2 A History of Railway Vehicle Dynamics

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CONTENTS

I. INTRODUCTION

The railway train running along a track is one of the most complicated dynamical systems in engineering. Many bodies comprise the system and so it has many degrees of freedom. The bodies that make up the vehicle can be connected in various ways and a moving interface connects the vehicle with the track. This interface involves the complex geometry of the wheel tread and the rail head and nonconservative frictional forces generated by relative motion in the contact area.

The technology of this complex system rests on a long history. In the late 18th and early 19th century, development concentrated on the prime mover and the possibility of traction using adhesion. Strength of materials presented a major problem. Even though speeds were low, dynamic loads applied to the track were of concern and so the earliest vehicles used elements of suspension adopted from horse carriage practice. Above all, the problem of guidance was resolved by the almost universal adoption of the flanged wheel in the early 19th century, the result of empirical development, and dependent on engineering intuition.

Operation of the early vehicles led to verbal descriptions of their dynamic behaviour, such as Stephenson's description of the kinematic oscillation, discussed below. Later in the 19th century the first simple mathematical models of the action of the coned wheelset were introduced by Redtenbacher and Klingel, but they had virtually no impact on engineering practice. Actually, the balancing of the reciprocating masses of the steam locomotive assumed much greater importance.

A catastrophic bridge failure led to the first analytical model in 1849 of the interaction between vehicle and flexible track.

The growing size of the steam locomotive increased the problem of the forces generated in negotiating curves, and in 1883 Mackenzie gave the first essentially correct description of curving. This became the basis of a standard calculation carried out in design offices throughout the era of the steam locomotive.

As train speeds increased, problems of ride quality, particularly in the lateral direction, became more important. The introduction of the electric locomotive at the end of the 19th century involved Carter, a mathematical electrical engineer, in the problem, with the result that a realistic model of the forces acting between wheel and rail was proposed and the first calculations of lateral stability carried out.

Generally, empirical engineering development was able to keep abreast of the requirements of ride quality and safety until the middle of the 20th century. Then, increasing speeds of trains and the greater potential risks arising from instability stimulated a more scientific approach to vehicle dynamics. Realistic calculations, supported by experiment, on which design decisions were based were achieved in the 1960s and as the power of the digital computer increased so did the scope of engineering calculations, leading to today's powerful modelling tools.

This chapter tells the story of this conceptual and analytical development. It concentrates on the most basic problems associated with stability, response to track geometry, and behaviour in curves of the railway vehicle and most attention is given to the formative stage in which an understanding was gained. Progress in the last 20 years is only sketchily discussed, as the salient points are considered later in the relevant chapters. Moreover, many important aspects such as track dynamics, noise generation, and other high frequency (in this context, above about 15 Hz) phenomena are excluded.

II. CONING AND THE KINEMATIC OSCILLATION

The conventional railway wheelset, which consists of two wheels mounted on a common axle, has a long history[1] and evolved empirically. In the early days of the railways, speeds were low, and the objectives were the reduction of rolling resistance (so that the useful load that could be hauled by horses could be multiplied) and solving problems of strength and wear.

The flanged wheel running on a rail existed as early as the 17th century. The position of the flanges was on the inside, outside, or even on both sides of the wheels, and was still being debated in the 1820s. Wheels were normally fixed to the axle, although freely rotating wheels were sometimes used in order to reduce friction in curves. To start with, the play allowed between wheel flange and rail was minimal.

Coning was introduced partly to reduce the rubbing of the flange on the rail, and partly to ease the motion of the vehicle around curves. It is not known when coning of the wheel tread was first introduced. It would be natural to provide a smooth curve uniting the flange with the wheel tread, and wear of the tread would contribute to this. Moreover, once wheels were made of cast iron, taper was normal foundry practice. In the early 1830s the flangeway clearance was opened up to reduce the lateral forces between wheel and rail so that, typically, in current practice about 7 to 10 mm of lateral displacement is allowed before flange contact.

Coning of the wheel tread was well-established by 1821. George Stephenson in his "Observations on Edge and Tram Railways"[2] stated that:

> It must be understood the form of edge railway wheels are conical that is the outer is rather less than the inner diameter about 3/16 of an inch. Then from a small irregularity of the railway the wheels may be thrown a little to the right or a little to the left, when the former happens the right wheel will expose a larger and the left one a smaller diameter to the bearing surface of the rail which will cause the latter to lose ground of the former but at the same time in moving forward it gradually exposes a greater diameter to the rail while the right one on the contrary is gradually exposing a lesser which will cause it to lose ground of the left one but will regain it on its progress as has been described alternately gaining and losing ground of each other which will cause the wheels to proceed in an oscillatory but easy motion on the rails.

This is a very clear description of what is now called the kinematic oscillation, as shown in Figure 2.1.

The rolling behaviour of the wheelset suggests why it adopted its present form. If the flange is on the inside the conicity is positive and as the flange approaches the rail there will be a strong steering action tending to return the wheelset to the centre of the track. If the flange is on the outside the conicity is negative and the wheelset will simply run into the flange and remain in contact as the wheelset moves along the track. Moreover, consider motion in a sharp curve in which the wheelset is in flange contact. If the flange is on the inside, the lateral force applied by the rail to the leading wheelset is applied to the outer wheel and will be combined with an enhanced vertical load thus diminishing the risk of derailment. If the flange is on the outside, the lateral force applied by the rail is applied to the inner wheel, which has a reduced vertical load, and thus the risk of derailment is increased.

As was explicitly stated by Brunel in 1838 (see Vaughan[3]) it can be seen that for small displacements from the centre of straight or slightly curved track the primary mode of guidance is conicity and it is on sharper curves, switches, and crossings that the flanges become the essential mode of guidance.

Lateral oscillations caused by coning were experienced from the early days of the railways. One solution to the oscillation problem that has been proposed from time to time, even down to modern times, was to fit wheels with cylindrical treads. However, in this case, if the wheels are rigidly mounted on the axle, very slight errors in parallelism would induce large lateral displacements that would be limited by flange contact. Thus, a wheelset with cylindrical treads tends to run in continuous flange contact.

In 1883 Klingel gave the first mathematical analysis of the kinematic oscillation[4] and derived the relationship between the wavelength Λ and the wheelset conicity λ, wheel radius r_0, and the lateral distance between contact points $2l$ as

$$\Lambda = 2\pi(r_0 l/\lambda)^{1/2} \tag{2.1}$$

Klingel's formula shows that as the speed is increased, so will the frequency of the kinematic oscillation. Any further aspects of the dynamical behaviour of railway vehicles must be deduced

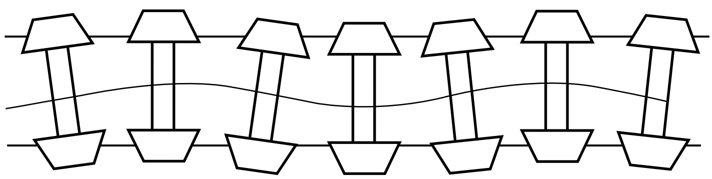


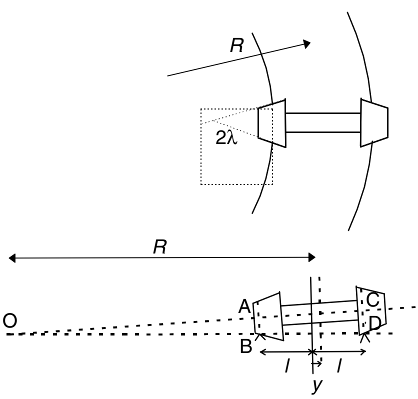
FIGURE 2.1 The kinematic oscillation of a wheelset.

from a consideration of the forces acting, and this had to wait for Carter's much later contribution to the subject.

III. CONCEPTS OF CURVING

The action of a wheelset with coned wheels in a curve was understood intuitively early in the development of the railways. For example, in 1829 Ross Winans took out a patent that stressed the importance of the axles taking up a radial position on curves,[5] a fundamental objective of running gear designers ever since, and Adams clearly understood the limitations of coning in curves.[6] Redtenbacher[7] provided the first theoretical analysis in 1855 and this is illustrated in Figure 2.2.

From the geometry in this figure it can be seen that there is a simple geometric relationship between the outwards movement of the wheel y, the radius of the curve R, the wheel radius r_0, the distance between the contact points $2l$ and the the conicity λ of the wheels in order to sustain pure rolling. The application of Redtenbacher's formula shows that a wheelset will only be able to move outwards to achieve pure rolling if either the radius of curvature or the flangeway clearance is sufficiently large. Otherwise, a realistic consideration of curving requires the analysis of the forces acting between the vehicle and the track. In 1883, Mackenzie[8] supplied the first essentially correct description of curving in a seminal paper (which was subsequently translated and published in both France and Germany). His work was suggested by an unintentional experiment, in which the springs of the driving wheels of a six-wheeled engine were tightened to increase the available adhesion. The leading wheel mounted the rail when the locomotive approached a curve. Mackenzie provided a numerical but nonmathematical treatment of the forces generated in curving. His discussion is based on sliding friction, neglects coning, so that it is appropriate for sharp curves, where guidance is provided by the flanges. Referring to Figure 2.3, Mackenzie explains: If the flange were removed from the outer wheel, the engine would run straight forwards, and this wheel, in making one revolution, would run from A to B; but it is compelled by the flange to move in the direction of the line AC, a tangent to the curve at A, so that it slides sideways through a distance equal to BC. If this wheel were loose on the axle, it would, in making a revolution, run along the rail to F; but the inner wheel, in making a revolution, would run from H to K, the centreline of the axle being KG; so that, if both wheels are keyed on the axle, either the outer wheel must slide forwards or the inner wheel backwards. Assuming that the engine is exerting no tractive force, and that both wheels revolve at the speed due to the inner wheel, then the outer wheel will slide forwards from F to G. Take AL equal to BC, and LM equal to FG, the diagonal AM is the distance which the outer wheel slides in making one revolution.



OAB = OCD

$$(r_0 - \lambda y)/(R - l) = (r_0 + \lambda y)/(R + l)$$

$$y = r_0 l / R\lambda$$

FIGURE 2.2 Redtenbacher's formula for the rolling of a coned wheelset on a curve.

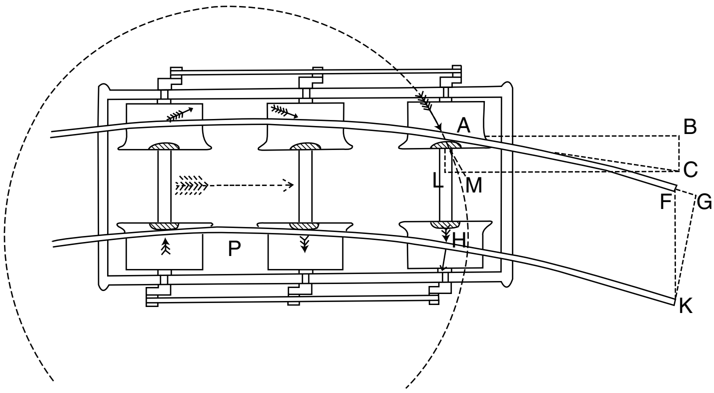


FIGURE 2.3 Forces acting on a vehicle in a curve according to Mackenzie.[8]

He then applies similar reasoning to the other wheels, assuming various positions for the wheelsets in relation to the rails. Thus, Mackenzie's calculations showed that the outer wheel flange exerts against the rail a force sufficient to overcome the friction of the wheel treads. Previously, centrifugal forces were regarded as the cause of many derailments. He also made the comment that "the vehicle seems to travel in the direction which causes the smallest amount of sliding," which foresaw a later analytical technique developed by Heumann.

Subsequent work by Boedecker, von Helmoltz, and Uebelacker (described by Gilchrist[9]) was dominated by the need to avoid excessive loads on both vehicle and track caused by steam locomotives with long rigid wheelbases traversing sharp curves. Hence, in these theories, the conicity of the wheelsets is ignored and the wheels are assumed to be in the sliding regime. The corresponding forces are then balanced by a resultant flange force or flange forces. This approach culminated in the work of Heumann in 1913,[10] and Porter in 1934 to 1935.[11]

Superelevation of tracks in curves was introduced on the Liverpool and Manchester Railway, and in the 1830s tables giving the relationship between superelevation of the outer rail and maximum speed were available.

IV. HUNTING AND THE EMPIRICAL DEVELOPMENT OF THE BOGIE

The inception of service on the Liverpool and Manchester Railway meant that, for the first time, railway vehicles operated at speeds at which dynamic effects became apparent. The coaches had a very short wheelbase and were reputed to hunt violently at any speed. One measure employed to control this was to close couple the vehicles. The instability of two-axle vehicles was an accepted and often unremarked occurrence throughout their employment on the railways. In the early days of the railways, it had become customary to link together two- and three-axle vehicles not only by couplings but also by side chains to provide yaw restraint between adjacent car bodies in order to stabilise lateral motions.

Two-axle vehicles, and other vehicles with a rigid wheelbase, had obvious limitations in curves. The first known proposal for the bogie was made by William Chapman in 1797, although it was in the United States that the concept was first exploited. As the performance of the rigid wheelbase British locomotives on the lightly built and curvaceous American track was very unsatisfactory, a locomotive with a leading swivelling bogie was introduced in 1832 and this radically improved both stability and curving behaviour. Similarly, the bogie passenger coach became general in North America in the 1840s. These early bogies had very short wheelbases, were free to swivel without restraint and tended to oscillate violently, which was the probable cause of many derailments. In the 1850s the bogie wheelbase was increased, thus improving stability significantly. In Britain, engineers were cautious about the use of the bogie. Fernihough pointed out the danger of bogie oscillation in his evidence before the Gauge Commission in 1845. He also suggested that it might be controlled by the frictional resistance of a bearing ring of large diameter and this (or, equivalently, friction at the side bearers) became established practice from the 1850s. This pragmatic measure enabled the bogies to follow sharp curves at low speeds while at the same time preventing bogie hunting on straight track.

It was appreciated that the function of the secondary suspension, connecting the bogie frame to the car body, was to isolate the car body from motions of the bogie, for the swing bolster was invented by Davenport in 1841. At a later stage, locomotives often incorporated lateral movement of the bogie pivot restrained by some form of spring, called a centring spring. Experience showed that excessive flexibility of the mounting of the axles gave rise to hunting, and consequently, axles were usually stiffly mounted in bogie frames.

V. INTERACTION BETWEEN VEHICLE AND TRACK

Although it was thought by some early engineers that the track would be so smooth that no vertical suspension would be necessary, experience soon showed that this was not so. George Stephenson built several locomotives with steam springs as early as 1816, but his first locomotive with steel springs was the Lancashire Witch of 1828. At this stage the suspension of locomotives was provided in order to reduce stresses on the track. Railway carriages followed road practice where laminated steel (leaf) springs had, from about 1770, become normal practice, replacing suspension by leather straps.

A major concern in early locomotive development was the ability to negotiate irregular track, and to maintain contact of all the wheels with the track. A system of equalisation, in which leaf springs connected to levers all attached to the locomotive frame distributed the vertical forces among the wheels, was first used by Timothy Hackworth on his Royal George. However, the most pressing need was, again, in the United States where track was much rougher than in Great Britain, and this resulted in patents covering equalization systems by Eastwick (in 1837) and Harrison (in 1838 and 1842). Such systems became a common feature of the vertical suspension on both locomotives and carriages in the United States.

Another source of vertical loading on the track was the impact caused by lack of balance of the revolving and reciprocating parts of the locomotive, which on occasion caused severe track damage. Initially, only the lack of balance of the revolving parts was corrected by balance weights attached to the rims of the driving wheels, and Fernihough appears to be the first to use weights heavy enough to approach complete balance. The theory of balancing was published by Le Chatelier in 1849, who laid down the rules practiced by Clark in his treatise.[12]

Generally, specific problems of interaction between vehicle and track were addressed by empirical and inventive measures. An exception to this was caused by the collapse of Stephenson's bridge across the River Dee at Chester in 1847. At that time, little was known about the dynamic effects of moving loads on bridges. In order to support the inquiry into the accident, a series of experiments was carried out by Willis on a dynamic test rig at Portsmouth dockyard. This was followed by further model tests at Cambridge, and in 1849, G.G. Stokes gave the first analysis of the travelling load problem, albeit with severe simplifying assumptions. This was the beginning of a long history of such investigations.[13]

VI. INNOVATIONS FOR IMPROVED STEERING

There is a long history of inventions that have attempted to ensure that wheelsets are steered so that they adopt a more or less radial position on curves, and many engineers have tried to improve curving performance by making the vehicle more flexible in plan view. Probably one of the first of

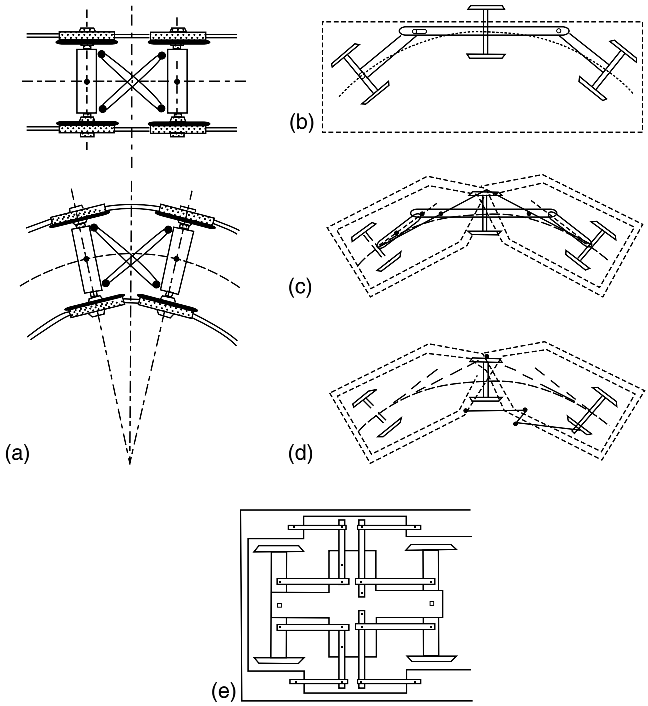


FIGURE 2.4 Innovations for improved steering: (a) direct connections between wheels by cross-bracing; (b) three-axle vehicle; (c) articulation with a steering beam; (d) articulation with linkage steering driven by angle between adjacent car bodies; (e) bogie with steered wheelsets driven by angle between bogie frame and car body.

these was the arrangement for the Linz-Budweis railway, in which the wheelsets were directly connected by cross-bracing (Figure 2.4a, 1827).

The first articulated locomotive was designed by Horatio Allen in 1832. Although it had a short career, it probably stimulated several of the articulated designs for the Semmering contest in 1851. Thereafter, there was a succession of articulated locomotives, the development of which is described by Weiner.[14] The conflict between the length of the locomotive made necessary by high power and the large curvature of many railway lines was resolved by providing several articulated sections thus reducing the effective wheelbase. The calculation methods described in Section III were applied as these became available.

Various forms of three-axle vehicle have been used widely in the past. In most of these designs the wheelsets were connected to the car body by a conventional suspension similar to that used in two-axle vehicles. Negotiation of curved track was catered for by allowing greater flexibility or clearances for the central wheelset. According to Liechty[15] a three-axle vehicle, in which the lateral displacement of the central axle steered the outer axles through a linkage, was tried out in 1826 on the Linz-Budweis railway. It was argued that three axles, connected by suitable linkages, would assume a radial position on curves and then realign themselves correctly on straight track. Other examples of inventions in which wheelsets are connected so as to achieve radial steering are the three-axle vehicles of Germain (1837), Themor (1844), and Fidler (1868), an example of which is shown in Figure 2.4b. In these schemes the outer wheelsets were pivoted to the car body. More refined arrangements, due to Robinson (1889) and Faye (1898), were much used in trams. More details of these configurations and many others can be found in Ref. 16.

In 1837, as an alternative to the use of the bogie, W.B. Adams proposed an articulated two-axle carriage. Adams invented a form of radial axle in 1863, which had no controlling force, with the result that on straight track there was considerable lateral oscillation of the axle. The idea of a controlling force was suggested by Phipps and this was subsequently applied by Webb (see Ahrons[17]). This was an early example of the fundamental conflict between stability and curving. In the case of locomotives, in 1859, Bissel moved the swivel pin behind the bogie centre in order to allow the wheels to take up a more radial attitude in curves. This idea was adopted quite widely when applied to two-wheel trucks although it was found that a centring spring was required to maintain stability at speed.[18]

Another form of steering exploited the angle between the bogie and the car body in order to steer the wheelsets relative to the bogie frame using a linkage (Figure 2.4e). A similar objective was achieved by mounting the outer wheelset on an arm pivoted on the car body and actuated by a steering beam (Figure 2.4c). An alternative approach was to steer the wheelsets using the angle between adjacent car bodies (Figure 2.4d).

All these developments were based on very simple ideas about the mechanics of vehicles in curves, and depended on systems of rigid linkages and pivots. Not surprisingly, in the light of modern knowledge, there is considerable evidence that when such schemes were built they exhibited an even wider spectrum of various hunting instabilities than the more conventional mainstream designs. This is probably why so few of these inventions achieved widespread adoption.

VII. CARTER

The configuration of the steam locomotive, originally evolved by Stephenson, was the result of the layout necessary for the boiler, cylinders, and drive to the wheels. This, combined with the guiding bogie at the front of the vehicle, provided a configuration which was unsymmetric fore-and-aft. This configuration was the norm for the steam locomotive intended for main-line operation throughout its history, and which, even if it had riding problems, was usually safe. On the other hand, symmetric configurations were used, but only at low speeds, as at higher speeds they were subject to riding problems, lateral oscillation, and sometimes, derailment. Experience had therefore shown that symmetric configurations were best avoided. This seems to have been forgotten when the first electric locomotives were designed, presumably because they evolved from trams and electric multiple units rather than steam locomotives, and the operational advantages of a symmetric configuration looked attractive. As a result the introduction of the symmetric electric locomotive had been accompanied by many occurrences of lateral instability at high speed, and consequently large lateral forces between vehicle and track. This was how Carter became involved in the problem.

Until then, railway engineering and theory had followed separate paths. The achievements of railway engineers, in the field of running gear at least, largely rested on empirical development and acute mechanical insight. Mackenzie's work in understanding the forces acting on a vehicle in a curve represents an excellent example. It is perhaps not surprising that the seminal development in railway vehicle dynamics was made not by a mechanical engineer but by an electrical engineer who had been exposed to the new analytical techniques necessary to further the application of electrification.

Carter (1870–1952) read mathematics at Cambridge, and after a four-year spell as a lecturer he decided to make electrical engineering his career and spent the following 3 years with General Electric at Schenectady, where he was employed in the testing department working on electric traction. He then returned to England and spent the rest of his career with British Thompson Houston (a company affiliated with General Electric) at Rugby. For most of his career he was consulting engineer to this company, dealing with problems that were beyond the ordinary engineering mathematics of the day. With his mathematical ability and working at the leading edge of railway electric traction he was able to bridge the gap between science, theory, and railway engineering.[19] After making many significant contributions to electric traction, Carter turned to the mechanical engineering problems of locomotives. The first realistic model of the lateral dynamics of a railway vehicle was that presented by Carter.[20] In this model, Carter introduced the fundamental concept of creep and included the effect of conicity. The paper showed that the combined effects of creep and conicity could lead to a dynamic instability.

Carter stated that the forces acting between wheels and rails can be assumed to be proportional to the creepages, without reference or derivation in this 1916 paper. The concept of creep had first been described by Osborne Reynolds in relation to the transmission of power by belts or straps, and he noted that the concept was equally applicable to rolling wheels.[21]

It was Carter's introduction of the creep mechanism into the theory of lateral dynamics that was the crucial step in identifying the cause of "hunting."

Carter derived equations of motion for the rigid bogie in which two wheelsets were connected by means of a stiff frame. They consist of the two coupled second-order linear differential equations in the variables lateral displacement y and yaw angle ψ of the bogie and they are equivalent to

$$\begin{aligned} m\ddot{y} + 4f(\dot{y}/V - \psi) &= Y \\ 4f\lambda l y/r_0 + I\ddot{\psi} + 4f(l^2 + h^2)\dot{\psi}/V &= G \end{aligned} \tag{2.2}$$

where m and I are the mass and yaw moment of inertia of the bogie, f is the creep coefficient (the creep force per unit creep), h is the semiwheelbase of the bogie and V is the forward speed. It can be seen that lateral displacements of the wheelset generate longitudinal creep. The corresponding creep forces are equivalent to a couple that is proportional to the difference in rolling radii or conicity, and which tends to steer the wheelset back into the centre of the track. This is the basic guidance mechanism of the wheelset. In addition, when the wheelset is yawed, a lateral creep force is generated. In effect, this coupling between the lateral displacement and yaw of the wheelset represents a form of feedback, and the achievement of guidance brings with it the possibility of instability. Klingel's solution for pure rolling follows from these equations as a special case in which the wheelset is unrestrained and rolling at low speed.

The theory of dynamic stability had been developed during the 19th century by scientists and mathematicians.[22] The behaviour of governors was analysed by Airy in 1840 in connection with design of a telescope. Maxwell analysed the stability of Saturn's rings in 1856 and derived conditions of stability for governors in 1868. The most significant step forward was Routh's essay for the 1877 Adams Prize,[23] which derived comprehensive conditions for stability of a system in steady motion. Routh incorporated discussion of the stability conditions into the various editions of his textbook.[24] However, in England, mechanical engineers were not familiar with these developments. On the other hand, in 1894 in Switzerland, Stodola studied the stability of steam turbine control systems and encouraged Hurwitz to formulate conditions for stability[25] that are equivalent to Routh's criteria. Carter's work was one of the first engineering applications of Routh's work, and it is interesting to note that Bryan and Williams's[26] pioneering work on the stability of aeroplanes using similar methods had only been published a few years before. Significantly, another early application of stability theory was made by another electrical engineer, Bertram Hopkinson, in his analysis of the hunting of alternating machinery published in 1904.[27] All these early publications refer to Routh's textbook. In contrast, the stability of the bicycle, another system involving rolling wheels, was analysed by Whipple[28] using a solution of the equations of motion.

Moreover, it is interesting that, concurrently with Carter, aeronautical engineers were grappling with the dynamic instabilities of aircraft structures. The first flutter analysis was made in 1916 by Bairstow and Page[29] and the imperatives of aeronautical progress ensured the development of many of the techniques that were to be brought to bear on the "hunting" problem in the future. By 1927 Frazer and Duncan had laid firm foundations for flutter analysis[30] and a foundation for the application of matrices in engineering dynamics was established in the text,[31] which eventually was to find application in the railway field.

As Carter's interest was in stability he considered that the flangeway clearance was not taken up and he therefore applied Routh's stability theory, not only to electric bogie locomotives, but also to a variety of steam locomotives. In his mathematical models, a bogie consists of two wheelsets rigidly mounted in a frame, and locomotives comprise wheelsets rigidly mounted in one or more frames. Following Carter's first paper of 1916 the theory was elaborated in a chapter of his book.[32]

So far, Carter had used an approximation to give the value of the creep coefficient (the constant of proportionality between the creep force and the creepage). In 1926, Carter analysed the creep of a locomotive driving wheel by extending Hertz's theory of elastic contact, as presented by Love.[33] He considered the case of creep in the longitudinal direction, treating the wheel as a two-dimensional cylinder.[34] This not only provided an expression for the creep coefficient but described how the creep force saturated with increasing creepage. Hertz, the German physicist, had become interested in 1881 in the theory of compression of elastic bodies as a result of his work on optics. By making some realistic assumptions he was able to give a theoretical solution for the size of the contact area and the stresses in the two contacting bodies as a function of the normal load between the bodies. This work attracted not only the attention of physicists but of engineers who persuaded Hertz to prepare another version of his paper including experimental results.[35]

Carter's next paper[36] gave a comprehensive analysis of stability within the assumptions mentioned above. As he was concerned with locomotives the emphasis of his analyses was on the lack of fore-and-aft symmetry characteristic of the configurations he was dealing with, and he derived both specific results and design criteria.

His analysis of the 0–6–0 locomotive found that such locomotives were unstable at all speeds if completely symmetric and he comments that this class of locomotive is "much used in working freight trains; but is not employed for high speed running on account of the proclivities indicated in the previous discussion."

Carter analysed the 4–6–0 locomotive both in forward and reverse motion and found that in forward motion beyond the limits shown (i.e., for sufficiently high speed or sufficiently stiff bogie centring spring) the bogie tends to lash the rails; but being comparatively light and connected with the main mass of the locomotive, the impacts are unlikely to be a source of danger at ordinary speeds.

Two of Carter's stability diagrams, the first of their kind in the railway field, are shown in Figure 2.5. As the system considered has four degrees of freedom (lateral translation and yaw of mainframe and bogie), substitution of a trial exponential solution and expansion of the resulting characteristic equation leads to an eighth order polynomial. As Carter writes "expansion of the determinant is long … but not difficult." Carter examines stability in two ways: first, by extracting the roots of the polynomial and second by Routh's scheme of cross-multiplication [Ref. 24, p. 226].

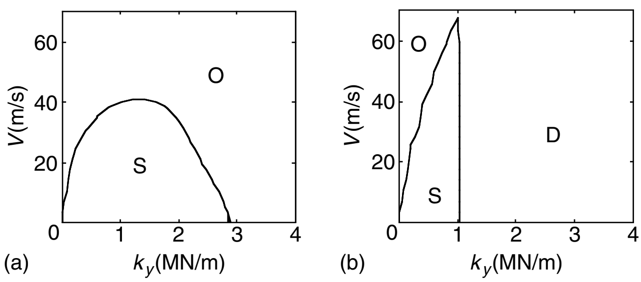


FIGURE 2.5 Carter's stability diagram for the 4–6–0 locomotive in: (a) forward motion and (b) reverse motion. k_y is the centring stiffness. (Recalculated in modern units from Ref. 36.) S = stable; O = oscillatory instability; D = divergence.

Either method involved tedious and lengthy calculations by hand, and tackling more complex cases "becomes, more appropriately, an office undertaking."

In reverse motion (Figure 2.5b), he found that beyond a certain value of the centring spring stiffness buckling of the wheelbase tending to cause derailment at a fore-wheel and moreover that the impacts of the flanges on the rail when the locomotive is running at speed are backed by the mass of the main frame and are accordingly liable to constitute a source of danger.

This was the explanation for a number of derailments at speed of tank engines such as the Lincoln to Tamworth mail train at Swinderby on June 6, 1928, as discussed in his final paper.[37]

Carter's analysis of the 2–8–0 with a leading Bissel similarly explained the need for a very strong aligning couple for stability at high speed, while noting that in reverse motion a trailing Bissel has a stabilising effect for a large and useful range of values of aligning couple.

Although Carter had exploited the stabilising influence of elastic elements in his analyses of unsymmetric locomotive configurations, his brief treatment of symmetric vehicles with two-axle bogies (by now a common configuration of passenger rolling stock) assumed that the bogies were pivoted to the car body. That this case received such brief mention is consistent with the fact that railway engineers had, by empirical development, achieved an acceptable standard of ride at the speeds then current. Moreover, as Carter says "the destructive effect of the instability is, however, limited on account of the comparatively small mass of the trucks."

Carter's work expressed, in scientific terms, what railway engineers had learnt by hard experience, that stability at speed required rigid-framed locomotives to be unsymmetric and unidirectional. A further practical result of his work was a series of design measures, the subject of various patents,[38] for the stabilisation of symmetric electric bogie locomotives.

The fact that the analyses, although only involving a few degrees of freedom, required heavy algebra and arithmetic, and involved techniques beyond the mechanical engineering training of the day, is perhaps one reason why Carter's work was not taken up much sooner.

Thus, while the theoretical foundations had been established, the need for vehicle dynamics was not, and practising railway engineers were largely sceptical of theory, particularly when the experimental basis was very limited. As a result, the next 20 years saw only a few significant contributions to the science of railway vehicle dynamics.

Rocard[39] employed the same form of equations of motion as Carter. In addition to covering much of the same ground as Carter, he considered the case of a massless bogie, which is connected by a lateral spring to the car body, and showed that the system could be stabilised. Rocard also considered the case of the unsymmetric bogie in which the wheelsets have different conicities. He found that the distribution of conicity can be arranged to give stability in one direction of motion, but not in both. Rocard states that a successful experiment was made by French National Railways in 1936.

There were also theoretical contributions by Langer and Shamberger[40] and Cain[41] that involved rather severe assumptions, but in general, papers concerned with bogie design published during this period were purely descriptive, reflecting the negligible role played by analysis in this branch of engineering practice. However, in 1939 Davies carried out significant model experiments of instability although stopping short of a complete analysis,[9] and pointed out the importance of worn wheel and rail profiles to wheelset dynamics.[42]

VIII. WHEEL–RAIL GEOMETRY

Carter assumed that the wheel treads were purely conical. In practice, it had been known from the earliest days of the railways that treads wear rapidly and assume a hollow form. It was also known that there was a connection between ride quality and the amount of wheel wear. An important further step in developing a realistic mathematical model was concerned with the treatment of actual wheel and rail profiles. While new wheel profiles were purely coned on the tread, usually to an angle of 1:20, in 1937 Heumann,[43] emphasised the importance of worn wheel and rail profiles to wheelset behaviour in curves. Heumann analysed the effect of the mutual wheel and rail geometry on the variation of the rolling radius as the wheelset is displaced laterally, and derived the formula for the effective conicity λ_0 of a wheel–rail combination for small displacements from the central running position, defined as the rate of change of the rolling radius with lateral displacement of the wheelset.

$$\lambda_0 = \delta_0 R_w/(R_w - R_r)(1 - r_0\delta_0/l) \tag{2.3}$$

where R_w and R_r are the wheel and rail radius of curvature and δ_0 is the slope of the tread at the contact point. Heumann's expression shows clearly that the effective conicity of a worn wheelset can be much greater than that of the corresponding purely coned wheelset. Moreover, Heumann suggested for the first time that profiles approximating to the fully worn should be used rather than the purely coned treads then standard. He argued that after reprofiling to a coned tread, tyre profiles tend to wear rapidly so that the running tread normally in contact with the rail head is worn to a uniform profile. This profile then tends to remain stable during further use, and is largely independent of the original profile and of the tyre steel. Similarly, rail head profiles are developed which also tend to remain stable after the initial period of wear is over. Heumann therefore suggested that vehicles should be designed so as to operate with these naturally worn profiles, as it is only with these profiles that any long-term stability of the wheel–rail geometrical parameters occurs. Moreover, a considerable reduction in the amount of wear would be possible by providing new rails and wheels with an approximation to worn profiles at the outset. Modern wheel and rail profiles are largely based on this concept.

IX. MATSUDAIRA

Tadashi Matsudaira studied marine engineering at the University of Tokyo and then joined the aircraft development department of the Japanese Imperial Navy where he was concerned with the vibration of aeroplanes. After the end of World War II, he moved to the Railway Technical Research Institute of Japanese National Railways to work on railway vehicle dynamics. During the years 1946 to 1957, Japanese National Railways were attempting to increase the speed of freight trains. The short wheelbase two-axle wagons then in use experienced hunting at low speeds and a high rate of derailment. Matsudaira introduced his experience of the flutter problem in aeroplanes (such as the Japanese Imperial Navy's "Zero" fighter), then using both analysis and scale model experiments on roller rigs, he showed that the hunting problem is one of self-excited vibration and not arising from external factors such as uneven rail geometry. This was the beginning of roller rig testing for vehicle dynamics.[44] In his paper[45] he departed from Carter's model by considering a single wheelset and demonstrated the stabilising effect of elastic restraint. As this paper was in Japanese it had little impact in the West. Subsequently, Matsudaira, for the first time, introduced into the mathematical model of the two-axle vehicle both longitudinal and lateral suspension flexibilities between wheelset and car body, a crucial step in understanding the stability of railway vehicles and based on this was able to suggest an improved suspension design.

In the 1950s planning started for the new Tokaido line or Shinkansen, the first purpose-built dedicated high-speed railway. Shima[46] identifies the bogies as one of the key enabling technologies of the Shinkansen as it made possible the concept of the high-speed multiple unit train in which every bogie is powered. It had been widely assumed that the powered bogies would not run as smoothly as the trailer bogies, but by studying closely the stability of bogies theoretically and experimentally it was possible to improve the riding quality of the powered bogies up to very high speeds. The analysis of these bogies by Matsudaira and his group led to the choice of suspension parameters that were subsequently validated by roller rig and track tests.

X. THE ORE COMPETITION

In the 1950s, the newly formed Office for Research and Experiments (ORE) of the International Union of Railways held a competition for the best analysis of the stability of a two-axle railway vehicle. The specification for the competition, drawn up by Committee C9 under the chairmanship of Robert Levi, emphasised worn wheel and rail profiles and nonlinear effects, for it was still widely held, in spite of Carter's work, that the explanation for instability lay in some way in the nonlinearities of the system.[47] The three prize winning papers (by de Possel, Boutefoy, and Matsudaira),[48] in fact, all gave linearised analyses. However, Matsudaira's paper was alone in incorporating both longitudinal and lateral suspension stiffness between wheelsets and frame. Surprisingly, it was awarded only the third prize.

Matsudaira's model has suspension stiffnesses but no suspension damping. It has six degrees of freedom, lateral displacement, and yaw of the wheelsets and car body so that roll of the car body is neglected. Worn wheel and rail profiles were approximated by circular arcs in order to give an approximation for the effective conicity similar to that of Heumann. At the critical state between stable and unstable motion sinusoidal oscillation is possible, and so with this assumption Matsudaira was able to reduce the order of the characteristic equation, making it possible to derive a stability chart by a graphical method (Figure 2.6). In this way Matsudaira avoided the onerous task of calculating the actual eigenvalues for each pair of parameters. In this chart the lines representing an eigenvalue with a zero real part are plotted in the plane of speed vs. lateral suspension stiffness (Figure 2.6). There are four of these lines in the chart on which purely sinusoidal oscillations are possible. Two of these, A and B, relate to relatively large excursions of the car body and two, C and D, relate to relatively large excursions of the wheelsets. Interpreting these as stability boundaries, Matsudaira proposed two approaches to achieve stability for a practical range of speeds. One was to make the lateral stiffness rather large and exploit the lower stable region; the other was to use what was interpreted as an upper stable region with the smallest possible value of the lateral stiffness. In this latter case, Matsudaira suggested that the vehicle goes through an unstable region at a very low speed but the ensuing hunting is not severe.

In fact, a later examination of the root locus shows that each eigenvalue has a positive real part above its critical speed, although the magnitude of the real part corresponding to the two lowest critical speeds decreases as the speed increases. The reason that this prescription may work for the two-axle vehicle is that inclusion of suspension damping in the lateral direction can, under certain conditions, eliminate the instability at low speeds completely as was discovered later.

Another factor, which emerged for the first time in de Possel's and Boutefoy's papers but was neglected in Matsudaira's paper, was that of the gravitational stiffness, the lateral resultant of the resolved normal forces at the contact points between wheel and rail (Figure 2.7). On its own, this effect would be strongly stabilising but, in fact, it is largely counteracted by the lateral force due to

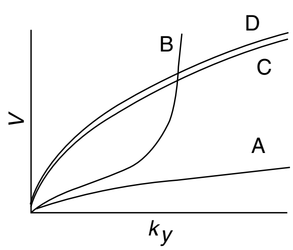


FIGURE 2.6 Stability chart in which the lines represent an eigenvalue with a zero real part. V = vehicle speed; k_y = lateral suspension stiffness; A and B correspond to mainly oscillations of the car body on the suspension and C and D correspond to wheelset oscillations.

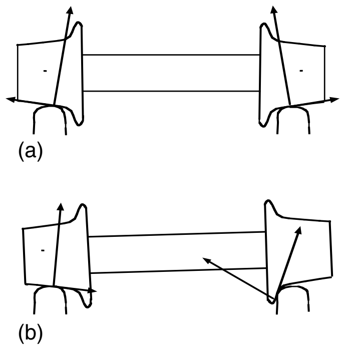


FIGURE 2.7 Normal and lateral tangential forces acting on wheelset: (a) in central position, (b) in laterally displaced position, illustrating the gravitational stiffness effect.

spin creep (discovered later, see below) for small lateral displacements. The resulting contact stiffness is therefore often ignored for motions within the flangeway clearance, although the correct representation of these forces in the case of flange contact, or in the case of freely rotating wheels is, of course, vital.

Thus, by the early 1960s, the basic ingredients of an analytical model of the lateral dynamics of a railway vehicle had been identified. Of these, few values of the creep coefficients had been measured, and in any case were not under the control of the designer; conicities could only be controlled within a narrow range by reprofiling, and the leading dimensions and number of wheelsets were largely dictated by the proposed duty of the vehicle. However, Matsudaira recognized that the designer could vary both the way in which wheelsets were connected and the corresponding stiffness properties and this pointed the way to future progress.

One of the members of Levi's ORE committee was de Pater, who considered the hunting problem and formulated it as a nonlinear problem.[49,50] Even though severe assumptions were made, interesting theoretical results emerged. In 1964, one of de Pater's students, P. van Bommel, published nonlinear calculations for a two-axle vehicle using wheel and rail profiles and a creep force-creepage law measured by Müller for the ORE C-9 committee.[51] However, lateral and longitudinal suspension flexibility was not considered.

XI. CREEP

Following Carter's analysis of creep, similar results were obtained by Poritsky,[52] and in the discussion therein Cain[53] pointed out that the region of adhesion must lie at the leading edge of the contact area. A three-dimensional case was solved approximately by Johnson,[54] who considered an elastic sphere rolling on an elastic plane. This solution was based on the assumption that the area of adhesion is circular and tangential to the area of contact (which is also circular) at the leading edge. Good agreement with experiment was obtained. The influence of spin about an axis normal to the contact area was first studied by Johnson,[55] who showed that spin could generate significant lateral force owing to the curvature of the strain field in the vicinity of the contact patch (the couple about the common normal is small and may be safely neglected). The general case where the contact area is elliptical was considered by Haines and Ollerton,[56] who confined their attention to creep in the direction of motion and assumed that Carter's two-dimensional stress distribution held in strips parallel to the direction of motion. A general theory for the elliptical contact area, based on similar assumptions to those made in Ref. 54, was developed by Vermeulen and Johnson,[57] yielding the relationship between creepage and tangential forces for arbitrary values of the semiaxes of the contact area. This, suitably modified to take account of spin, was later much used in vehicle dynamics studies. De Pater[58] initiated the complete solution of the problem by considering the case where the contact area is circular, and derived solutions for both small and large creepages, without making assumptions about the shape of the area of adhesion. However, this analysis was confined to the case where Poisson's ratio was zero; Kalker[59] gave a complete analytical treatment for the case in which Poisson's ratio is not zero. The agreement between these theoretical results and the experimental results of Johnson[54] is very good. Kalker gave a full solution of the general three-dimensional case in Ref. 60, covering arbitrary creepage and spin for the case of dry friction and ideal elastic bodies, and subsequently gave simpler approximate solution methods.[61] Kalker's theory is described in Ref. 62.

XII. THE COMPLETE SOLUTION OF THE HUNTING PROBLEM

Matsudaira's work was not well known in Europe and the results of the ORE competition were inconclusive. This was partly due to the fact that railway engineers were reluctant to accept the results of theory, perhaps justifiably so in view of the draconian assumptions that at that date were necessary to complete an analysis. In the early 1960s, British Railways, similar to Japanese National Railways, faced an increasing incidence of derailments of short wheelbase two-axle wagons as freight train speeds increased. Various modifications to wagon suspensions were made and tested but to no avail and it was obvious that there was a lack of understanding of the basic dynamics of these vehicles. As a result, a team was formed at British Rail Research Department, Derby to undertake research into railway vehicle dynamics. (Editor's note: the writer was recruited to lead this team, having been an aeroelastician in the aerospace industry.) Armed with a description of Carter's work in Rocard's textbook,[39] a ride on a test coach with hunting bogies[63] made it clear that the origin of hunting was dynamic instability. It was decided to mount a combined theoretical and experimental attack on the problem in order to understand it, find practical solutions, and to convince railway engineers of the relevance of the results.

Although some simple studies were made of a rigid bogie it was soon realised that the influence of the primary suspension, as Matsudaira had shown in his 1960 paper, was all important. Therefore, the two-axle vehicle was chosen for detailed study. Not only was this important in its own right, and relevant to the derailment problem mentioned above, but it could represent a bogie and was the simplest complete realistic vehicle. Relying on symmetry considerations, stability is governed by a set of seven differential equations, involving lateral displacement and yaw of the wheelsets and car body, and roll of the car body. Solutions of these equations were obtained numerically using digital computers, by simulation using analogue computers, and by a variety of analytical techniques. Insight into the behaviour of the system was gained by the derivation of solutions for special cases and for particular subsystems.

For example, comprehensive details of the behaviour of a simple elastically restrained wheelset were derived. Not only is this a good approximation at high speeds for certain vehicles, but as it is a system with only two degrees of freedom analytical analysis is straightforward, and the basic mechanics are revealed. As the equations of motion are not symmetric and the system is nonconservative, the wheelset is able to convert energy from the forward motion to the energy of the lateral motion.[64] Moreover, the representation and analysis of the wheelset as a feedback system was introduced.

A significant result of these studies was the scope for improvement of the stability of the two-axle vehicle by appropriate selection of the suspension parameters. Two approaches are possible, both of which make it possible to achieve quite high critical speeds. In the first of these, as exploited by Matsudaira in the Shinkansen bogies, both lateral and yaw primary stiffnesses are increased, there being an optimum at which stability is a maximum. This optimum depends in a complex way on the creep coefficients and conicity. This approach is most appropriate for bogies where the wheelbase is comparatively small.

The second approach employs a relatively flexible suspension. It depends on extending the analysis of stability by introducing a new feature, lateral suspension damping, and by reintroducing the gravitational stiffness effect which de Possel and Boutefoy had already used.[48] As a result it was shown that, with a careful choice of lateral suspension damping and the lateral and longitudinal stiffnesses so that they satisfied certain inequalities and were neither too small nor too large, it was possible to eliminate the low-speed body instability (a strongly contributory factor in wagon derailments) so that the vehicle operating speed was only limited by the wheelset instability.

So far, in the studies of stability, only the creepage due to longitudinal and lateral relative motion between wheel and rail had been considered, and the relative angular motion about the normal to the contact plane, the spin, had been neglected. When the effect of spin creep was included in the equations,[65] the stabilising influence of gravitational stiffness was found to be much reduced and had to be counteracted by an increase in yaw stiffness.

In this work, the application of analytical and both analogue and digital computer techniques marched hand-in-hand with experimental work on models and full-scale vehicles. It was fortunate that this was the heyday of corporate research, with centralised facilities and, most importantly, at British Rail Research, the ability to carry out full-scale experiments on a real railway.

On the full-scale experimental side, the earliest measurements of critical speeds and mode shapes associated with the hunting limit cycle of a range of vehicles were made by King[63] and by Pooley.[66]

The earliest model experiments were at one fifth scale on a roller rig. The model was dynamically scaled, longitudinal and lateral stiffnesses were provided by adjustable independent swing links. Satisfactory agreement was obtained between theory and experiment. The same model was tested on a model test track.

A striking validation of the theory came from a series of full-scale experiments with two kinds of standard two-axle vehicles (Gilchrist et al.).[67] These experiments were sponsored by the chief civil engineer of British Railways, but significantly, not the chief mechanical engineer. The dynamic response of the vehicles was measured on a full-scale test track that featured a series of track imperfections. In addition, as the linear critical speeds of these vehicles were low, it was possible to measure the fully developed hunting limit cycle. Careful measurement of the vehicle parameters followed by a nonlinear analogue simulation led to a successful replication of the fully developed hunting motion of these vehicles by Hobbs (published in Ref. 67). Quite apart from the highly nonlinear suspension characteristics, which were realistically modelled, two major limitations of linear theory were faced. These were creep saturation and wheel–rail geometry. Crude representations of creep saturation had long been available, but consideration was given to the representation of the graph of rolling radius difference against lateral displacement of the wheelset, which determines the yaw moment of the longitudinal creep forces in the equations of motion, as mentioned above. Hence the concept of "equivalent conicity" was introduced. For a coned wheelset the equivalent conicity is simply the cone angle of the tread. For a wheelset with worn or profiled treads the equivalent conicity is defined as that cone angle which for purely coned wheels would produce the same wavelength of kinematic oscillation and is approximately equal to the mean slope out to the amplitude in question. In other words, the equivalent conicity is a "describing function," a method of dealing with nonlinear control system components introduced by Kochenburger.[68] The circular arc theory of Matsudaira and others is accurate only for extremely small lateral displacements of the wheelset. An example of the comparative results by Gilchrist et al. are shown in Figure 2.8, and demonstrate that both the onset of instability and the fully developed hunting limit cycle were satisfactorily modelled.

The next step was to design and build the full-scale variable parameter test vehicle HSFV-1, which was designed in accordance with the flexible suspension concept described above. Experimental verification of the stability boundaries predicted by the above prescription was obtained from full-scale roller rig and track testing of the specially constructed vehicle HSFV-1.

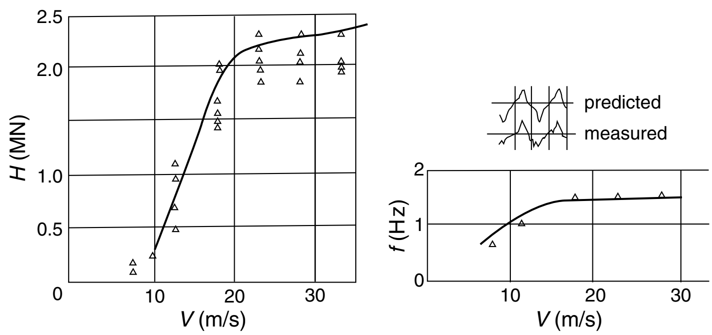


FIGURE 2.8 Measured and predicted lateral forces H and frequency f during hunting of a two-axle vehicle as a function of forward speed V. Inset shows waveforms of H. Predicted results indicated by full lines. From Ref. 67, in modern units.

Some results are shown in Figure 2.9,[69] showing the elimination of the low-speed body instability, for a suitable choice of the parameters.

The operating speed was then limited only by the high-speed wheelset instability so that the critical speed was 225 km/h. The results of Figure 2.9 show that qualitatively agreement is excellent, but it is clear that refinements to the theory to take account of the influence of the rollers on the joint wheel–rail geometry are necessary.

Although a larger roller rig was built at Derby, the use of roller rigs there was soon entirely superseded by track tests using random process methods of analysis.

Various approaches to the analysis of nonlinear hunting motions have been developed. Cooperrider et al.[70] introduced the more formal method of "quasilinearisation" in which the nonlinear functions are replaced by linear functions so chosen to minimise the mean-square error between the nonlinear and the quasilinear response. They also introduced the limit cycle or bifurcation diagram, an example of which is shown in Figure 2.10. This procedure was extended by

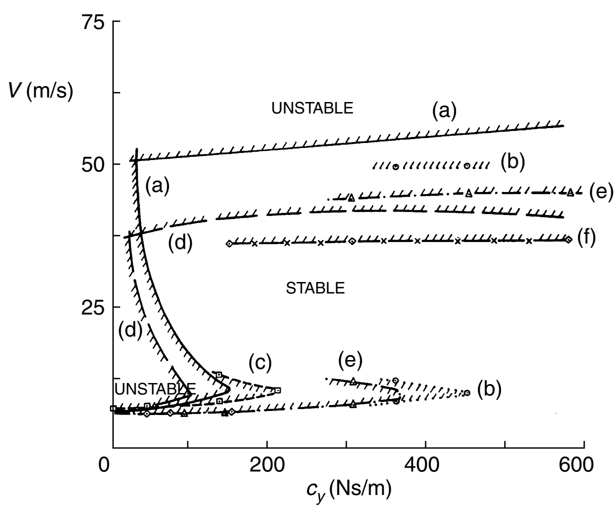


FIGURE 2.9 Comparison between predicted stability boundaries and roller rig and track test measurements as lateral suspension damping c_y is varied (from Ref. 69 in modern units). (a) Predicted (equivalent conicity $\lambda = 0.27$, yaw stiffness $k_\psi = 5.88$ MNm), (b) roller rig ($k_\psi = 5.88$ MNm), (c) track tests ($k_\psi = 5.88$ MNm), (d) predicted ($\lambda = 0.27$, $k_\psi = 3.06$ MNm), (e) roller rig ($k_\psi = 5.20$ MNm), (f) roller rig ($k_\psi = 3.06$ MNm).

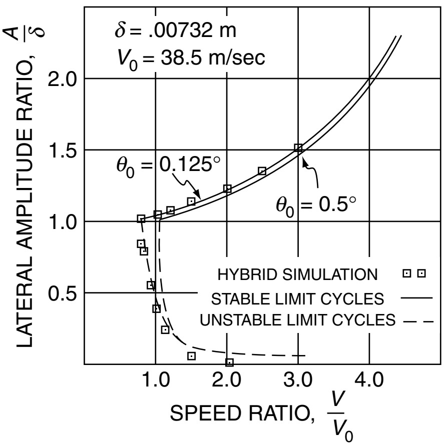


FIGURE 2.10 Limit cycle or bifurcation diagram.[70] δ = nominal flangeway clearance; A = lateral wheelset amplitude of oscillation; V_0 = nonlinear critical speed; θ_0 = breakaway yaw angle in yaw spring in series with dry friction.

Gasche, Moelle, and Knothe[71,72] who approximated the limit cycle by a Fourier series and used a Galerkin method to solve the equations. This made it possible to establish much detail about the limit cycle.

Developments in nonlinear dynamics revealed that apparently simple dynamical systems with strong nonlinearities can respond to a disturbance in complex ways. In fact, for certain ranges of parameters no periodic solution may exist. Moreover, systems with large nonlinearities may respond to a disturbance in an apparently random way. In this case, the response is deterministic but is very sensitive to the initial conditions. Such chaotic motions have been studied for railway vehicles by True.[73,74]

XIII. MODERN RESEARCH ON CURVING

In the late sixties, Boocock (at British Rail Research)[75] and Newland[76] independently considered the curving of a vehicle using the same equations of motion used in stability analyses but with terms on the right-hand side representing the input due to curvature and cant deficiency. As the wheelsets are constrained by the longitudinal and lateral springs connecting them to the rest of the vehicle, the wheelsets are not able to take up the radial attitude of perfect steering envisaged by Redtenbacher. Instead, a wheelset will balance a yaw couple applied to it by the suspension by moving further in a radial direction so as to generate equal and opposite longitudinal creep forces, and it will balance a lateral force by yawing further. For the complete vehicle, the attitude of the vehicle in the curve and the set of forces acting on it, are obtained by solving the equations of equilibrium. Newland's model made useful simplifications, but Boocock analysed several configurations including a complete bogie vehicle, a two-axle vehicle and vehicles with cross-braced bogies. The bogie vehicle had 14 degrees of freedom representing lateral displacement and yaw of the wheelsets, bogie frames, and car body. He also included the effects of gravitational stiffness and spin creep. Most important of all, Boocock was able to obtain experimental full-scale confirmation of his theory using the two-axle research vehicle HSFV-1 (Figure 2.11).

These linear theories are valid only for large radius curves. On most curves, the curving of conventional vehicles involves the same nonlinearities due to creep saturation and wheel–rail geometry that were noted in the case of hunting. The first comprehensive nonlinear treatment of practical vehicles in curves was given by Elkins and Gostling (also at British Rail Research).[77] Their treatment covers the movement of the contact patch across the wheel tread through the flange root and on to the flange and its subsequent change in shape, assuming a single point of contact, appropriate for worn or profiled wheels. At this stage the complication of two-point contact was to be the subject of much future research. As the contact moves across wheel and rail, account is taken of the increasing inclination of the normal force and the lateral creep force generated by spin. They used Kalker's results for the tangential creep forces for arbitrary values of creepage and spin and for a wide range of contact ellipticities, Kalker having issued his results numerically in a tablebook. Elkins and Gostling installed this table in their computer program so that values could be read by interpolation as needed. The resulting equations were solved by iterative numerical procedures, of which two alternatives were given. Elkins and Gostling's program required input in numerical form of the wheel and rail cross-sectional profiles, and much research was carried out on the measurement and analysis of profiles.

The accuracy of the predictions of Elkins and Gostling was demonstrated by experiments carried out on HSFV-1 and the tilting train research vehicle APT-E, an example of their results being given in Figure 2.12. Both vehicles were heavily instrumented, including load-measuring wheels,[78] allowing individual wheel forces to be measured and the limiting value of friction to be identified.

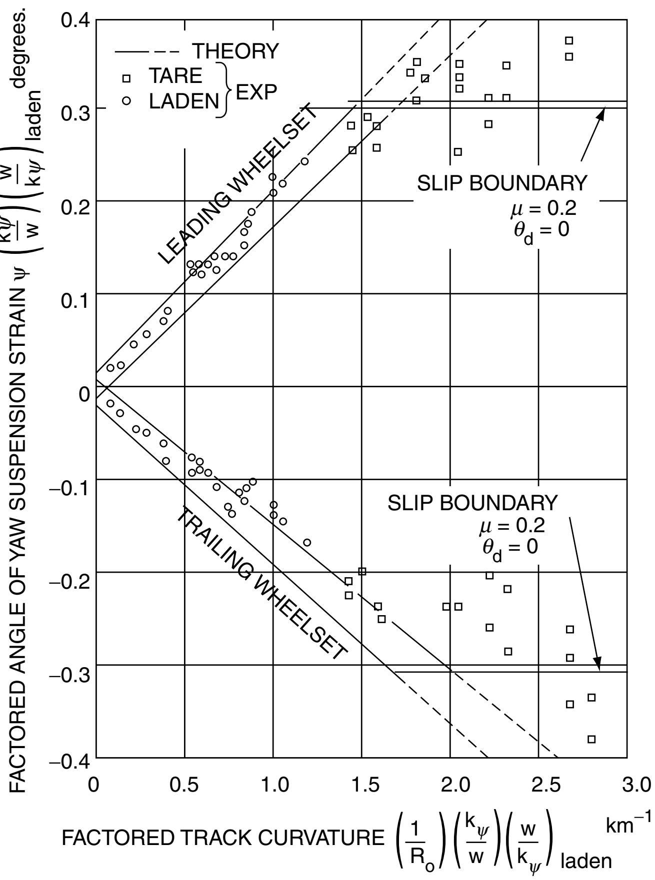


FIGURE 2.11 Comparison between theory and experiment for the steering behaviour of a two-axle vehicle. (HSFV-1,[75] results are factored to account for two loading conditions).

This comprehensive theory encompassed the two extremes envisaged in the earlier theories. Large radius curves, high conicity, and a high coefficient of friction produced agreement with the linear theory, and small radius curves and flange contacts produced agreement with Porter's results.

The importance of wheel and rail profiles was, as discussed above, recognised in the 1930s. Müller gave a detailed analysis of the wheel–rail contact geometry[79] and he tabulated geometric data,[80] which was measured for a combination of worn wheels and rails. In the early 1960s, King evaluated the contact conditions between a pair of worn wheels and worn rails,[81] between worn wheels and new rails[82] and on the basis of this work designed a new standard wheel profile for British Railways which was subsequently adopted.[83] In this work it was shown that the graph of rolling radius difference vs. wheelset lateral displacement was extremely sensitive to the gauge of the track, rail inclination, and small variations in profile geometry. As a result more refined measuring and computational techniques were developed.[84] Such methods were subsequently used very widely (see for example[85,86]).

The neglect of the effect of wheelset yaw on the wheel–rail geometry is a realistic assumption apart from in the case of flange contact at large angles of wheelset yaw. In addition to Müller's pioneering analysis, three-dimensional geometry analyses were developed in the 1970s by Cooperrider et al.[86] and Hauschild.[87] Research on the topic continues (see for example Duffek,[88] de Pater[89] and Guang[90]).

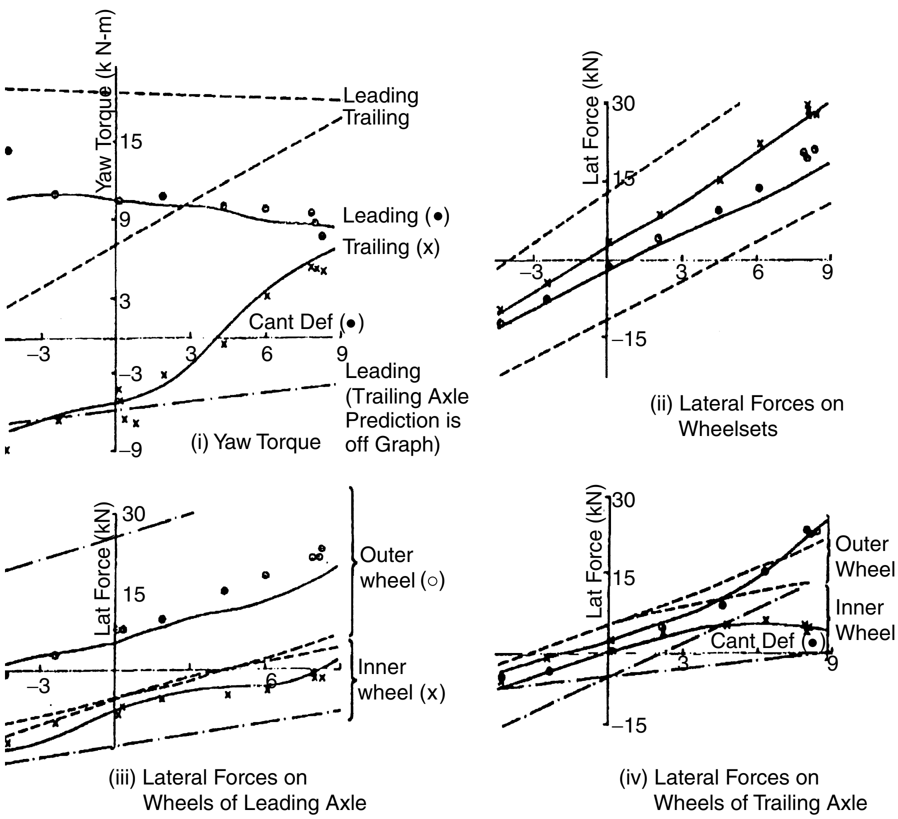


FIGURE 2.12 Curving test results for APT-E[77] for curve radius $R_0 = 650$ m and cant $\phi_0 = 150$ mm. Linear theory – – – – – –; Porter's theory —— ; Nonlinear theory ——.

Many wheel–rail combinations experience contact at two points on one wheel for certain values of the lateral wheelset displacement. This commonly occurs, for example, when contact is made between the throat of the flange and the gauge corner of the rail. If the wheels and rails are considered to be rigid, as in the case of single-point contact, discontinuities occur in the geometric characteristics such as the rolling radius difference and slope difference graphs. The mathematical aspects of two-point contact in this case have been considered by Guang.[90] However, in this case the distribution of forces between the points of contact depends on the elasticity in the contact areas and the formulation of the equations of motion becomes more complicated.[91,92]

XIV. DYNAMIC RESPONSE TO TRACK GEOMETRY

Before vehicle dynamics became established, it was engineering practice to carry out a simple static analysis and tests to measure the amount of wheel unloading on track with a defined degree of twist. In 1964, Gilchrist et al.[67] computed the dynamic response of two-axle vehicles to a dipped rail joint, and compared the results with experiment. Jenkins et al.[93] analysed the vertical response of a vehicle to a dipped rail joint in 1974 and showed that the response involved two distinct peaks. The first fast transient involves the rail mass and the contact stiffness and the second slower transient involves the unsprung mass and the track stiffness. Subsequently, quite complex models of track and vehicle have been used to establish transient stresses resulting from geometric defects in both track and wheels.

For the lateral motions of railway vehicles, the excitation terms in the equations of motion that had been derived for stability analysis first originated with Hobbs.[94] These were validated by Illingworth, using a model roller rig.[95] A comprehensive approach to the dynamic response to large discrete inputs, including both suspension and wheel–rail contact nonlinearities, was carried out by Clark et al.,[96] in 1980 and includes full-scale experimental validation. Similar calculations have been carried out on the response of vehicles to switch and crossing work.

In the case where it can be considered that irregularities are distributed continuously along the track the approach offered by stochastic process theory is appropriate, and this was first applied by Hobbs[94] to the lateral motions of a restrained wheelset. The response of complete vehicles became established as an indication of ride quality, and for passenger comfort assessments existing international standards, which define frequency weighting characteristics, were extended to lower frequencies to cover the railway case. The first measurements of power spectral density using a specially developed trolley-based measurement system were carried out by Gilchrist.[97] Subsequently, extensive measurements of the power spectra of irregularities of track have been made on railways in many countries, resulting in inputs used in design.

With the advent of research into innovative transport systems involving air cushions and magnetic levitation in the late 1960s, the problem of interaction between a flexible track and the vehicle received renewed attention. A review paper by Kortüm and Wormley[98] indicates the progress achieved by 1981 in developing appropriate computer models.

The development of the heavy-haul railway with extremely long trains, often with locomotives attached at various points along the train, introduced serious problems arising from the longitudinal response of trains to hills and to braking. The availability of the digital computer made it possible to develop dynamic models and also to support the development of train-driving simulators.

XV. SUSPENSION DESIGN CONCEPTS AND OPTIMISATION

It has been discussed above that in the period before adequate mathematical models existed, evolution of the suspension of the railway vehicle had been based on rather general ideas. Based on empirical development and simple calculations, surprisingly good results had been obtained before

1960, providing that conicities were kept low by re-turning wheel treads and speeds were moderate, for example below 160 km/h.

When linear theories of the curving of railway vehicles became available it became possible for the first time to consider the best compromise between the requirements of stability and curving on a numerate basis. This, of course, had been a continuing design consideration from the earliest days of the railways but attempts to resolve it often resulted in intuitive schemes of articulation, as discussed in Section VI. The derivation of configurations and sets of suspension parameters, which are optimal with respect to the needs of stability and curving, are a continuing task for the vehicle designer. Not only are the parameters that are associated with wheel–rail contact, both geometrical and frictional, not under the control of the designer or operator, but they are not known exactly and can vary over a wide range. It follows that practical designs must be very robust in relation to such parameters. On the other hand, there is enormous scope for the design of the suspension system in terms of the way in which the wheelsets and car bodies in a train are connected.

Boocock[75] defined the bending and shear stiffnesses that characterise the elastic properties of the connections (actual or equivalent) between the wheelsets of a two-axle vehicle. It was shown[99] that such a vehicle should possess zero bending stiffness to achieve radial steering, but would be dynamically unstable at low speeds. The design of a two-axle vehicle with a purely elastic suspension therefore requires a compromise between stability and curving. However, Hobbs[100] showed that the use of yaw relaxation dampers could provide sufficient flexibility at low frequencies in curves and sufficient elastic restraint at high frequencies to prevent wheelset instability.

It was also shown by Boocock that for conventional bogies, in which there are primary longitudinal and lateral springs connecting the wheelsets to a frame, there is a limit to the overall shear stiffness that can be provided in relation to the bending stiffness and therefore the stability/curving trade-off in which the bending stiffness must be minimised is constrained. This limitation is removed if the wheelsets are connected directly by diagonal elastic elements or cross-bracing, or interconnections which are structurally equivalent. Such an arrangement is termed a self-steering bogie. Superficially, this arrangement is similar to the systems of articulation between axles by means of rigid linkages which have been discussed in Section VI. In order to discover optimal configurations, various representations of generic two-axle vehicles or bogies have been considered,[101–106] and an example is shown in Figure 2.13.

In the 1970s the self-steering bogie was successfully developed and put into service, notably by Scheffel.[107] Self-steering bogies have been applied to locomotives (with benefits to the maximum exploitation of adhesion), passenger vehicles, and freight vehicles.[108] It should be noted that inter-wheelset connections can be provided by means other than springs and dampers. In Ref. 109 the equivalent of cross-bracing was provided by means of a passive hydrostatic circuit which has a number of potential design advantages.

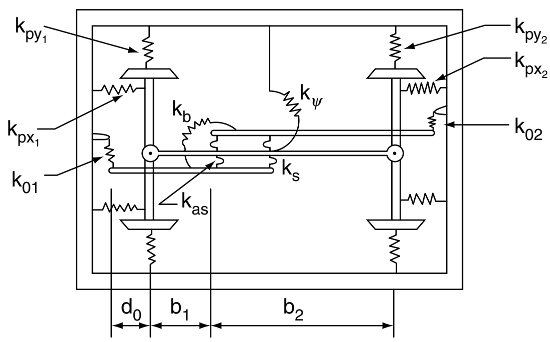


FIGURE 2.13 A generic bogie configuration.[102]

An alternative to providing self-steering by means of elastic or rigid linkages directly between wheelsets is to use a linkage system which allows the wheelsets to take up a radial position but provides stabilising elastic restraint from the vehicle body. This is so-called forced steering as it can be considered that the vehicle body imposes a radial position on the wheelsets. It was early in the 20th century that significant and successful development was carried out by Liechty.[110,111] Schwanck[112] reported on service experience with a particular design of body-steered bogie and its advantages of reduced wheel and rail wear, reduced energy consumption, and increased safety against derailment. Analytical studies of forced-steered bogie vehicles were initiated in 1981 by Bell and Hedrick[113] and Gilmore,[114] who identified various instabilities which were promoted by low conicities and reduced creep coefficients. A considerable body of work by Anderson and Smith and colleagues[115–120] covers the analysis of a vehicle with bogies having separately steered wheelsets. Weeks[121] described dynamic modelling and track testing of vehicles with steered bogies, noting the enhanced sensitivity of this type of configuration to constructional misalignments. Many examples of body steering are in current use.

It was shown[122] that for a vehicle with three or more axles it is possible, in principle, to arrange the suspension so that radial steering and dynamic stability are both achieved. The three-axle vehicle was examined in this context in a series of papers.[122–124] A similar approach, although with slightly different assumptions, has been followed by de Pater[125,126] and Keizer.[127] Although various forms of instability became evident, no radical avenue for improvement emerged from this configuration.

All the configurations discussed so far have been symmetric fore-and-aft. Unsymmetric configurations make it possible, in principle, to achieve a better compromise between curving and dynamic stability, at least in one direction of motion. However, additional forms of instability can occur.

A general theory for the stability of unsymmetric vehicles and the derivation of theorems relating the stability characteristics in forward motion with those in reverse motion has been provided.[128,129] In the case of articulated two-axle vehicles at low speeds it was shown that a suitable choice of elastic restraint in the inter-wheelset connections results in static and dynamic stability in forward and reverse motion, which will steer perfectly, without any modification dependent on the direction of motion. However, the margin of stability is small.

In the case of the three-piece freight truck, calculations taking account of the effects of unsymmetric wear have been described by Tuten et al.[130] Illingworth[131] suggested the use of unsymmetric stiffness in steering bogies. Elkins[132] showed both by calculation and experiment that a configuration of bogie, with the trailing axle having independently rotating wheels and the leading axle conventional, significantly improved stability and curving performance and reduced rolling resistance.

Suda[133,134] studied bogies with unsymmetric stiffnesses and symmetric conicity, and this development work has led to application in service. The concept has been extended to include lack of symmetry of the wheelsets by equipping the trailing axle with freely rotating wheels. This provides an example of a reconfigurable design as the wheelsets are provided with a lock which is released on the trailing wheelset (allowing free rotation of the wheels) and locked on the leading wheelset (providing a solid axle). The lock is switched depending on the direction of motion.

Independently rotating wheels have been frequently proposed as they eliminate the classic hunting problem. Some of the possibilities have been surveyed by Frederich.[135] The essential difference between a conventional wheelset and independent wheels lies in the ability of the two wheels to rotate at different speeds and thus the kinematic oscillation of a conventional wheelset is therefore eliminated. A measure of guidance is then provided by the lateral component of the gravitational stiffness (reduced by the lateral force due to spin creep) which becomes the flange force when the flangeway clearance is taken up, but this leads to slow self-centring action. Extensive experimental experience has shown that, indeed, the kinematic oscillation is absent but that one or other of the wheels tends to run in continuous flange contact.[136] Good agreement between calculation and experiment is demonstrated.[137–139]

An attempt to increase the effect of the lateral resultant gravitational force but reduce the amount of spin is to incline substantially from the horizontal the axis of rotation of the wheels, as suggested by Wiesinger (see de Pater[140]). A generic wheelset model including the effect of modest amounts of camber has been studied theoretically and experimentally by Jaschinski and Netter.[141]

Another important modern development is the use of articulation on vehicles with single-axle running gear. A current example of a train which has single-axle running gear is the Copenhagen S-Tog which embodies forced steering of the wheelsets through hydraulic actuators driven by the angle between adjacent car bodies. Extensive design calculations have been carried out on this train[142] and its lateral stability has also been discussed.[143]

These developments demonstrate that the availability of computational tools in vehicle dynamics has made it possible to depart from conventional configurations tried and tested by empirical means.

XVI. DERAILMENT

The conditions necessary to sustain equilibrium of the forces in flange contact were considered in 1908 by Nadal in a classical analysis[144] which provided a derailment criterion. Gilchrist and Brickle[145] applied Kalker's theory of creep in a reexamination of Nadal's analysis and they have shown that Nadal's formula is correct only for the most pessimistic case when the angle of attack is large and the longitudinal creep on the flange is small.

The dynamics of the derailment process was considered by Matsui[146] and Sweet et al.,[147–150] who obtained good agreement with model experiments. The mechanics of derailment remains a topic for research and presents a challenge to the modelling of vehicle dynamics.

XVII. THE DEVELOPMENT OF COMPUTER SIMULATION

Carter, as discussed above, formulated equations of motion which he was able to use to examine stability of rigid-framed locomotives. The analysis of the restrained wheelset, perhaps the most fruitful model with which to understand stability, could be carried out analytically. Once the importance of the suspension stiffnesses was recognised, the solution of the equations of motion of a system with many degrees of freedom presented a problem that, if attempted, was time-consuming and error-prone even using the calculating machines of the early 1950s. As already mentioned, Matsudaira resorted to a graphical method to establish stability boundaries for a two-axle vehicle. The advent of the digital computer provided the means to calculate the eigenvalues for complete vehicles, while the general purpose electronic analogue computer could be used to compute stability boundaries. Among the first to use these computers on a large scale was the aircraft industry, and it is interesting that in 1962 the first eigenvalue analysis of a complete two-axle vehicle carried out by British Rail Research used a flutter routine and computer at English Electric Aviation. (The linear equations of motion of a railway vehicle and the aeroelastic equations of an aircraft wing are formally the same, provided that vehicle speed is interpreted appropriately.) The work by Gilchrist et al.,[67] in which the simulation of two-axle vehicles involved severe nonlinearities, was carried out using an analogue computer.

With the application of the computer on a large scale in all branches of engineering,[151] interest in numerical methods quickened and efficient methods of computation became available in the form of libraries of standard routines for the eigenvalue problem and the step-by-step solution of differential equations. Thus, in the 1960s and 1970s, simulations of complex nonlinear railway vehicle models with many degrees of freedom were developed that exploited the increase in computer power becoming available. These simulations were based on equations of motion derived manually which were then incorporated into computer programs to solve specific problems such as stability, response to track irregularities, and curving.

The derivation of equations of motion, their reduction to first-order differential equations suitable for numerical integration or eigenvalue analysis, and the calculation of the parameters in the equations were lengthy and error prone tasks. Moreover, the application of railway vehicle dynamics was moving from the research laboratory to the industrial design office. Many companies set up groups specialising in bogie and suspension design dependent on dynamics calculations. This motivated the development of complete packages, which covered a range of dynamics calculations, using the same consistent model of the vehicle. In these programs, the input data consist of basic dimensions, masses, and the type of interconnections, such as massless force elements representing springs and dampers that connect the bodies that make up the vehicle. The variety and scope of programs available was reviewed in 1993.[152] Two different approaches to simulation emerged.

In the first approach, the equations embodied in the package are restricted to more-or-less standard configurations, such as symmetric bogie vehicles and to specific situations, such as stability or curving. Approximations are made in the formulation of the equations which are consistent with the expected behaviour of these configurations. The subsequent limits of applicability are typically validated by full-scale experiment for the specific situations for which the package is applicable. Such software is extensively used in industry where the emphasis is on design.

The second approach to simulation involves the use of general vehicle models in general situations, so-called multibody programs. The theoretical basis of the multibody approach had its origin in work carried out on satellite dynamics in the 1960s when satellites became more complex and could no longer be considered as single rigid bodies. The degree of generality varies: for example, some packages cater for large rotational angles. Software has been developed that allows the automatic formulation of the complex equations of motion by the computer. A further development is that the formulation of the equations of motion may be carried symbolically instead of numerically.

Another variation in methods lies in the treatment of constraints associated with the motion of the wheelset rolling along the track. Some packages work with a set of differential equations (involving all the states, i.e., displacements and velocities, of the wheelset) and algebraic equations expressing the constraints, and the constraints are not used to reduce the order of the system. Alternatively, many packages work with generalised coordinates and a minimal set of differential equations. With the choice of a moving axis system orientated with reference to the track, the wheelset reference axes are not fixed in the wheelset and are themselves moving with respect to inertial space. The formulation of the equations of motion therefore requires special care and their logical derivation has been considered by de Pater,[153] Guang,[90] and Schiehlen.[154]

Packages also vary in the extent to which they interface to finite-element structural packages, necessary for studies involving structural flexibility, and to control system analysis software, necessary for studies involving active suspensions.

As computer power has increased and vehicle dynamics computations have been used more widely for engineering design purposes it has become necessary to refine the modelling of the suspension components themselves.[155–157] For example, components such as air springs and dampers are complex and cannot be represented adequately by simple springs and viscous dampers. As an example of the sophistication of modelling now considered necessary, the modelling of an articulated vehicle consisting of three car bodies and four single-axle bogie frames and wheelsets[158] involves 167 states to cover these bodies and the suspension details. The maturity of the subject is indicated by the successful results of the Manchester benchmarks for rail simulation[159] in which the performance of five packages was compared.

XVIII. ACTIVE SUSPENSIONS

As defined by Goodall[162] the concept of an active suspension is to add sensors, a controller, and actuators to an existing mechanical system, and usually involves feedback action so that the dynamics of the system is modified. The concept of feedback is an ancient one although Watts governor was the first widespread application. In railways there are three principal areas of application of active suspensions — car body tilting systems, secondary, and primary suspensions. Attention in the following will be confined to the earliest developments and their relationship with vehicle dynamics, as modern development is thoroughly reported in a number of reviews.[162–164]

As mentioned above, superelevation of the track in curves was used at an early date, and tilting the car body to achieve the same effect was demonstrated in monorail systems, such as that at Wuppertal with cars suspended from an overhead rail, and gyroscopically stabilised cars, such as that of Brennan (1906). In the latter case the developments in dynamics exemplified by Routh's textbook had promoted intense scientific and popular interest in gyroscopes in the 1890s. Various demonstrations were made of monorail systems which exploited gyroscopic stabilisation, that of Brennan being perhaps the most successful within the limits of contemporary technology.

Bogies with provision for pendular suspension of the car body were put into service in limited numbers in the United States.[160] Experiments were carried out on a vehicle with full passive tilting in France,[161] and in the late 1960s the United Aircraft turbotrain had full passive tilting. The limited dynamic performance of passive tilt systems, together with advances in control system analysis techniques and the availability of sensors and actuators from other fields of technology, stimulated the development of active tilt starting in 1967. The evolution of the various configurations of tilt systems has been described by Goodall.[162] After a long period of development this technology became established in commercial operation, representing the first service application of active suspensions, the Pendolino having started operation in Italy in 1988. This has been followed by operation of tilting trains, developed by several manufacturers, in a number of countries.[162]

Also starting in the late 1960s, research was carried out on the applications of active systems to vertical and lateral vibration isolation and stability augmentation involving the secondary suspension, initially largely motivated by development activity on air cushion vehicles. The basic theoretical considerations have been outlined by Hedrick.[163] For conventional rail vehicles, a variety of active systems were the subject of full-scale experimentation,[164] but even though significant benefits in terms of ride quality were demonstrated, the additional cost of the equipment has deterred commercial application, so that there are only a few examples of service operation.[162]

The suspension and guidance of vehicles by magnetic suspension provides another example of an active suspension. Magnetic suspension relies on active control for stabilisation. Albertson, Bachelet, and Graeminger all proposed schemes for magnetic levitation in the 1900s, and in 1938 Kemper[165] demonstrated a model showing the feasibility of a wheelless train. Subsequently, with the advent of high power solid-state electronic devices in the mid-1960s, and the application of various forms of linear motors, research and development on the topic has flourished. There has been considerable cross-fertilisation between the dynamics of rail and magnetically levitated vehicles as a result, both in terms of technique and personnel.

Turning to primary suspensions, feedback control system methods were used in the stability analysis of the wheelset in 1962.[166] This approach revealed some of the deficiencies of the wheelset as a guidance element. Bennington[167] used control system techniques to propose an active torque connection between the two wheels of a wheelset, and subsequently, a wheelset with an active torque connection using a magnetic coupling was developed.[168] Various control laws were used with the object of providing a good torque connection between the wheels at low frequencies so that curving ability is maintained but at high frequencies the wheels are more or less uncoupled so that instability does not arise. Pascal and Petit[169] carried out experiments in which active steering, using freely rotating wheels, was achieved by the yaw moment generated by electromagnets which react against a guide rail. Currently, many possibilities are being considered.[162] It can be seen that, increasingly, control engineering techniques will exercise a strong influence on the dynamics of railway vehicles (see for example[170]), either by improving the dynamics of vehicles using the conventional wheelset, or by supporting the development of more innovative systems.

XIX. THE EXPANDING DOMAIN OF RAIL VEHICLE DYNAMICS

The proceedings of recent IAVSD symposia reveal the growing range of dynamics studies undertaken in the railway field. This chapter has reviewed the development of ideas about the basic problems associated with stability, response to track geometry, and behaviour in curves of the railway vehicle. A few final examples will indicate how rail vehicle dynamics is growing in scope, both in the range of frequencies considered and the detail of the models used, to match the challenge of higher speeds and the application of new technology.

The extrapolation of conventional railway technology to higher speeds has led in many cases to increased traction forces, increased wheelset mass, and greater track stiffness. New problems of interaction between vehicle and track have emerged, such as irregular ballast settlement and deterioration, increased levels of rail corrugation, and out-of-round wheels. The solution of these problems requires the consideration of structural dynamics of both vehicle and track in the frequency range of about 40 to 400 Hz together with the analysis of the long-term behaviour of wheel and track components.[171]

The analysis of mechanisms of noise generation requires the consideration of an even higher range of frequencies of structural oscillations of wheelsets and track, up to 5 kHz or more. At these frequencies a nonsteady-state analysis of the contact forces is needed. Moreover, for problems such as corrugation and squealing in curves it is necessary to account for the contact forces at large values of the creepages.[172]

Both tractive and guidance forces are provided by the same wheel–rail interface. It has gradually been recognised that the interaction between traction and guidance due to the contact forces at the wheel–rail interface may severely affect overall performance. Although traction and guidance have usually been considered separately, a systems approach is needed for design in which the control of the drive system is combined with the needs of guidance.[173]

At high speeds, particularly in tunnels, aerodynamic forces are significant not only as a generator of drag but in affecting the lateral response of vehicles. With the further increases in speed, and reductions in mass of car bodies, lateral oscillations have been experienced that have been created by pressure fluctuations caused by unsteady flow separations from the car body surface. This requires simulation in which the vehicle dynamics is combined with the aerodynamics of the flow field with moving boundaries and which is dependent on the car body motion.[174]

Increasing use of electronics and active controls makes it necessary to adopt a mechatronic approach in which the mechanical parts and the electronics are seen as integral parts of the system to be analysed and designed concurrently. Industrial applications demand integration of software tools with design and manufacturing systems.[175]

Although there is much to be understood about the behaviour of apparently simple systems with strong nonlinearities,[176] the history of railway vehicle dynamics suggests that, in most cases, the subject rests on a sound conceptual basis with satisfactory full-scale experimental validation.

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