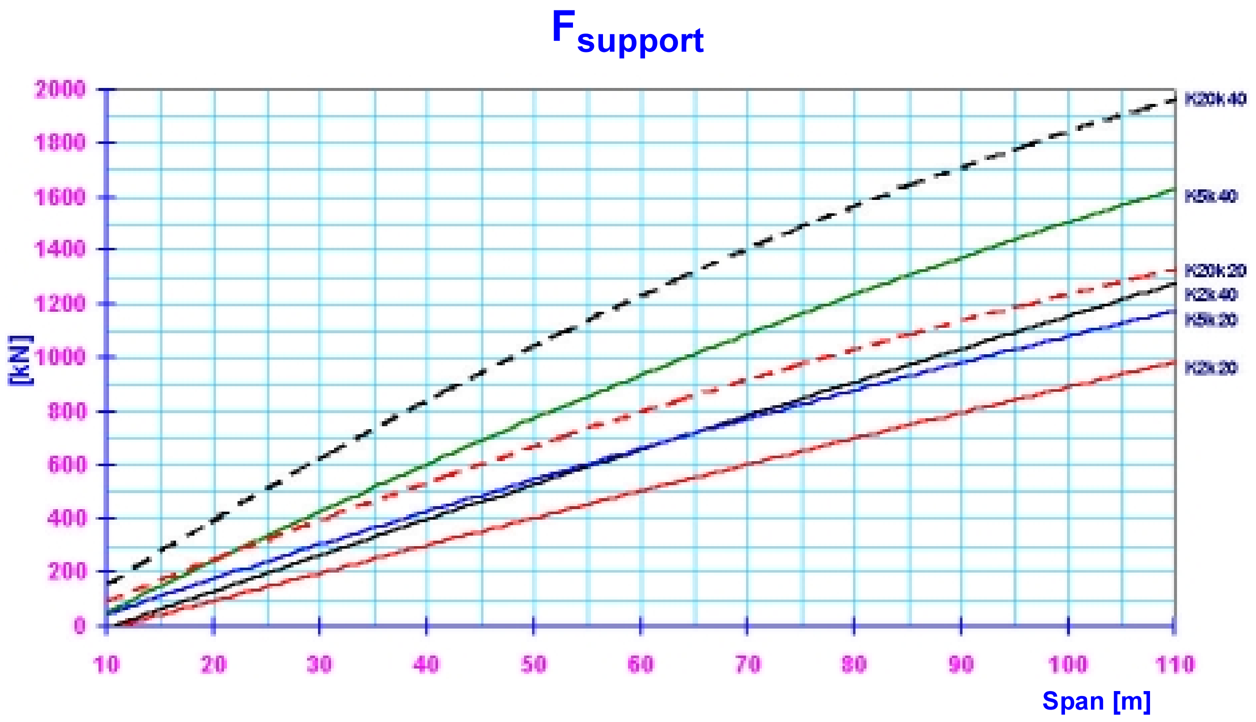


Fig. 6 - Temperature-variation load-case (ΔT$_{deck}$ *= 35°C) (support reaction)*

A.2 - With an expansion device

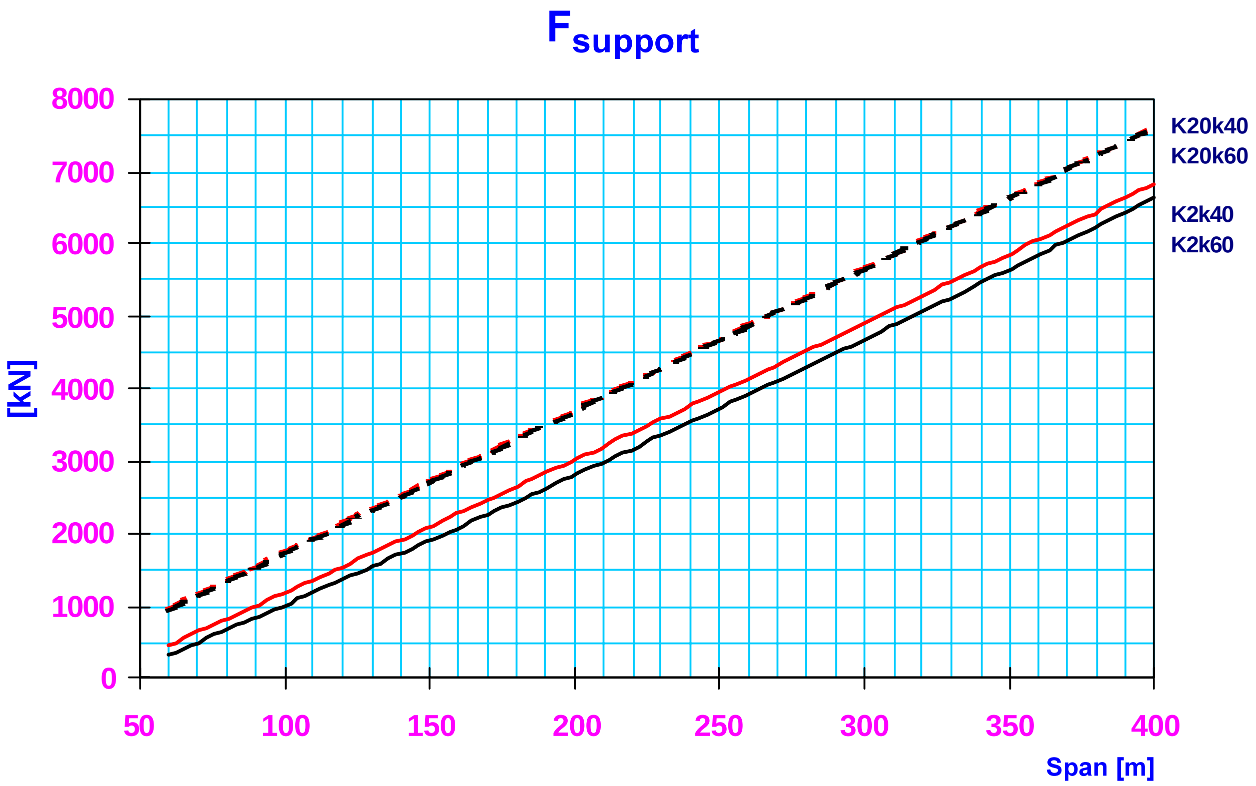


Fig. 7 - Braking load-case (20kN/m') (reaction of the fixed support)

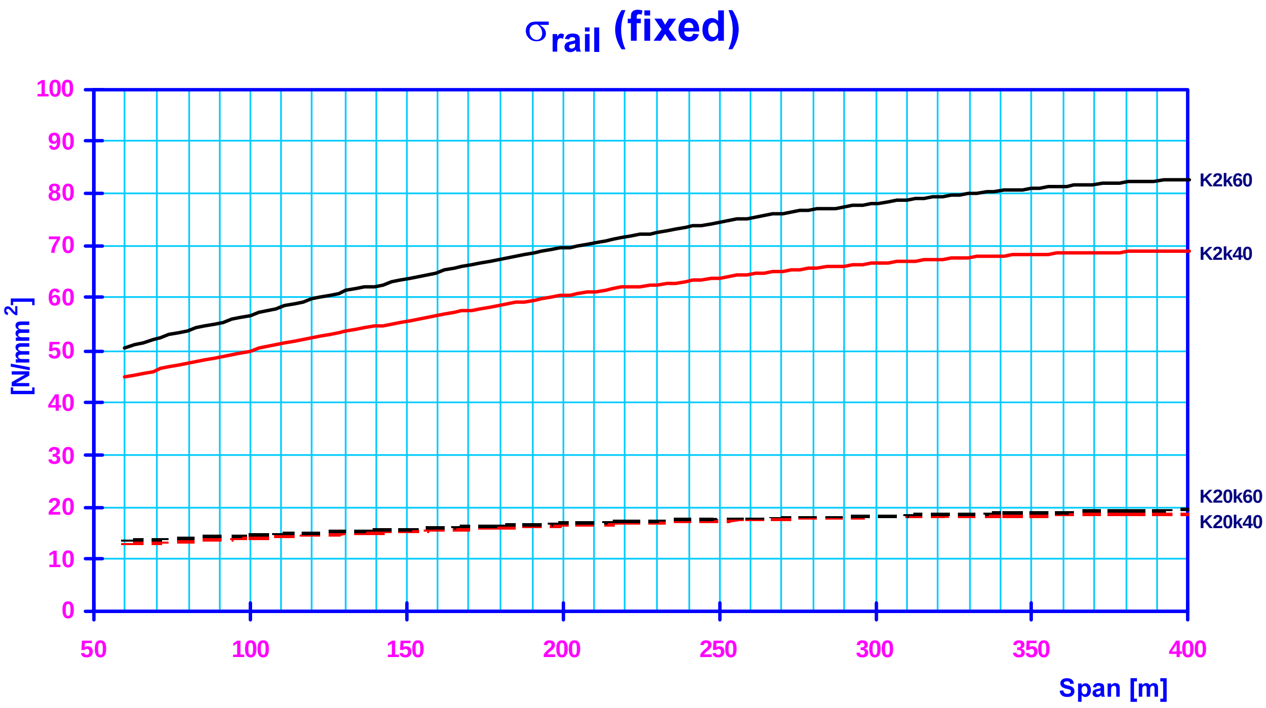


Fig. 8 - Braking load-case (20kN/m') (fixed support)

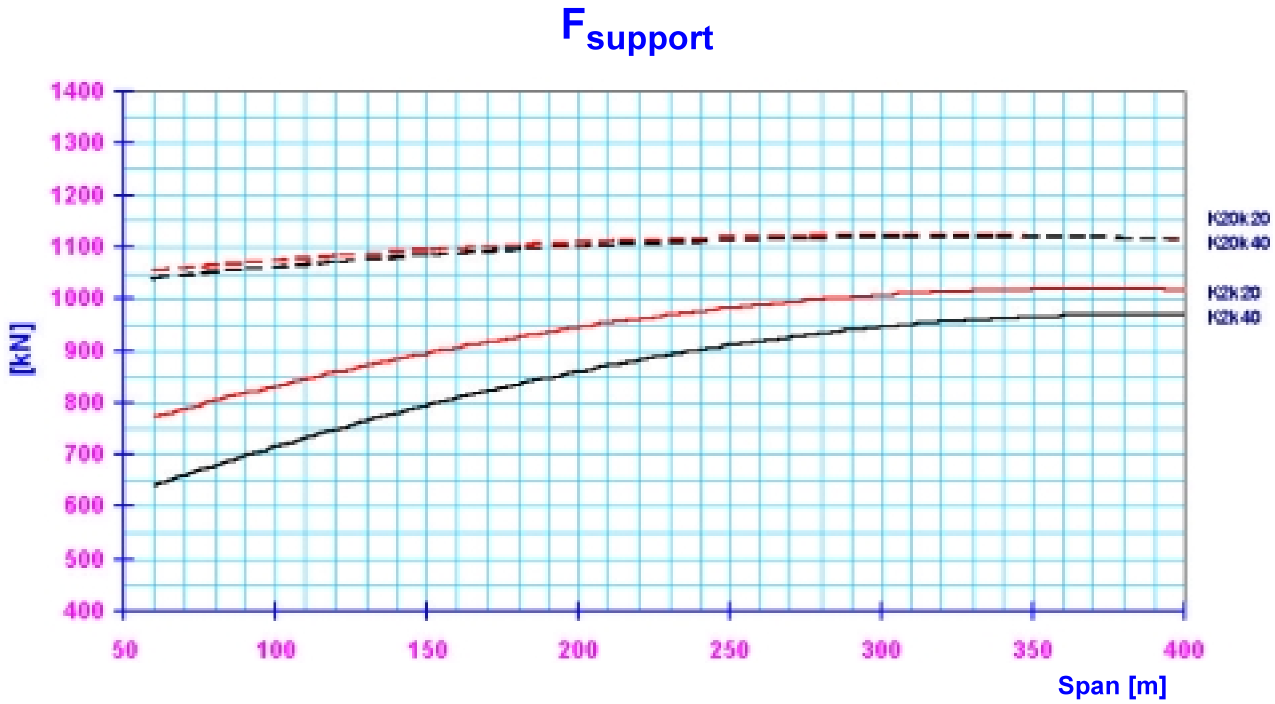
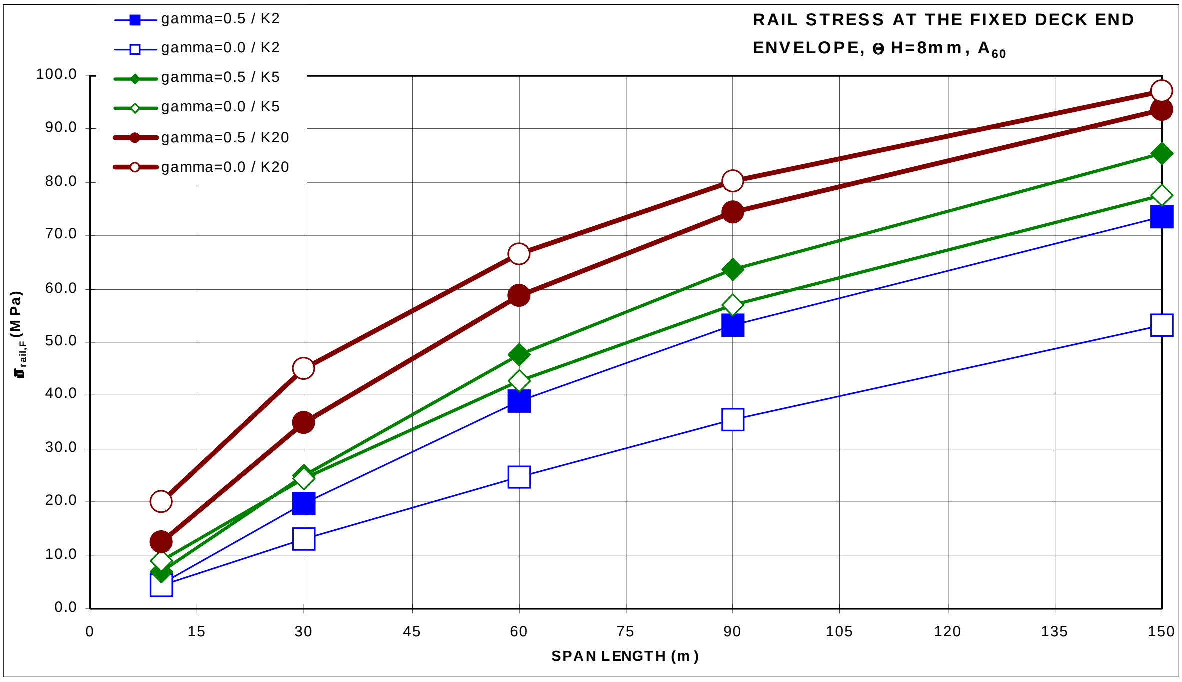


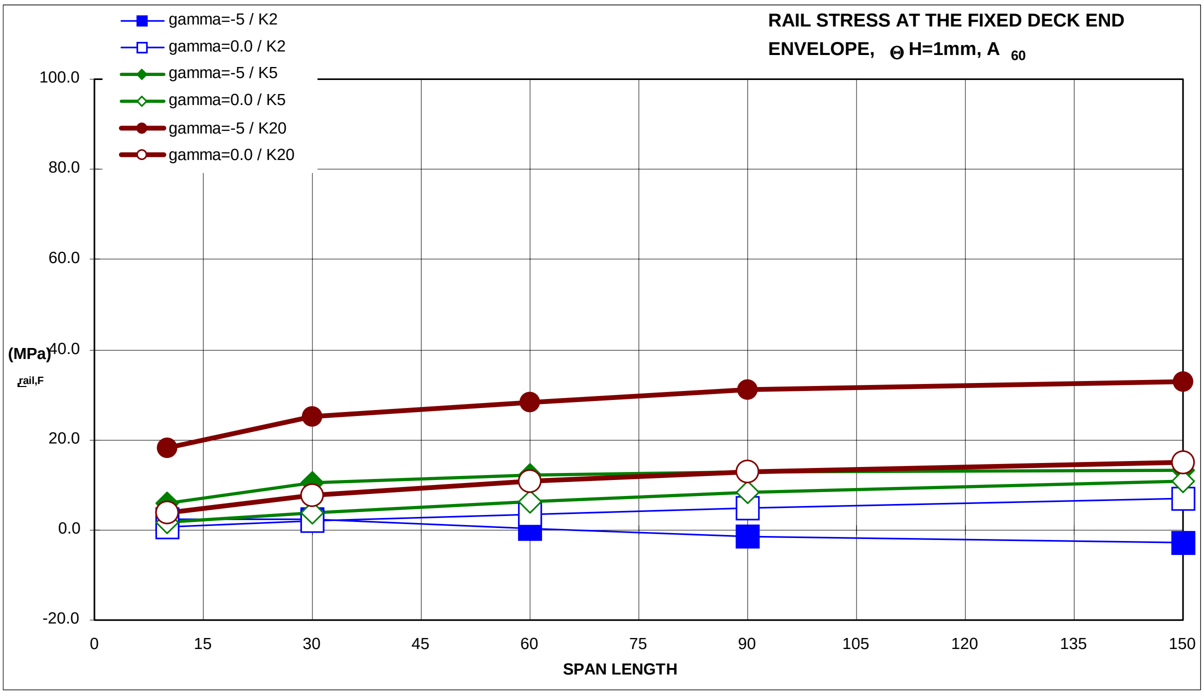
Fig. 9 - Temperature-variation load-case (ΔT_{deck} = 35°C; ΔT_{rail} = 50°C)

Appendix B - Diagrams of interaction effects under vertical bending

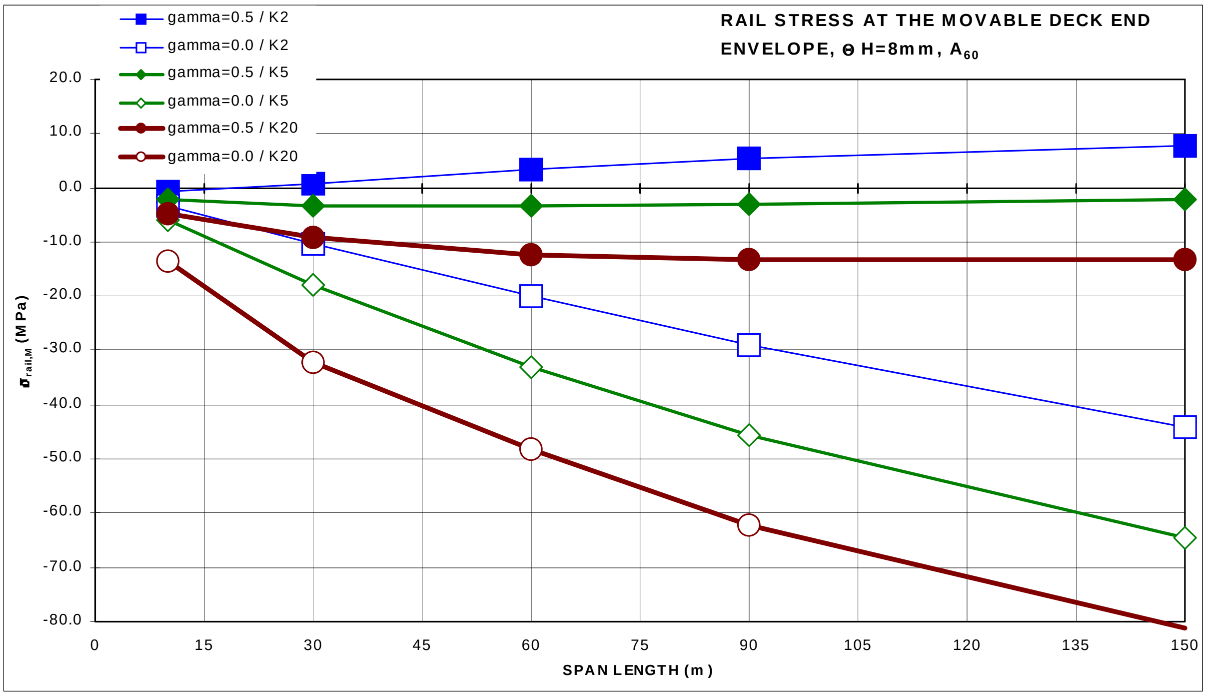
B.1 - Deck bridge - σ_{rail} (fixed)



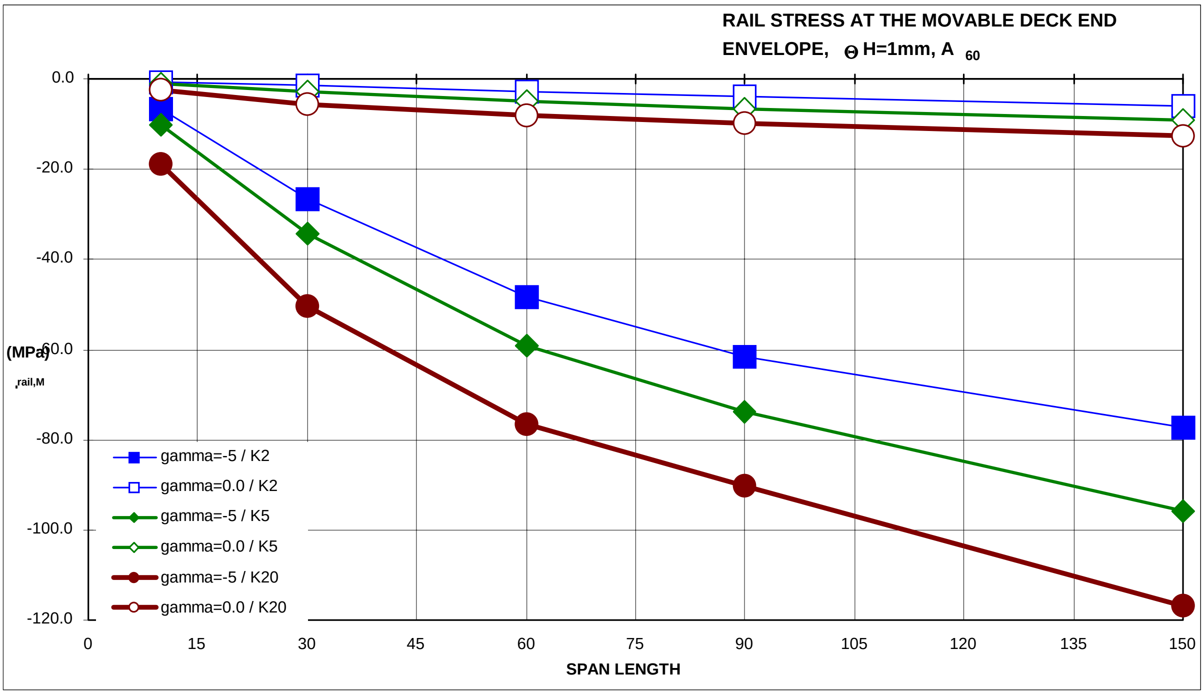
B.2 - Through girder bridge - σ_{rail} (fixed)



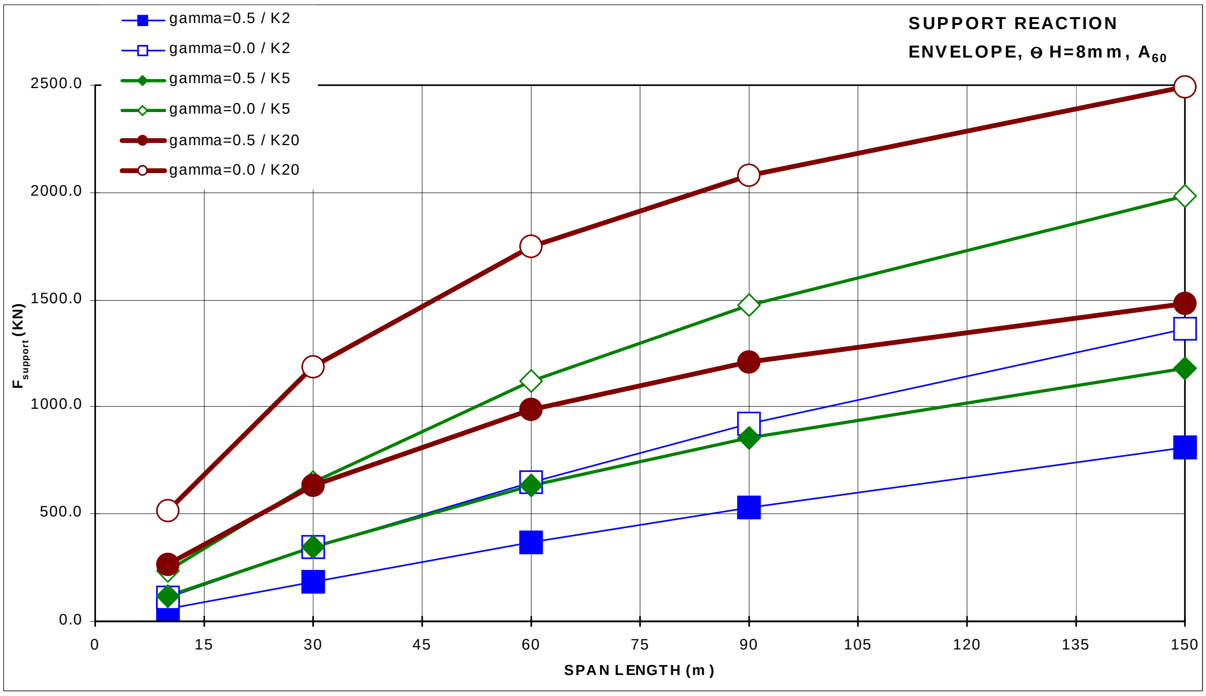
B.3 - Deck bridge - σ_{rail} (movable)



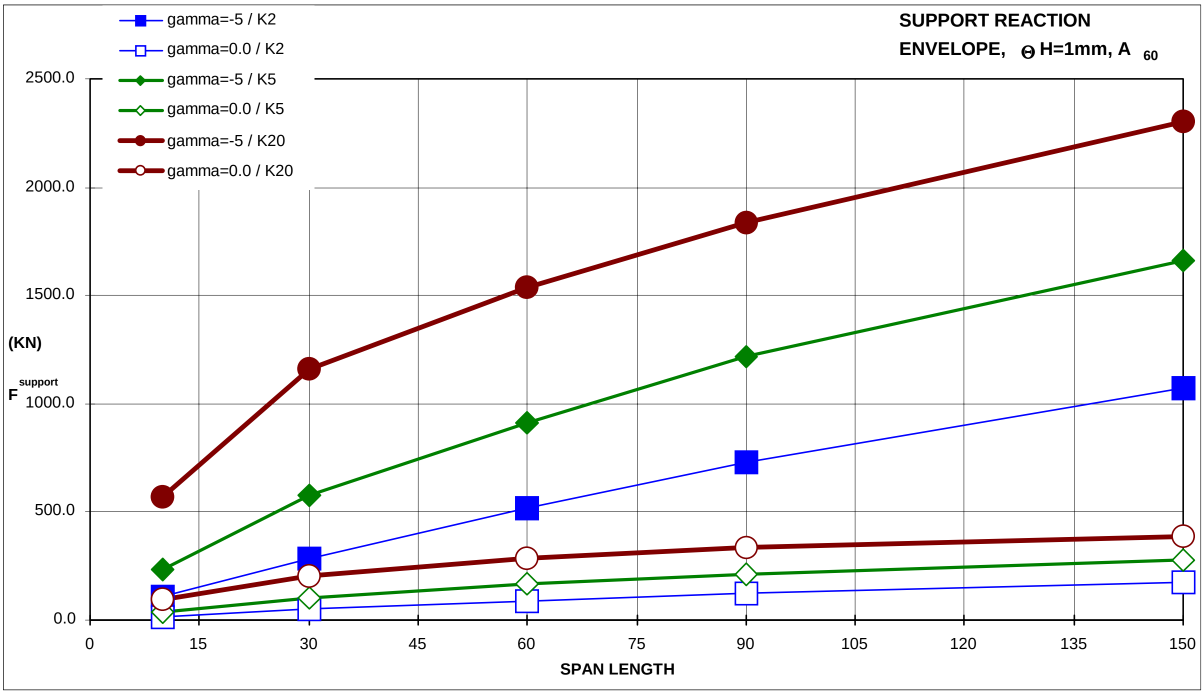
B.4 - Through girder bridge - σ_{rail} (movable)



B.5 - Deck bridge - $F_{support}$



B.6 - Through girder bridge - $F_{support}$



Appendix C - Examples

C.1 - Example 1: Simply-supported deck with one elastic fixed support (deck slab bridge) / no expansion device (use of the predimensioning method/case of a rail stress higher than permissible)

Input data:

Simply-supported deck	
one track	
deck length	L = 75 m
fixed support at one end	
no friction in the movable bearing	
deck bridge with	E = 21 E7 kN/m^2; I = 2,59 m^4; A = 0,74 m^2; H = 6,0 m; ω = 1,21 m; $\gamma = \omega/H$ = 0,20; θH [LM 71] = 7,9 mm
UIC 60 continuous welded rail	

resistance of the unloaded track: a displacement of 2 mm between the elastic zone and the plastic zone and a resistance of 20 kN/m of the plastic phase $\Rightarrow$ k (unloaded) = 20

resistance of the loaded track: a displacement of 2 mm between the elastic zone and the plastic zone and a resistance of 40 kN/m of the plastic phase $\Rightarrow$ k (loaded) = 40

coefficient of thermal expansion	$\alpha = 1{,}0 \times 10^{-5}$
temperature variation of the rails	$\Delta T_{rails} = \pm 50°C$
temperature variation of the deck	$\Delta T_{deck} = \pm 35°C$
load due to braking and acceleration	20 kN/m

The stiffness of the fixed support K is determined in a first step with point 1.6.1 - page 15:

The braking force transferred from the rails to the bearing is:	75 x 20 x 0,55 = 825 kN
and the minimum required stiffness of the fixed bearing is:	825 kN/5 mm = 165 kN/mm $\neq$ 2,2 L

Verification of σ_{rail}:

σ_{rail} due to the variation of the temperature in the rail is not an additional stress for the verification

Diagrams in figures 1, 2, 4 and 5 from point A.1 (Appendix A - page 36) for the interaction due to the temperature variation and due to braking; without an expansion device

temperature with Δ T_{deck} + 35°C, K = 2,2. L (between K_2 and K_5), k = 20 kN/2 mm = k_{20} and L = 75 m

braking with k = 2,2. L (between K_2 and K_5), k = 40 kN/2 mm = k_{40} and L = 75 m

σ_{rail} (fixed) [Δ T] = 11 N/mm^2

σ_{rail} (movable) [Δ T] = 34 N/mm^2

σ_{rail} (fixed) [braking] = 32,5 N/mm^2

σ_{rail} (movable) [braking] = 32,5 N/mm^2

Diagrams for the interaction due to the vertical bending; deck bridge and θH = 8 mm; L = 75 m; K = 2,2. L; γ = 0,20 (Interpolation between γ = 0,5/K2 and γ = 0,0/K2, then between K2 and K5)

σ_{rail} (fixed) [8 mm] = 36,5 N/mm^2 (tension)

σ_{rail} (movable) [8 mm] = - 13 N/mm^2 (compression)

So representative value: additional compressive rail stress of 13 N/mm^2

σ_{rail} (fixed) [θH] = 0

σ_{rail} (movable) [θH] = 13 x $\{7,9/8\}^{0,86}$ = 13 N/mm^2

Verification:

1,0 x σ_{rail} (fixed) [ΔT] + 1,0 x σ_{rail} (fixed) [braking] + 1,0 x σ_{rail} (fixed) [θH] =
(11 + 32,5 + 0) = 43,5 N/mm^2 < 72 N/mm^2

1,0 x σ_{rail} (mov) [ΔT] + 1,0 x σ_{rail} (mov) [braking] + 1,0 x σ_{rail} (mov) [Hθ] =
(34 + 32,5 + 13) = 79,5 N/mm^2 > 72 N/mm^2

The stress condition has not been met. A greater stiffness has to be found in order to satisfy this condition. Using the preceding calculations and changing K = 2,2 L to K = 5 L gives 25 N/mm^2 for braking, 37 N/mm^2 for temperature variation, and 20 N/mm^2 for vertical bending: (37 + 25 + 20) = 82 N/mm^2 > 72 N/mm^2. Due to the increased stress at the movable support for the vertical-bending case, the total stress is greater than previously.

For this reason it seems preferable to replace the 75 m simply supported structure by a continuous structure or to seek a significant reduction in the value θ.H.

C.2 - Example 1a: Simply-supported deck bridge with one elastic fixed support (deck slab bridge) / no expansion device

Input data:

Simply-supported deck

one track

deck length	$L = 60$ m
fixed support at one end, stiffness	$K = 120$ kN/mm $= 2\ L$ kN/mm
no friction in the movable bearing	
deck bridge with	$E = 21$ E7 kN/m^2; $I = 2{,}59$ m^4; $A = 0{,}74$ m^2; $H = 6{,}0$ m; $\omega = 1{,}21$ m; $\gamma = \omega/H = 0{,}20$; θH [LM 71] = 7,9 mm

UIC 60 continuous welded rail

resistance of the unloaded track: a displacement of 2 mm between the elastic zone and the plastic zone and a resistance of 20 kN/m of the plastic phase $\Rightarrow$ k (unloaded) = 20

resistance of the loaded track: a displacement of 2 mm between the elastic zone and the plastic zone and a resistance of 40 kN/m of the plastic phase $\Rightarrow$ k (loaded) = 40

coefficient of thermal expansion	$\alpha = 1{,}0 \times 10^{-5}$
temperature variation of the rails	$\Delta T_{rails} = \pm 50°C$
temperature variation of the deck	$\Delta T_{deck} = \pm 35°C$
load due to braking	20 kN/m

Verification of σ_{rail}:

σ_{rail} due to the variation of the temperature in the rail is not an additional stress for the verification

Diagrams in figures 1, 2, 4 and 5 of point A.1 (Appendix A - page 36) for the interaction due to the temperature variation and due to braking, without an expansion device

temperature with $\Delta\ T_{deck} + 35°C$, $K = 2\ L = K_2$, $k = 20$ kN/2 mm $= k_{20}$ and $L = 60$ m

braking with $K = 2\ L = K_2$, $k = 40$ kN/2 mm $= k_{40}$ and $L = 60$ m

σ_{rail} (fixed) [Δ T] = 8 N/mm^2

σ_{rail} (movable) [Δ T] = 26 N/mm^2

σ_{rail} (fixed) [braking] = 28 N/mm^2

σ_{rail} (movable) [braking] = 28 N/mm^2

Diagrams in figures of points B.1, B.3 and B.5 (Appendix B - page 42) for the interaction due to the vertical bending; deck bridge and $\theta H = 8$ mm; $L = 60$ m ; $K = 2L$; $\gamma = 0{,}20$ (interpolation between $\gamma = 0{,}5/K2$ and $\gamma = 0{,}0/K2$)

σ_{rail} (fixed) [8 mm] = 30,6 N/mm^2 (tension)

σ_{rail} (movable) [8 mm] = - 10,8 N/mm^2 (compression)

So representative value: additional compressive rail stress of 10,8 N/mm^2

σ_{rail} (fixed) [θH] = 0

σ_{rail} (movable) [θH] = 10,8 x $\{7{,}9/8\}^{0{,}86}$ = 10,8 N/mm^2

For the support reaction:

$F_{support}[\theta H] = F_{support}[8\text{ mm}] \times (\theta H/8)^{0{,}86} = 530 \times (7{,}9/8)^{0{,}86} = 530$ kN

Verification of σ_{rail}:

1,0 x σ_{rail} (fixed) [ΔT] + 1,0 x σ_{rail} (fixed) [braking] + 1,0 x σ_{rail} (fixed) [θH] =
(10,8 + 28 + 0) = 38,8 N/mm^2 $\leq$ 72 N/mm^2

1,0 x σ_{rail} (mov) [ΔT] + 1,0 x σ_{rail} (mov) [braking] + 1,0 x σ_{rail} (mov) [Hθ] =
(26 + 28 + 10,8) = 64,8 N/mm^2 $\leq$ 72 N/mm^2

Verification of δ-relative:

The verification of the relative displacement between the rail and the deck or the platform due to braking and acceleration is assured when the verification of δ-deck (fixed) is satisfied.

Verification of δ_{deck}(fixed):

$\delta_{deck} = F_{supp}[\text{braking}] / K_{supp} = 350 / 120 = 2{,}9$ mm < 5 mm

Verification of $\delta(\theta H)$:

$\delta(\theta H)$< 10 mm (the criteria is satisfied)

Horizontal support reactions:

$F_{support}[\Delta T]$ = 530 kN

$F_{support}[\text{braking}]$ = 350 kN

$F_{support}[\theta H]$ = 500 kN

$F_{support}[\text{total}] = \alpha\, F_{support}[\Delta T] + \beta\, F_{support}[\text{braking}] + \gamma\, F_{support}[\Theta H]$

where $\alpha = \beta = \gamma = 1$

$F_{support}[\text{total}]$ = 1380 kN

C.3 - Example 1b: Simply-supported deck with one elastic fixed support (through-girder bridge) / no expansion device

Input data:

Simply-supported deck

one track

deck length	L = 60 m
fixed support at one end, stiffness	K = 120 kN/mm = 2 L
no friction in the movable bearing	
through-girder bridge with	E = 21 E7 kN/m^2; I = 2,59 m^4; A = 0,7 m^2; H = 0,7 m; ω = - 0,6 m; $\gamma = \omega/H$ = - 0,86; θH [LM 71] = 0,9 mm

UIC 60 continuous welded rail

resistance of the unloaded track: a displacement of 2 mm between the elastic zone and the plastic zone and a resistance of 20 kN/m of the plastic phase $\Rightarrow$ k (unloaded) = 20

resistance of the loaded track: a displacement of 2 mm between the elastic zone and the plastic zone and a resistance of 40 kN/m of the plastic phase $\Rightarrow$ k (loaded) = 40

coefficient of thermal expansion	$\alpha = 1{,}0 \times 10^{-5}$
temperature variation of the rails	$\Delta T_{rails} = \pm 50°C$
temperature variation of the deck	$\Delta T_{deck} = \pm 35°C$
load due to braking	20 kN/m

Verification of σ_{rail}:

σ_{rail} due to the variation of the temperature in the rail is not an additional stress for the verification

Diagrams in figures 1, 2, 4 and 5 of point A.1 (Appendix A - page 36) for the interaction due to the temperature variation and due to braking; without an expansion device

temperature with Δ T_{deck} + 35°C, K = 2 L = K_2, k = 20 kN/2 mm = k_{20} and L = 60 m

braking with K = 2 L = K_2, k = 40 kN/2 mm = k_{40} and L = 60 m

σ_{rail} (fixed) [Δ T] = 8 N/mm^2

σ_{rail} (movable) [Δ T] = 26 N/mm^2

σ_{rail} (fixed) [braking] = 28 N/mm^2

σ_{rail} (movable) [braking] = 28 N/mm^2

Diagrams of points B.2, B.4 and B.6 (Appendix B - page 42) for the interaction due to the vertical bending; through-girder bridge θH = 1 mm; L = 60 m; K = 2.L; γ = - 0,9 (Interpolation between γ = - 5/K2 and γ = 0,0/K2)

σ_{rail} (fixed) [1 mm] = 5 N/mm^2 (tension)

σ_{rail} (movable) [1 mm] = - 11 N/mm^2 (compression)

So representative value: additional compressive rail stress of 11 N/mm^2

σ_{rail} (fixed) [θH] = 5 x [0,9] = 4,5 N/mm^2

σ_{rail} (movable) [θH] = - 11 x {0,9} = - 10 N/mm^2

For the support reaction:

$F_{support}[\theta H] = F_{support}[1\text{ mm}] \times [\theta H] = 170 \times (0,9) = 153$ kN

Verification of σ_{rail}:

1,0 x σ_{rail} (fixed) [ΔT] + 1,0 x σ_{rail} (fixed) [braking] + 1,0 x σ_{rail} (fixed) [θH] =
(8 + 28 + 0) = 36 N/mm^2 $\leq$ 72 N/mm^2

1,0 x σ_{rail} (mov) [ΔT] + 1,0 x σ_{rail} (mov) [braking] + 1,0 x σ_{rail} (mov) [Hθ] =
(26 + 28 + 10) = 64 N/mm^2 $\leq$ 72 N/mm^2

Verification of δ-relative:

The verification of the relative displacement between the rail and the deck or the platform due to braking and acceleration is assured when the verification of δ-deck (fixed) is satisfied.

Verification of δ_{deck}(fixed):

$\delta_{deck} = F_{supp}[\text{braking}] / K_{supp} = 350 / 120 = 2,9\text{ mm} < 5\text{ mm}$

Verification of $\delta(\theta H)$:

$\delta(\theta H)$ = 0,9 mm < 10 mm (the criterion is met)

Support reactions:

$F_{support}[\Delta T]$ = 500 kN

$F_{support}[\text{braking}]$ = 350 kN

$F_{support}[\theta H]$ = 153 kN

$F_{support}[\text{total}] = \alpha\, F_{support}[\Delta T] + \beta\, F_{support}[\text{braking}] + \gamma\, F_{support}[\Theta H]$

$F_{support}[\text{total}]$ = 1003 kN

C.4 - Example 2: Continuous deck bridge with one intermediate fixed support (deck slab bridge) / no expansion device

Input data:

Multi-span continuous deck	
double-track	
deck length	$L = 90$ m, 3 spans of 30 m
intermediate fixed support, stiffness	$K = 900$ kN/mm = 10 L
no friction in the movable bearings	
deck bridge with	$E = 21$ E7 kN/m^2; $I = 0{,}165$ m^4; $A = 0{,}57$ m^2; $H = 3{,}0$ m; $\omega = 0{,}36$ m; $\gamma = \omega/H = 0{,}12$; $\theta H_L[\max] = 4{,}7$ mm, $\theta H_R[\max] = 3{,}1$ mm
UIC 54 continuous welded rail	

resistance of the unloaded track: a displacement of 2,0 mm between the elastic zone and the plastic zone and a resistance of 20 kN/m of the plastic phase $\Rightarrow$ k (unloaded) = 20

resistance of the loaded track: a displacement of 2,0 mm between the elastic zone and the plastic zone and a resistance of 40 kN/m of the plastic phase $\Rightarrow$ k (loaded) = 40

coefficient of thermal expansion	$\alpha = 1{,}0 \times 10^{-5}$
temperature variation of the rails	$\Delta T_{rails} = \pm 45°C$
temperature variation of the deck	$\Delta T_{deck} = \pm 30°C$
load due to braking and acceleration	one track acceleration of 33 kN/m with a total of maximum 1000 kN and one track braking of 20 kN/m with a maximum of 6000 kN

Verification of σ_{rail}:

σ_{rail} due to the variation of the temperature in the rail is not an additional stress for the verification

Temperature variation:

Diagram in figure 5 - page 38 point A.1 (Appendix A) for the interaction due to the temperature variation; without an expansion device; single track

$\Delta T_{deck} = +35°C$; UIC 60 with $L = 60$ m, $K = 0{,}5 \times 10L = 5L$, $k = 20$ kN/2 mm

σ_{rail} [Δ T] (60 m, 5L, k_{20}) = 30 N/mm^2, σ_{rail} [Δ T] (60 m, ∞, k_{20}) = 34 N/mm^2

σ_{rail} [Δ T] (UIC 60, 35°C) = 30 + (34 - 30)30/60 = 32 N/mm^2

σ_{rail} [Δ T] (UIC 54, 35°C) = 32 x {1 + 1,05(1 - A_{54}/A_{60})} = 35 N/mm^2

σ_{rail} [Δ T] (UIC 54, 30°C) = 35 x $(40/45)^{1/2}$ = 33 N/mm^2

σ_{rail} [Δ T] (double track, 10L) = σ_{rail} [Δ T] (single track, 5L)= 33 N/mm^2

Braking:

Diagram in figure 2 - page 37 of point A.1 (Appendix A) for the interaction due to braking; without an expansion device; single track; UIC 60

with L = 90 m, K = 0,5 x 10L = 5L, k = 40 kN/2 mm

σ_{rail} [braking] (single track, 5L, UIC 60) = 28 N/mm^2 (single track)

σ_{rail} [braking] (single track, 5L, UIC 54) = 28 + (1 + 0,9 (1 - A_{54}/A_{60}) = 31 N/mm^2

σ_{rail} [braking] (double track, 10L) = (2800/3600) x 30 + 5 = 29 N/mm^2

Vertical bending:

Diagram in figure of point B.1 - page 42 for the interaction due to the vertical bending; deck bridge and θH = 8 mm
with K = 10L; γ = 0,12; L = 30 m

σ_{rail} (θH = 8) = 21 N/mm^2

σ_{rail} (θH = 3,1) = 15 x $\{3,1/8\}^{0,86}$ = 9 N/mm^2

σ_{rail} (UIC 54) = 9 x $(A_{60}/A_{54})^{0,9}$ = 10 N/mm^2

Verification:

1,0 x $\sigma_{rail}[\Delta T]$ + 1,0 x σ_{rail}[braking] + 1,0 x $\sigma_{rail}[\theta H]$ = (33 + 29 + 10) N/mm^2

= 72 N/mm^2 $\leq$ 72 N/mm^2

Verification of δ-relative:

The verification of the relative displacement between the rail and the deck or the platform due to braking and acceleration is assured when the verification of δ-deck (fixed) is satisfied.

Verification of δ_{deck} (fixed):

Diagram in figure 3 - page 37 of point A.1 (Appendix A) for the interaction due to braking; without an expansion device; single track; UIC 60
with L = 90 m, K = 0,5 x 10L = 5L, k = 40 kN/2 mm

F_{supp} [braking] (single track, 5L, UIC 60) = 950 kN
F_{supp} [braking] (single track, 5L, UIC 54) = 950 x {1 + 0,36(1 - A_{54}/A_{60})} = 984 kN
F_{supp} [braking] (double track, 10L, UIC 54) = (2800/1800) x 984 = 1530 kN

Verification:

δ_{deck}(fixed) = F_{supp} / K_{supp} = 1530/900 = 1,7 mm $\leq$ 5 mm

Verification of $\delta[\theta H]$:

$\delta[\theta H]$(max) = 4,7 mm $\leq$ 10 mm

Support reactions:

Temperature:

Diagram in figure 6 - page 39 of point A.1 (Appendix A) for the interaction due to the temperature variation; without an expansion device; single track

ΔT_{deck} = + 35°C; UIC 60: with L = 60 m and 30 m, K = 0,5 x 10L = 5L, k = 20 kN/2mm

F_{supp} [ΔT] (60 m, 5L, k_{20}) = 650 kN, F_{supp} [ΔT] (30 m, 5L, k_{20}) = 300 kN

F_{supp} [ΔT] (UIC 60, 35°C) = 650 + 300 = 950 kN

F_{supp} [ΔT] (UIC 54, 35°C) = 950 x {1 - 0,24 (1 - A_{54} /A_{60})} = 927 kN

F_{supp} [ΔT] (UIC 54, 30°C) = 927 x $(30/35)^{1/2}$ = 858 kN

F_{supp} [ΔT] (double track, 10L) = F_{supp} [ΔT] (single track, 5L) = 858 kN

Braking:

F_{supp} [braking] (double track, 10L, UIC 54) = 1530 kN

Vertical bending:

Diagram in figure of point B.5 - page 44 for the interaction due to the vertical bending; deck bridge and θH = 8 mm; K = 10L; γ = 0,12; L = 30 m

F_{supp} (θH = 8) = 850 kN

F_{supp} (θH = 3,1) = 850 x $\{3, 1/8\}^{0,86}$ = 375 kN

F_{supp} (UIC 54) = 375 x $(A_{60}/A_{54})^{0,9}$ = 422 kN

F_{supp} [total] = α F_{supp}[ΔT] + β F_{supp}[braking] + γ F_{supp}[θH]

(if $\alpha = \beta = \gamma = 1$): F_{supp} = 858 + 1530 + 422 = 2810 kN

C.5 - Example 3: Continuous deck with one fixed support at the end (deck slab bridge) / with expansion device

Input data:

Multi-span continuous deck	
single track	
deck length	L = 180 m, 3 spans of 60 m
intermediate fixed support, stiffness	K = 900 kN/mm = 5
no friction in the movable bearings	
deck bridge with	E = 21 E7 kN/m^2; I = 2,59 m^4; A = 0,74 m^2; H = 6,0 m; ω = 1,21 m; $\gamma = \omega/H$ = 0,20; θH [80 kN/m] = 7,9 mm

expansion device at one end and UIC 60 rails

resistance of the unloaded track: a displacement of 2,0 mm between the elastic zone and the plastic zone and a resistance of 20 kN/m of the plastic phase $\Rightarrow$ k (unloaded) = 20

resistance of the loaded track: a displacement of 2,0 mm between the elastic zone and the plastic zone and a resistance of 40 kN/m of the plastic phase $\Rightarrow$ k (loaded) = 40

coefficient of thermal expansion	$\alpha = 1{,}0 \times 10^{-5}$
temperature variation of the rails	$\Delta T_{rail} = \pm 50°C$
temperature variation of the deck	$\Delta T_{deck} = \pm 30°C$
load due to braking and acceleration	one-track acceleration of 33 kN/m with a total of maximum 1000 kN and one-track braking of 20 kN/m with a maximum of 6000 kN

Verification of σ_{rail}:

Diagram in figure 8 - page 40 of point A.2 (Appendix A) for the interaction due to the temperature variation and due to braking; with an expansion device

temperature with Δ T_{deck} + 30°C; K = 5L; k = 20 kN/2 mm = k_1 and L = 180 m

braking with K = 5L, k = 40 kN/2 mm = k_{40} and L = 180 m

σ_{rail} (fixed) [Δ T] is less than in the case of continuous welded rail on the embankments; take

σ_{rail} (fixed) [Δ T] = 0 N/mm^2

σ_{rail} (mobile) [Δ T] = 0 N/mm^2

σ_{rail} (fixed) [braking] = 51 N/mm^2 (Interpolation between 58 and 15)

σ_{rail} (mobile) [braking] = 0 N/mm^2

Diagram in figures of points B.1 - page 42 and B.3 - page 43 (Appendix B) for the interaction due to the vertical bending; deck bridge and θH = 8 mm; L = 60 m; K = 5L; γ = 0,20 (Interpolation between γ = 0,5/K5 and γ = 0,0/K5)

σ_{rail} (fixed) [8 mm] = 45 N/mm^2 (tensile)

σ_{rail} (mobile) [8 mm] = 0 N/mm^2

So representative value: additional compressive rail stress of 0 N/mm^2

σ_{rail} (fixed) [θH] = 0

σ_{rail} (mobile) [θH] = 0 N/mm^2

Verification:

1,0 x σ_{rail}(fix.)[ΔT] + 1,0 x σ_{rail}(fix.)[braking] + 1,0 x σ_{rail}(fix.)[θH] = (0 + 51 + 0) = 51N/mm^2 $\leq$ 72 N/mm^2 (compression)

Verification of δ-relative:

The verification of the relative displacement between the rail and the deck or the platform due to braking and acceleration is assured when the verification δ-deck (fixed) is satisfied.

Verification of δ_{deck} (fixed):

Diagram for the interaction due to braking; with an expansion device: with K = 5L, k_{40} and L = 180 m

$F_{support}$ [braking] = 2800 kN (interpolation between 2700 and 3300)

δ_{deck} (fixed) = $F_{support}$ [braking]/K_2 = 2800/900 = 3,1 mm

Verification: δ_{deck} (fixed) $\leq$ 5 mm

Verification of $\delta[\theta H]$:

$\delta[\theta H]$ = 7,9 mm < 10 mm

Support reactions:

Diagrams in figures 7 - page 40 and 9 - page 41 of point A.2 (Appendix A) for the interaction due to the temperature variation and due to braking; with an expansion device

temperature with K = 5L, k = k_{20} and L = 180 m

braking with K_1 = 5*60 kN/2 mm, k = k_{40} and L = 180 m

$F_{support}$ [Δ T = 30/50] = 955 kN (interpolation between 925 and 1100)

$F_{support}$ [Δ T = 35/50] = 955 + 16(35-30) = 1035 kN

$F_{support}$ [braking] = 2800 kN (interpolation between 2700 and 3300)

Diagram in figure of point B.5 - page 44 (Appendix B) for the interaction due to the vertical bending; deck bridge and θH = 8 mm; L = 60 m; K = 5L; γ = 0,20 (interpolation between γ = 0,5/K5 and γ = 0,0/K5)

$F_{support}$ [8 mm] = 900 kN

$F_{support}$ [θH] = $F_{support}$ [8 mm] * $(\theta H/8)^{0,86}$ = 890 kN

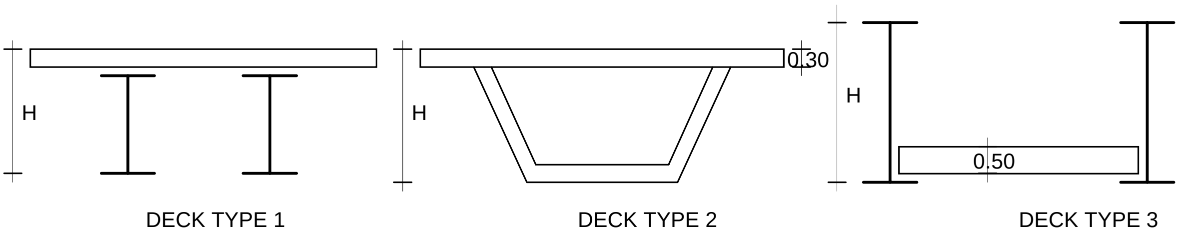
$F_{support}$ [total]
= α $F_{support}$ [Δ T] + β $F_{support}$ [braking] + γ $F_{support}$[θH]
= α 1035 kN + β 2800 kN + γ 890 kN

$F_{support}$ = 4725 kN

Appendix D - Validation of the computer programs

D.1 - Test-case No. 1

16 single-span bridges with different deck types, span lengths and longitudinal stiffnesses were analysed. For each structure both train-running directions were considered. The 32 cases listed in the table below were analysed. E is the Young's modulus, I is the moment of inertia, H is the height, S is the cross-sectional area, vi is the neutral axis ordinate. The deck types are schematically depicted in the figure below. The position of the centre of gravity of the rails is considered, for simplicity to coincide with the top of the R/C slab in all cases.



Case No.	Name	Deck type	Span (m)	Long. displ. (mm)	Direct.	K long. (KN/m)	E (KN/m^2)	I (m^4)	H (m)	S (m^2)	vi (m)
A1-3	1-30-1-1	1	30	1	1	300000	2,1E+08	0,165	3,00	0,57	2,64
A4-6	1-30-1-2	1	30	1	2	300000	2,1E+08	0,165	3,00	0,57	2,64
B1-3	1-30-5-1	1	30	5	1	60000	2,1E+08	0,165	3,00	0,57	2,64
B4-6	1-30-5-2	1	30	5	2	60000	2,1E+08	0,165	3,00	0,57	2,64
C1-3	1-45-1-1	1	45	1	1	450000	2,1E+08	0,830	4,50	0,64	3,84
C4-6	1-45-1-2	1	45	1	2	450000	2,1E+08	0,830	4,50	0,64	3,84
D1-3	1-45-5-1	1	45	5	1	90000	2,1E+08	0,830	4,50	0,64	3,84
D4-6	1-45-5-2	1	45	5	2	90000	2,1E+08	0,830	4,50	0,64	3,84
E1-3	1-60-1-1	1	60	1	1	600000	2,1E+08	2,590	6,00	0,74	4,79
E4-6	1-60-1-2	1	60	1	2	600000	2,1E+08	2,590	6,00	0,74	4,79
F1-3	1-60-5-1	1	60	5	1	120000	2,1E+08	2,590	6,00	0,74	4,79
F4-6	1-60-5-2	1	60	5	2	120000	2,1E+08	2,590	6,00	0,74	4,79
G1-3	1-90-1-1	2	90	1	1	1080000	3,42E+07	80,060	9,00	7,20	5,07
G4-6	1-90-1-2	2	90	1	2	1080000	3,42E+07	80,060	9,00	7,20	5,07
H1-3	1-90-5-1	2	90	5	1	216000	3,42E+07	80,060	9,00	7,20	5,07
H4-6	1-90-5-2	2	90	5	2	216000	3,42E+07	80,060	9,00	7,20	5,07
I1-3	3-30-3-1	3	30	1	1	300000	2,1E+08	0,180	3,00	0,50	0,42
I4-6	3-30-3-2	3	30	1	2	300000	2,1E+08	0,180	3,00	0,50	0,42
J1-3	3-30-5-1	3	30	5	1	60000	2,1E+08	0,180	3,00	0,50	0,42
J4-6	3-30-5-2	3	30	5	2	60000	2,1E+08	0,180	3,00	0,50	0,42
K1-3	3-45-3-1	3	45	1	1	450000	2,1E+08	0,820	4,50	0,57	0,74
K4-6	3-45-3-2	3	45	1	2	450000	2,1E+08	0,820	4,50	0,57	0,74
L1-3	3-45-5-1	3	45	5	1	90000	2,1E+08	0,820	4,50	0,57	0,74
L4-6	3-45-5-2	3	45	5	2	90000	2,1E+08	0,820	4,50	0,57	0,74
M1-3	3-60-3-1	3	60	1	1	600000	2,1E+08	2,560	6,00	0,70	1,31
M4-6	3-60-3-2	3	60	1	2	600000	2,1E+08	2,560	6,00	0,70	1,31
N1-3	3-60-5-1	3	60	5	1	120000	2,1E+08	2,560	6,00	0,70	1,31
N4-6	3-60-5-2	3	60	5	2	120000	2,1E+08	2,560	6,00	0,70	1,31
O1-3	3-90-3-1	3	90	1	1	1080000	2,1E+08	13,340	9,00	1,30	3,18
O4-6	3-90-3-2	3	90	1	2	1080000	2,1E+08	13,340	9,00	1,30	3,18
P1-3	3-90-5-1	3	90	5	1	216000	2,1E+08	13,340	9,00	1,30	3,18
P4-6	3-90-5-2	3	90	5	2	216000	2,1E+08	13,340	9,00	1,30	3,18

As can be seen from the table, two values for the horizontal flexibility of the fixed support were examined for each structure.

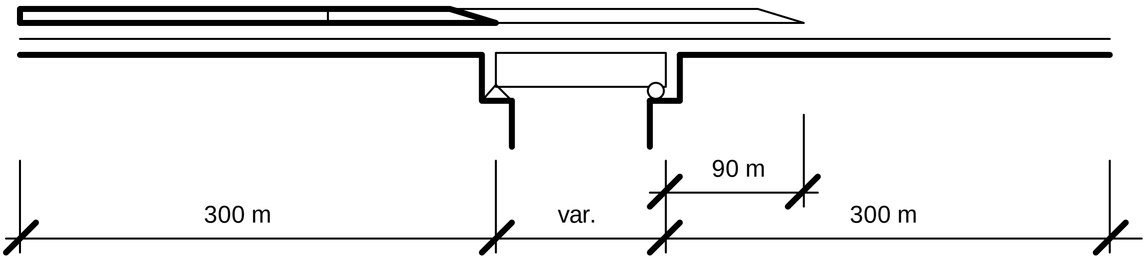
Ballasted track was assumed, with the following pair of resistance parameters:

20 KN/m for unloaded track	60 KN/m for loaded track (vertical load: 80 KN/m)

Thermal variations equal to 35°C for the deck and 50°C for the rail were considered. 80 KN/m vertical loads and 20 KN/m horizontal braking forces were assumed. The train was assumed to be 300 m long.

The braking forces, as well as the direction of travel, were considered to be acting from the fixed support towards the movable support (direct. 1) in half of the cases and the reverse (direct. 2) for the remaining half. With a positive thermal variation being applied to the deck, it was expected that the maximum and minimum rail stresses would be found in the former situation, and the maximum absolute support reactions in the latter.

The model was based on the configuration shown below with 300 m of embankment considered on each side of the bridge.



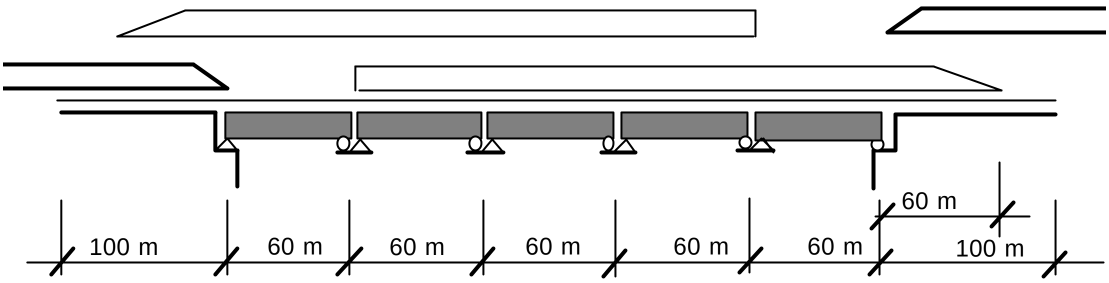
In order to evaluate the influence of each component of interaction, three analyses were carried out for each case. In the first analysis (analyses 1 or 4) only the vertical loads travelled along the bridge, in order to find the net effect of the end rotation. In the second analysis (analyses 2 or 5) the deck was assumed rigid, in order to evaluate the effects of braking forces alone. In the third analysis (analyses 3 or 6) the three effects were evaluated, applying the temperature variation first and then the moving train loads. Having calculated the three contributions separately, it was then possible to compare the simple sum of the contributions with the overall effects.

The tables below show the results relevant to all the cases examined. The first 5 columns describe the structural systems, the sixth and seventh column specify the quantities considered and the relevant units, the following four columns give the maximum effects of temperature variations in the rail, temperature variation in the deck, braking actions and end rotation respectively. The next two columns give the sum of the maximum effects evaluated separately and the total effects, i.e. the envelope of the results obtained in the third analysis. The comparison of these two columns allows the percentage error to be evaluated. This is shown in the following column. Since the maximum effect is sometimes given by the temperature variation alone, the following two columns give the absolute maximum between the sum of effects and the temperature variation effect, along with the percentage error on the absolute values. The effects of the track-structure interaction are given in terms of additional rail stresses, relative displacement between rail and deck, absolute displacements of the deck, reaction at the fixed support. The final two columns allow the total rail stresses to be compared directly, i.e. including the rail stresses due to the rail temperature variation.

The fundamental cases to be tested are those labelled E1-3 and E4-6. These cases should be satisfied within the tolerances given above. The other cases are given for a supplementary check, when the results for the fundamental cases are not satisfactory.

D.2 - Test-case No. 2

The bridge configuration is shown below:



The main characteristics and the considered effects of the 5-span bridge models are given in the following table:

Case No.	Type of analysis	Deck type	Span (m)	Action	Way of travelling	K long. (KN/m)	E (KN/m2)	I (m4)	H (m)	S (m2)	vi (m)
G1	multistep	1	60	bending + braking	F-M	2000L	2,1E+08	2,590	6,00	0,74	4,79
G2	multistep	1	60	bending + braking	M-F	2000L	2,1E+08	2,590	6,00	0,74	4,79
G3	multistep	1	60	bending + braking	F-M	20000L	2,1E+08	2,590	6,00	0,74	4,79
G4	multistep	1	60	bending + braking	M-F	20000L	2,1E+08	2,590	6,00	0,74	4,79
GT	one-step	1	60	temperature		20000L	2,1E+08	2,590	6,00	0,74	4,79
G5	one-step	1	60	braking	F-M	20000L	2,1E+08	2,590	6,00	0,74	4,79
G6	one-step	1	60	bending	M-F	20000L	2,1E+08	2,590	6,00	0,74	4,79

E is Young's modulus, I is the moment of inertia, H is the section height, S is the cross-sectional area, vi is the neutral axis ordinate. The position of the barycentre of the rails is considered, for simplicity, to coincide with the top of the R/C slab in all cases.

Two types of analyses have been used. In the multistep analyses the train is travelling along the bridge and the ballast characteristics are automatically varied by the program according to the vertical loads applied. In these analyses the temperature effects were considered separately, since they can increase or decrease the single effect according to the sign of the variation and it has been clearly shown in practice that their effect is practically additive to the train load effects.

The following assumptions have also been made:

- the mechanical characteristics of the ballast are taken equal to 20 KN/m for unloaded ballast and 60 KN/m for loaded ballast (80 KN/m vertical load), while the displacement between the elastic and plastic zones is taken equal to 2 mm,
- the elastic-plastic law is assumed for the ballast behaviour under longitudinal actions,

- the temperature variation of the deck is taken equal to 35°C and the thermal coefficient is taken equal to 0,00001, while zero thermal variation is assumed for the rails,

- the train is assumed to be 300 m long.
  The model is based on the configuration shown in Figure 2 (see page 60), where 100 m of embankment is considered on each side of the bridge. Analyses G5 and G6 are one-step analyses made for the purposes of comparison, with the train exactly centred on the bridge.

case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
A1-3	1	30	1	1	rail stress	MPa	-126	-13,22	-12,76	-14,98	-40,96	-38,89	-5,3		41,0	5,3		Sum	-166,96
A1-3	1	30	1	1	rel.displac.	m	0	-7,72E-03	1,61E-03	-2,11E-03	-8,22E-03	-8,82E-03	6,8		8,22E-03	-6,8		Global	-164,89
A1-3	1	30	1	1	abs.displac.	m	0	-1,59E-03	1,08E-03	2,99E-03	2,49E-03	3,17E-03	21,5		2,49E-03	-21,5		%error	-1,3
A1-3	1	30	1	1	supp. react.	KN	0	306,22	-324,2	706,34	688,36	736,82	6,6		688,4	-6,6			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
A4-6	1	30	1	2	rail stress	MPa	-126	-13,22	-12,7	-24,75	-50,67	-23,9	-112,0		50,7	112,0		Sum	-176,67
A4-6	1	30	1	2	rel.displac.	m	0	-7,72E-03	-1,61E-03	-2,39E-03	-1,17E-02	-9,24E-03	-26,8		1,17E-02	26,8		Global	-149,9
A4-6	1	30	1	2	abs.displac.	m	0	-1,59E-03	-1,08E-03	3,28E-03	6,11E-04	-1,59E-03	61,5		1,59E-03	0,0		%error	-17,9
A4-6	1	30	1	2	supp.react.	KN	0	306,22	324,41	666,97	1297,6	1199,5	-8,2		1297,6	8,2			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
B1-3	1	30	5	1	rail stress	MPa	-126	-9,63	-16,84	-6,38	-32,85	-30,79	-6,7		32,9	6,7		Sum	-158,85
B1-3	1	30	5	1	rel.displac.	m	0	-6,45E-03	2,03E-03	-1,20E-03	-5,62E-03	-6,82E-03	17,6		6,45E-03	-5,5		Global	-156,79
B1-3	1	30	5	1	abs.displac.	m	0	-3,16E-03	2,01E-03	1,12E-03	-3,40E-05	-3,16E-03	98,9		3,16E-03	0,0		%error	-1,3
B1-3	1	30	5	1	supp.react.	KN	0	172,6	-120,37	303,4	355,63	380,32	6,5		355,6	-6,5			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
B4-6	1	30	5	2	rail stress	MPa	-126	-9,63	-12,37	-8,99	-30,99	-9,63	-221,8		31,0	221,8		Sum	-156,99
B4-6	1	30	5	2	rel.displac.	m	0	-6,45E-03	-2,02E-03	-1,56E-03	-1,00E-02	-6,81E-03	-47,3		1,00E-02	47,3		Global	-135,63
B4-6	1	30	5	2	abs.displac.	m	0	-3,16E-03	-2,01E-03	1,54E-03	-3,64E-03	-3,59E-03	-1,3		3,64E-03	1,3		%error	-15,7
B4-6	1	30	5	2	supp.react.	KN	0	172,6	120,76	283,31	576,67	559,62	-3,0		576,7	3,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
C1-3	1	45	1	1	rail stress	MPa	-126	-22,02	-14,92	-17,47	-54,41	-49,36	-10,2		54,4	10,2		Sum	-180,41
C1-3	1	45	1	1	rel.displac.	m	0	-1,20E-02	1,78E-03	-2,38E-03	-1,26E-02	-1,31E-02	3,9		1,26E-02	-3,9		Global	-175,36
C1-3	1	45	1	1	abs.displac.	m	0	-1,71E-03	1,24E-03	3,51E-03	3,05E-03	4,06E-03	24,9		3,05E-03	-24,9		%error	-2,9
C1-3	1	45	1	1	supp.react.	KN	0	504,34	-557,79	883,63	830,18	857,91	3,2		830,2	-3,2			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
C4-6	1	45	1	2	rail stress	MPa	-126	-22,02	-14,7	-28,93	-65,65	-31,7	-107,1		65,7	107,1		Sum	-191,65
C4-6	1	45	1	2	rel.displac.	m	0	-1,20E-02	-1,76E-03	-2,70E-03	-1,65E-02	-1,36E-02	-21,1		1,65E-02	21,1		Global	-157,7
C4-6	1	45	1	2	abs.displac.	m	0	-1,71E-03	-1,24E-03	3,84E-03	8,92E-04	-1,71E-03	47,7		1,71E-03	0,0		%error	-21,5
C4-6	1	45	1	2	supp.react.	KN	0	504,34	557,59	812,14	1874,07	1727,72	-8,5		1874,1	8,5			

Simply-supported one-span bridges

case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
D1-3	1	45	5	1	**rail stress**	MPa	-126	**-17,57**	**-21,7**	**-8,03**	**-47,3**	**-42,52**	-11,2		47,3	11,2		**Sum**	-173,3
D1-3	1	45	5	1	**rel.displac.**	m	0	**-1,02E-02**	**2,43E-03**	**-1,39E-03**	**-9,16E-03**	**-1,09E-02**	15,9		1,02E-02	-6,3		**Global**	-168,52
D1-3	1	45	5	1	**abs.displac.**	m	0	**-3,91E-03**	**2,61E-03**	**1,46E-03**	**1,63E-04**	**-3,91E-03**	95,8		3,91E-03	0,0		**%error**	-2,8
D1-3	1	45	5	1	**supp.react.**	KN	0	**321,13**	**-235,15**	**417,32**	**503,3**	**520,44**	3,3		503,3	-3,3			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
D4-6	1	45	5	2	**rail stress**	MPa	-126	**-17,57**	**-20,65**	**-10,98**	**-49,2**	**-17,57**	-180,0		49,2	180,0		**Sum**	-175,2
D4-6	1	45	5	2	**rel.displac.**	m	0	**-1,02E-02**	**-2,42E-03**	**-1,79E-03**	**-1,44E-02**	**-1,07E-02**	-34,7		1,44E-02	34,7		**Global**	-143,57
D4-6	1	45	5	2	**abs.displac.**	m	0	**-3,91E-03**	**-2,62E-03**	**1,95E-03**	**-4,58E-03**	**-4,48E-03**	-2,3		4,58E-03	2,3		**%error**	-22,0
D4-6	1	45	5	2	**supp.react.**	KN	0	**321,13**	**236,2**	**380,77**	**938,1**	**903,68**	-3,8		938,1	3,8			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
E1-3	1	60	1	1	**rail stress**	MPa	-126	**-30,67**	**-16,42**	**-16,98**	**-64,07**	**-56,4**	-13,6		64,1	13,6		**Sum**	-190,07
E1-3	1	60	1	1	**rel.displac.**	m	0	**-1,64E-02**	**1,90E-03**	**-2,70E-03**	**-1,72E-02**	**-1,73E-02**	0,8		1,72E-02	-0,8		**Global**	-182,4
E1-3	1	60	1	1	**abs.displac.**	m	0	**-1,69E-03**	**1,36E-03**	**3,77E-03**	**3,43E-03**	**4,56E-03**	24,8		3,43E-03	-24,8		**%error**	-4,2
E1-3	1	60	1	1	**supp.react.**	KN	0	**700,12**	**-813,22**	**977,7**	**864,6**	**874,42**	1,1		864,6	-1,1			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
E4-6	1	60	1	2	**rail stress**	MPa	-126	**-30,67**	**-15,95**	**-28,22**	**-74,84**	**-36,06**	-107,5		74,8	107,5		**Sum**	-200,84
E4-6	1	60	1	2	**rel.displac.**	m	0	**-1,64E-02**	**-1,85E-03**	**-3,12E-03**	**-2,13E-02**	**-1,78E-02**	-19,8		2,13E-02	19,8		**Global**	-162,06
E4-6	1	60	1	2	**abs.displac.**	m	0	**-1,69E-03**	**-1,36E-03**	**4,16E-03**	**1,11E-03**	**-1,69E-03**	34,3		1,69E-03	0,0		**%error**	-23,9
E4-6	1	60	1	2	**supp.react.**	KN	0	**700,12**	**817,74**	**855,61**	**2373,47**	**2196,1**	-8,1		2373,5	8,1			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
F1-3	1	60	5	1	**rail stress**	MPa	-126	**-25,97**	**-25,85**	**-10,11**	**-61,93**	**-51,71**	-19,8		61,9	19,8		**Sum**	-187,93
F1-3	1	60	5	1	**rel.displac.**	m	0	**-1,42E-02**	**2,77E-03**	**-1,67E-03**	**-1,31E-02**	**-1,50E-02**	12,6		1,42E-02	-5,2		**Global**	-177,71
F1-3	1	60	5	1	**abs.displac.**	m	0	**-4,35E-03**	**3,18E-03**	**1,64E-03**	**4,74E-04**	**-4,35E-03**	89,1		4,35E-03	0,0		**%error**	-5,8
F1-3	1	60	5	1	**supp.react.**	KN	0	**482,92**	**-381,29**	**501,84**	**603,47**	**609,33**	1,0		603,5	-1,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
F4-6	1	60	5	2	**rail stress**	MPa	-126	**-25,97**	**-25,58**	**-10,23**	**-61,78**	**-26,97**	-129,1		61,8	129,1		**Sum**	-187,78
F4-6	1	60	5	2	**rel.displac.**	m	0	**-1,42E-02**	**-2,75E-03**	**-2,18E-03**	**-1,91E-02**	**-1,48E-02**	-29,5		1,91E-02	29,5		**Global**	-152,97
F4-6	1	60	5	2	**abs.displac.**	m	0	**-4,35E-03**	**-3,19E-03**	**2,29E-03**	**-5,25E-03**	**-5,13E-03**	-2,4		5,25E-03	2,4		**%error**	-22,8
F4-6	1	60	5	2	**supp.react.**	KN	0	**482,92**	**383,07**	**436,75**	**1302,74**	**1250,6**	-4,2		1302,7	4,2			

Simply-supported one-span bridges

case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
G1-3	2	90	1	1	rail stress	MPa	-126	-47,33	-17,13	-18,03	-82,49	-61,63	-33,8		82,5	33,8		Sum	-208,49
G1-3	2	90	1	1	rel.displac.	m	0	-2,48E-02	1,93E-03	-3,35E-03	-2,62E-02	-2,51E-02	-4,5		2,62E-02	4,5		Global	-187,63
G1-3	2	90	1	1	abs.displac.	m	0	-1,22E-03	1,31E-03	3,20E-03	3,29E-03	4,03E-03	18,5		3,29E-03	-18,5		%error	-11,1
G1-3	2	90	1	1	supp.react.	KN	0	1094,7	-1410,8	923,59	607,49	1094,7	44,5		1094,7	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
G4-6	2	90	1	2	rail stress	MPa	-126	-47,33	-16,16	-17,51	-81	-47,33	-71,1		81,0	71,1		Sum	-207
G4-6	2	90	1	2	rel.displac.	m	0	-2,48E-02	-1,84E-03	-4,10E-03	-3,08E-02	-2,57E-02	-19,6		3,08E-02	19,6		Global	-173,33
G4-6	2	90	1	2	abs.displac.	m	0	-1,22E-03	-1,30E-03	3,69E-03	1,17E-03	-1,22E-03	3,9		1,22E-03	0,0		%error	-19,4
G4-6	2	90	1	2	supp.react.	KN	0	1094,7	1404,7	590,52	3089,92	2909,3	-6,2		3089,9	6,2			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
H1-3	2	90	5	1	rail stress	MPa	-126	-43,14	-24,58	-17,65	-85,37	-62,23	-37,2		85,4	37,2		Sum	-211,37
H1-3	2	90	5	1	rel.displac.	m	0	-2,26E-02	3,20E-03	-2,47E-03	-2,19E-02	-2,31E-02	5,2		2,26E-02	-2,0		Global	-188,23
H1-3	2	90	5	1	abs.displac.	m	0	-4,09E-03	3,84E-03	1,47E-03	1,22E-03	-4,09E-03	70,2		4,09E-03	0,0		%error	-12,3
H1-3	2	90	5	1	supp.react.	KN	0	858,67	-828,67	580,61	610,61	858,67	28,9		858,7	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
H4-6	2	90	5	2	rail stress	MPa	-126	-43,14	-29,62	-17,46	-90,22	-43,14	-109,1		90,2	109,1		Sum	-216,22
H4-6	2	90	5	2	rel.displac.	m	0	-2,26E-02	-3,12E-03	-3,52E-03	-2,93E-02	-2,33E-02	-25,4		2,93E-02	25,4		Global	-169,14
H4-6	2	90	5	2	abs.displac.	m	0	-4,09E-03	-3,84E-03	2,60E-03	-5,33E-03	-5,24E-03	-1,8		5,33E-03	1,8		%error	-27,8
H4-6	2	90	5	2	supp.react.	KN	0	858,67	830,22	364,35	2053,24	1975,7	-3,9		2053,2	3,9			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
I1-3	3	30	1	1	rail stress	MPa	-126	-14,14	-12,77	-2,71	-29,62	-29,15	-1,6		29,6	1,6		Sum	-155,62
I1-3	3	30	1	1	rel.displac.	m	0	-8,02E-03	1,62E-03	-4,27E-04	-6,83E-03	-8,15E-03	16,2		8,02E-03	-1,6		Global	-155,15
I1-3	3	30	1	1	abs.displac.	m	0	-1,16E-03	1,08E-03	5,49E-04	4,65E-04	-1,16E-03	60,0		1,16E-03	0,0		%error	-0,3
I1-3	3	30	1	1	supp.react.	KN	0	344,42	-323,72	132,82	153,52	344,42	55,4		344,4	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
I4-6	3	30	1	2	rail stress	MPa	-126	-14,14	-12,7	-4,29	-31,13	-14,14	-120,2		31,1	120,2		Sum	-157,13
I4-6	3	30	1	2	rel.displac.	m	0	-8,02E-03	-1,61E-03	-4,76E-04	-1,01E-02	-8,73E-03	-15,7		1,01E-02	15,7		Global	-140,14
I4-6	3	30	1	2	abs.displac.	m	0	-1,16E-03	-1,08E-03	5,96E-04	-1,65E-03	-1,56E-03	-5,9		1,65E-03	5,9		%error	-12,1
I4-6	3	30	1	2	supp.react.	KN	0	344,42	323,96	119,3	787,68	755,81	-4,2		787,7	4,2			

Simply-supported one-span bridges

case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
J1-3	3	30	5	1	**rail stress**	MPa	-126	**-9,91**	**-11,55**	**-1,22**	**-22,68**	**-26,98**	15,9		22,7	-15,9		**Sum**	-148,68
J1-3	3	30	5	1	**rel.displac.**	m	0	**-6,51E-03**	**2,03E-03**	**-2,23E-04**	**-4,71E-03**	**-6,84E-03**	31,1		6,51E-03	-4,8		**Global**	-152,98
J1-3	3	30	5	1	**abs.displac.**	m	0	**-3,05E-03**	**2,00E-03**	**1,69E-04**	**-8,77E-04**	**-3,05E-03**	71,2		3,05E-03	0,0		**%error**	2,8
J1-3	3	30	5	1	**supp.react.**	KN	0	**182,58**	**-120,28**	**49,06**	**111,36**	**182,58**	39,0		182,6	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
J4-6	3	30	5	2	**rail stress**	MPa	-126	**-9,91**	**-12,37**	**-1,35**	**-23,63**	**-13,12**	-80,1		23,6	80,1		**Sum**	-149,63
J4-6	3	30	5	2	**rel.displac.**	m	0	**-6,51E-03**	**-2,02E-03**	**-3,02E-04**	**-8,84E-03**	**-6,87E-03**	-28,6		8,84E-03	28,6		**Global**	-139,12
J4-6	3	30	5	2	**abs.displac.**	m	0	**-3,05E-03**	**-2,01E-03**	**2,56E-04**	**-4,81E-03**	**-4,71E-03**	-2,0		4,81E-03	2,0		**%error**	-7,6
J4-6	3	30	5	2	**supp.react.**	KN	0	**182,58**	**120,72**	**44,45**	**347,75**	**341,8**	-1,7		347,8	1,7			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
K1-3	3	45	1	1	**rail stress**	MPa	-126	**-22,95**	**-14,96**	**-6**	**-43,91**	**-39,69**	-10,6		43,9	10,6		**Sum**	-169,91
K1-3	3	45	1	1	**rel.displac.**	m	0	**-1,23E-02**	**1,79E-03**	**-7,52E-04**	**-1,13E-02**	**-1,32E-02**	14,4		1,23E-02	-6,5		**Global**	-165,69
K1-3	3	45	1	1	**abs.displac.**	m	0	**-1,25E-03**	**1,24E-03**	**8,97E-04**	**8,87E-04**	**-1,25E-03**	28,8		1,25E-03	0,0		**%error**	-2,5
K1-3	3	45	1	1	**supp.react.**	KN	0	**547,49**	**-556,7**	**179,12**	**169,91**	**547,49**	69,0		547,5	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
K4-6	3	45	1	2	**rail stress**	MPa	-126	**-22,95**	**-14,72**	**-10,32**	**-47,99**	**-22,95**	-109,1		48,0	109,1		**Sum**	-173,99
K4-6	3	45	1	2	**rel.displac.**	m	0	**-1,23E-02**	**-1,76E-03**	**-6,39E-04**	**-1,47E-02**	**-1,35E-02**	-9,2		1,47E-02	9,2		**Global**	-148,95
K4-6	3	45	1	2	**abs.displac.**	m	0	**-1,25E-03**	**-1,24E-03**	**8,16E-04**	**-1,67E-03**	**-1,57E-03**	-6,3		1,67E-03	6,3		**%error**	-16,8
K4-6	3	45	1	2	**supp.react.**	KN	0	**547,49**	**556,48**	**214,71**	**1318,68**	**1264**	-4,3		1318,7	4,3			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
L1-3	3	45	5	1	**rail stress**	MPa	-126	**-17,88**	**-21,79**	**-3,52**	**-43,19**	**-38,78**	-11,4		43,2	11,4		**Sum**	-169,19
L1-3	3	45	5	1	**rel.displac.**	m	0	**-1,03E-02**	**2,44E-03**	**-5,44E-04**	**-8,41E-03**	**-1,17E-02**	28,0		1,03E-02	-11,8		**Global**	-164,78
L1-3	3	45	5	1	**abs.displac.**	m	0	**-3,70E-03**	**2,61E-03**	**4,56E-04**	**-6,37E-04**	**-3,73E-03**	82,9		3,70E-03	-0,8		**%error**	-2,7
L1-3	3	45	5	1	**supp.react.**	KN	0	**334,11**	**-234,88**	**75,99**	**175,22**	**334,11**	47,6		334,1	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
L4-6	3	45	5	2	**rail stress**	MPa	-126	**-17,88**	**-21,65**	**-5,37**	**-44,9**	**-17,88**	-151,1		44,9	151,1		**Sum**	-170,9
L4-6	3	45	5	2	**rel.displac.**	m	0	**-1,03E-02**	**-2,42E-03**	**-3,92E-04**	**-1,31E-02**	**-1,09E-02**	-20,7		1,31E-02	20,7		**Global**	-143,88
L4-6	3	45	5	2	**abs.displac.**	m	0	**-3,70E-03**	**-2,62E-03**	**2,83E-04**	**-6,04E-03**	**-5,98E-03**	-1,1		6,04E-03	1,1		**%error**	-18,8
L4-6	3	45	5	2	**supp.react.**	KN	0	**334,11**	**236,06**	**91,52**	**661,69**	**652,36**	-1,4		661,7	1,4			

Simply-supported one-span bridges

case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
M1-3	3	60	1	1	**rail stress**	MPa	-126	**-31,48**	**-16,45**	**-10,91**	**-58,84**	**-42,44**	-38,6		58,8	38,6		**Sum**	-184,84
M1-3	3	60	1	1	**rel.displac.**	m	0	**-1,66E-02**	**1,90E-03**	**-1,37E-03**	**-1,60E-02**	**-1,81E-02**	11,5		1,66E-02	-8,6		**Global**	-168,44
M1-3	3	60	1	1	**abs.displac.**	m	0	**-1,30E-03**	**1,35E-03**	**1,34E-03**	**1,39E-03**	**1,68E-03**	17,0		1,39E-03	-17,0		**%error**	-9,7
M1-3	3	60	1	1	**supp.react.**	KN	0	**742,17**	**-812,33**	**238,92**	**168,76**	**742,17**	77,3		742,2	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
M4-6	3	60	1	2	**rail stress**	MPa	-126	**-31,48**	**-15,96**	**-18,8**	**-66,24**	**-30,44**	-117,6		66,2	117,6		**Sum**	-192,24
M4-6	3	60	1	2	**rel.displac.**	m	0	**-1,66E-02**	**-1,85E-03**	**-1,13E-03**	**-1,95E-02**	**-1,82E-02**	-7,6		1,95E-02	7,6		**Global**	-156,44
M4-6	3	60	1	2	**abs.displac.**	m	0	**-1,30E-03**	**-1,35E-03**	**1,16E-03**	**-1,49E-03**	**-1,41E-03**	-5,8		1,49E-03	5,8		**%error**	-22,9
M4-6	3	60	1	2	**supp.react.**	KN	0	**742,17**	**810,78**	**338,35**	**1891,3**	**1816,5**	-4,1		1891,3	4,1			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
N1-3	3	60	5	1	**rail stress**	MPa	-126	**-26,31**	**-21,36**	**-7,89**	**-55,56**	**-49,4**	-12,5		55,6	12,5		**Sum**	-181,56
N1-3	3	60	5	1	**rel.displac.**	m	0	**-1,43E-02**	**2,78E-03**	**-1,13E-03**	**-1,26E-02**	**-1,67E-02**	24,6		1,43E-02	-14,8		**Global**	-175,4
N1-3	3	60	5	1	**abs.displac.**	m	0	**-4,19E-03**	**3,18E-03**	**8,06E-04**	**-2,12E-04**	**-4,19E-03**	94,9		4,19E-03	0,0		**%error**	-3,5
N1-3	3	60	5	1	**supp.react.**	KN	0	**497,35**	**-381,04**	**112,22**	**228,53**	**497,35**	54,1		497,4	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
N4-6	3	60	5	2	**rail stress**	MPa	-126	**-26,31**	**-25,59**	**-11,66**	**-63,56**	**-26,31**	-141,6		63,6	141,6		**Sum**	-189,56
N4-6	3	60	5	2	**rel.displac.**	m	0	**-1,43E-02**	**-2,75E-03**	**-7,85E-04**	**-1,78E-02**	**-1,50E-02**	-18,9		1,78E-02	18,9		**Global**	-152,31
N4-6	3	60	5	2	**abs.displac.**	m	0	**-4,19E-03**	**-3,19E-03**	**3,99E-04**	**-6,98E-03**	**-6,92E-03**	-1,0		6,98E-03	1,0		**%error**	-24,5
N4-6	3	60	5	2	**supp.react.**	KN	0	**497,35**	**382,92**	**160,43**	**1040,7**	**1030,7**	-1,0		1040,7	1,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
O1-3	3	90	1	1	**rail stress**	MPa	-126	**-47,52**	**-17,01**	**-22,31**	**-86,84**	**-65,56**	-32,5		86,8	32,5		**Sum**	-212,84
O1-3	3	90	1	1	**rel.displac.**	m	0	**-2,47E-02**	**1,92E-03**	**-2,51E-03**	**-2,53E-02**	**-2,79E-02**	9,2		2,53E-02	-9,2		**Global**	-191,56
O1-3	3	90	1	1	**abs.displac.**	m	0	**-1,19E-03**	**1,31E-03**	**2,38E-03**	**2,49E-03**	**2,94E-03**	15,3		2,49E-03	-15,3		**%error**	-11,1
O1-3	3	90	1	1	**supp.react.**	KN	0	**1108,7**	**-1413,5**	**389,82**	**85,02**	**1108,7**	92,3		1108,7	0,0			
case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error		mx(sm,T°)	%Error		total stress	
O4-6	3	90	1	2	**rail stress**	MPa	-126	**-47,52**	**-16,14**	**-38,68**	**-102,34**	**-59,4**	-72,3		102,3	72,3		**Sum**	-228,34
O4-6	3	90	1	2	**rel.displac.**	m	0	**-2,47E-02**	**-1,84E-03**	**-2,10E-03**	**-2,86E-02**	**-2,75E-02**	-4,0		2,86E-02	4,0		**Global**	-185,4
O4-6	3	90	1	2	**abs.displac.**	m	0	**-1,19E-03**	**-1,30E-03**	**2,07E-03**	**-4,32E-04**	**-1,23E-03**	64,9		1,19E-03	-3,1		**%error**	-23,2
O4-6	3	90	1	2	**supp.react.**	KN	0	**1108,7**	**1407,9**	**631,14**	**3147,74**	**2990,2**	-5,3		3147,7	5,3			

Simply-supported one-span bridges

case No.	deck type	span	long.dis.	direction	Quantity	Unit	T° rail	T° deck	Braking	End rot.	Sum	Global effect	%Error	mx(sm,T°)	%Error	total stress	
P1-3	3	90	5	1	**rail stress**	MPa	-126	**-43,14**	**-30,16**	**-18,76**	**-92,06**	**-67,52**	-36,3	92,1	36,3	**Sum**	-218,06
P1-3	3	90	5	1	**rel.displac.**	m	0	**-2,25E-02**	**3,19E-03**	**-2,16E-03**	**-2,15E-02**	**-2,69E-02**	20,0	2,25E-02	-16,2	**Global**	-193,52
P1-3	3	90	5	1	**abs.displac.**	m	0	**-4,13E-03**	**3,84E-03**	**1,64E-03**	**1,35E-03**	**-4,13E-03**	67,3	4,13E-03	0,0	**%error**	-12,7
P1-3	3	90	5	1	**supp.react.**	KN	0	**860,3**	**-829,67**	**228,7**	**259,33**	**860,3**	69,9	860,3	0,0		
case No.	**deck type**	**span**	**long.dis.**	**direction**	**Quantity**	**Unit**	**T° rail**	**T° deck**	**Braking**	**End rot.**	**Sum**	**Global effect**	**%Error**	**mx(sm,T°)**	**%Error**	**total stress**	
P4-6	3	90	5	2	**rail stress**	MPa	-126	**-43,14**	**-29,6**	**-28,3**	**-101,04**	**-43,14**	-134,2	101,0	134,2	**Sum**	-227,04
P4-6	3	90	5	2	**rel.displac.**	m	0	**-2,25E-02**	**-3,12E-03**	**-1,60E-03**	**-2,72E-02**	**-2,37E-02**	-15,1	2,72E-02	15,1	**Global**	-169,14
P4-6	3	90	5	2	**abs.displac.**	m	0	**-4,13E-03**	**-3,85E-03**	**9,63E-04**	**-7,02E-03**	**-6,97E-03**	-0,6	7,02E-03	0,6	**%error**	-34,2
P4-6	3	90	5	2	**supp.react.**	KN	0	**860,3**	**831,06**	**371**	**2062,36**	**2040**	-1,1	2062,4	1,1		

Simply-supported one-span bridges

List of abbreviations

A_{60}: Combined cross section of two UIC 60 rails

E: Young's modulus

F_{rails}: Force in the rails

$F_{support}$: Support reaction (kN)

H: Distance between the upper side of the deck slab, on which the track lies, and the centre of rotation of the fixed support

I: Moment of inertia of the deck

K: Support stiffness

K_2: Support stiffness $K_{support} = 2 \times L$ [m] [kN/mm]

K_5: Support stiffness $K_{support} = 5 \times L$ [m] [kN/mm]

K_{20}: Support stiffness $K_{support} = 20 \times L$ [m] [kN/mm]

L: Expansion length of the deck [m]

Q_L: Total horizontal load on the deck due to braking or acceleration

f: Friction coefficient

k: Longitudinal resistance of the track to the displacement

k_{20}: Longitudinal resistance of the track $k_{rail} = 20$ [kN/m track]

k_{40}: Longitudinal resistance of the track $k_{rail} = 40$ [kN/m track]

k_{60}: Longitudinal resistance of the track $k_{rail} = 60$ [kN/m track]

q_{lak}: Characteristic horizontal acceleration forces

q_{lbk}: Characteristic horizontal braking forces

x: Distance of the neutral axis to the top or to the bottom of the deck

ΔT_{rail}: Temperature variation in the rail

ΔT_{deck}: Temperature variation in the deck

$\Delta\Delta T$: Difference between the temperature variations in the rail and in the deck

θ: Rotation of the cross section at the end of the deck

α:	Coefficient of thermal expansion
β:	Reduction coefficient for the horizontal forces on the rails
γ:	ω/H value used in calibrating diagrams
δ_{rail}:	Displacement of the rail
δ_{abs}:	Absolute displacement of the deck
δ_{rel}:	Relative displacement between the deck and the rail
δ_{p}:	Displacement at the head of the support due to elastic deformation
δ_{ϕ}:	Displacement at the head of the support due to rotation of the foundation
δ_{h}:	Displacement of the support due to the horizontal movement of the foundation
δ_{a}:	Relative displacement between the upper and lower parts of the bearing
$\delta(\theta H)$:	Displacement of the slab due to the rotation of the end section of the deck
σ_{rail}:	Rail stress
σ_{rail} **(fixed):**	Rail stress at the end near the fixed support [N/mm^2]
σ_{rail} **(movable):**	Rail stress at the end near the movable support [N/mm^2]
ω:	Distance between the upper surface of the deck slab and the deck neutral axis (positive if the neutral axis is below the upper surface)

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